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Dynamic analysis of the influence of fiber orientation in composite laminated plates

Adriana Diacenco*, Ariosto Britain Jorge and Patricia da Silva

Mechanical Engineering Institute, Federal University of Itajubá - UNIFEI, Campus José Rodrigues Seabra, CP 50, CEP 37500-902, Itajubá, MG, Brazil.

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This paper evaluates numerically the dynamic behavior of structural composite laminate materials in relation to the angular change in fiber layers of the laminated composite. The behavior of the material is modeled through finite element method, where the First Order Shear Deformation Theory (FSDT) is used which is implemented on a rectangular element serendipity containing eight nodes. The mathematical modeling has been implemented using the commercial available software MATLAB®. Through numerical simulations, it will be possible to obtain the natural frequencies. And we will present a sensitivity analysis with respect to the fiber orientation parameter.

Key words: Dynamic behavior, composite laminated plates, First Order Shear Deformation Theory (FSDT), fiber orientation parameter.

INTRODUCTION

Composite materials have been increasingly highlighted in recent decades due to its advantages to the traditional engineering materials (steel or aluminum), characterized by its low density associated with high strength/stiffness relation characteristics, and its anti-corrosion properties. A composite material can be defined when two or more different materials are combined together to create a superior and unique material, to obtain a set of properties that none of the components individual features (Mendonça, 2005). There are different classifications for composite materials available in the literature. They can be classified according to morphology of the dispersed phase in particle reinforced composites, fiber reinforced composites and composites structural (Reddy, 1997).

The composite structural laminate consists of a layer stack attached to each other with the fibers oriented in different directions. And laminate typically consists of several layers, often identical, varying its guidelines to better meet the design requirements and manufacturing (Diacenco, 2010).

Composite materials are increasingly present in different areas of Engineering where it is necessary to study and analyze the behavior of these materials because, it can be stated that the engineering structures are subject to disturbances that affect static or dynamic response characteristics and performance of the structural system and control actions or vibration monitoring natural frequencies become necessary.

In the case of structural composites becomes relevant perform the analysis of stacking sequence of layers of the

*Corresponding author. E-mail:aadiacenco@yahoo.com.br.

composite structure in the context of attenuation of vibration levels, as high vibrations are to be avoided within the structural integrity, and this can be analyzed to obtain the functions of frequency response of the composite.

The use of numerical methods to assess the structural stability have been widely studied and applied by Reddy (1997), Faria et al. (2006), Lima et al. (2009) and Diacenco et al. (2013). Among these methods, the finite element method has been shown to be the most suitable, due to its characteristics of modeling flexibility and relative ease of implementation numerical computational complex problems in engineering and in the literature can be found a wide variety of theories used in the formulation of elements applied to finite composite materials. For the purposes of this paper, the well-known First-Order Shear Deformation Theory (FSDT), proposed by Reddy (1997), such theory presents significant results when working with moderately thick plate where is implemented in a rectangular Serendipity element containing eight nodes and five degrees of freedom per node. Based on what was stated above, the main objective of this article is the implementation numerical computation using the finite element method for composite plates and evaluates the influence of the stacking sequence of the layers composite in their dynamic behavior in terms of characterization of natural frequencies and vibration amplitudes.

FINITE ELEMENT MODELING

The mechanical behavior of the composite structure can be modeled by using the FSDT, in which the displacements at an arbitrary point in such a composite is expressed as follows:

$$\boldsymbol{U}(x,y,z,t)=\boldsymbol{A}(z)\boldsymbol{u}(x,y,t) \tag{1}$$

In Equation (1):

$$\boldsymbol{U}(x,y,z,t)=[u(x,y,z,t)\ v(x,y,z,t)\ w(x,y,z,t)]^T \tag{2}$$

$$A(z)=\begin{bmatrix}1&0&0&z&0\\0&1&0&0&z\\0&0&1&0&0\end{bmatrix} \tag{3}$$

$$\boldsymbol{u}(x,y,t)=\left[u_0(x,y,t)\ v_0(x,y,t)\ w_0(x,y,t)\ \psi_x(x,y,t)\ \psi_y(x,y,t)\right]^T \tag{4}$$

where $u(x,y,z,t)$, $v(x,y,z,t)$, e $w(x,y,z,t)$ denote the displacements in directions *x*, *y* and *z*. respectively. (u_0,v_0,w_0) and (ψ_x,ψ_y) are, respectively, the mid-plane displacements and the cross-section rotations in x and y directions. The usual strain–displacement relations are used and the resulting strains are separated in bending and transverse shear strains, $\boldsymbol{\varepsilon}_b$ and $\boldsymbol{\varepsilon}_s$, respectively, as follows:

$$\boldsymbol{\varepsilon}_b(x,y,z,t)=\left[\boldsymbol{D}_0\ +\ z\boldsymbol{D}_1\right]\ \boldsymbol{u}(x,y,t)=\boldsymbol{D}_b(z)\boldsymbol{u}(x,y,t) \tag{5}$$

$$\boldsymbol{\varepsilon}_s(x,y,z,t)=\left[\boldsymbol{D}_2\right]\ \boldsymbol{u}(x,y,t)=\boldsymbol{D}_s\boldsymbol{u}(x,y,t) \tag{6}$$

Where $\boldsymbol{\varepsilon}_b(x,y,z,t)=\left[\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{xy}\right]^T$ and $\boldsymbol{\varepsilon}_s(x,y,z,t)=\left[\gamma_{yz}\ \gamma_{zx}\right]^T$. $\varepsilon_{xx}=\partial u/\partial x$, $\varepsilon_{yy}=\partial v/\partial y$, $\varepsilon_{zz}=\partial w/\partial z$, $\gamma_{xy}=(\partial u/\partial y+\partial v/\partial x)$, $\gamma_{yz}=(\partial v/\partial z+\partial w/\partial y)$ and $\gamma_{zx}=(\partial u/\partial z+\partial w/\partial x)$.Matrices $\boldsymbol{D}_i\ (i=0,...,3)$ are formed by differential operators appearing in the strain–displacement relations.

Discretization of the displacement variables is made by using appropriate interpolation functions. Hence, for the eight nodes rectangular plate element, the five mechanical variables included in vector *u(x, y, t)* are interpolated from their corresponding 40 nodal values through the following relation:

$$\boldsymbol{u}(\xi,\eta,t)=\boldsymbol{N}(\xi,\eta)\boldsymbol{u}(t) \tag{7}$$

Where:

$\boldsymbol{u}(t)=\left[\boldsymbol{u}_1^T(t)\,\boldsymbol{u}_2^T(t)\ldots\boldsymbol{u}_8^T(t)\right]^T$ and

$\boldsymbol{u}_i(t)=\left[u_i\ v_i\ w_i\ \psi_{xi}\ \psi_{yi}\right]^T\ (i=1\ to\ 8)$.

$\boldsymbol{N}(\xi,\eta)$ of dimensions *5 x 40*, is the matrix formed by the standard serendipity eight-node shape interpolation functions formulated in local coordinates $(\xi,\eta), -1\le\xi\le 1$. And this was illustrated in Figure 1. By associating Equations (1) to (4), the displacement and strain fields are found to be expressed in terms of the nodal values as follows:

$$\boldsymbol{U}(x,y,z,t)=\boldsymbol{A}(z)\boldsymbol{N}(\xi,\eta)\boldsymbol{u}(t) \tag{8}$$

$$\boldsymbol{\varepsilon}_b(x,y,z,t)=\boldsymbol{D}_b(z)\boldsymbol{N}(\xi,\eta)\boldsymbol{u}(t)=\boldsymbol{B}_b(\xi,\eta,z)\boldsymbol{u}(t) \tag{9}$$

$$\boldsymbol{\varepsilon}_s(x,y,z,t)=\boldsymbol{D}_s\boldsymbol{N}(\xi,\eta)\boldsymbol{u}(t)=\boldsymbol{B}_s(\xi,\eta)\boldsymbol{u}(t) \tag{10}$$

Based on the stress-strain relations, the strain and kinetic energies of the composite plate element can be formulated

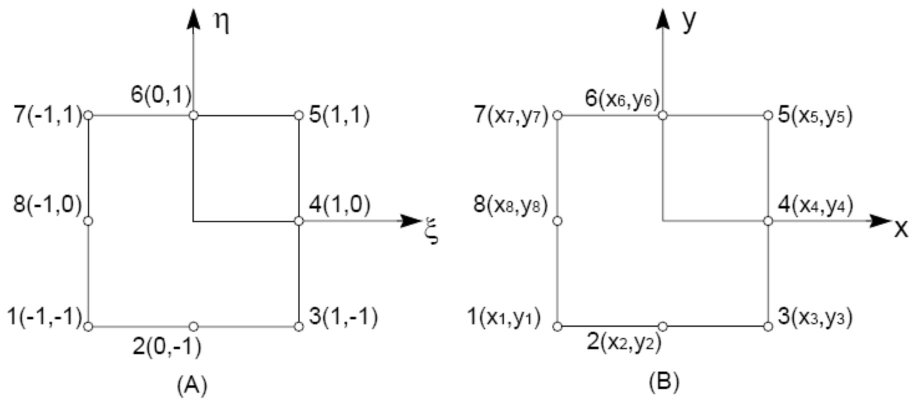

Figure 1. Serendipity family element used in the finite element formulation of laminated composite plates: (a) local coordinates, (b) coordinates.

in terms of the natural variables of strain field and the mechanical material properties. After, Lagrange's equations are used, considering the nodal displacements and rotations as generalized coordinates, to obtain the following elementary mass and stiffnesses matrices, respectively:

$$\boldsymbol{M}^{(e)} = \sum_{k=1}^{n} \int_{z=z_k}^{z_{k+1}} \int_{\xi=-1}^{\xi=+1} \int_{\eta=-1}^{\eta=+1} \rho_k \boldsymbol{N}^T(\xi,\eta)\boldsymbol{A}^T(z)\boldsymbol{A}(z)\boldsymbol{N}(\xi,\eta)\det(\boldsymbol{J})\, d\eta\, d\xi dz \quad (11)$$

$$\boldsymbol{K}_b^{(e)} = \sum_{k=1}^{n} \int_{z=z_k}^{z_{k+1}} \int_{\xi=-1}^{\xi=+1} \int_{\eta=-1}^{\eta=+1} \boldsymbol{B}_b^T(\xi,\eta,z)\boldsymbol{C}_b^{(k)}(\theta_k)\boldsymbol{B}_b(\xi,\eta,z)\det(\boldsymbol{J})d\eta\, d\xi\, dz \quad (12)$$

$$\boldsymbol{K}_s^{(e)} = \sum_{k=1}^{n} \int_{z=z_k}^{z_{k+1}} \int_{\xi=-1}^{\xi=+1} \int_{\eta=-1}^{\eta=-1} \boldsymbol{B}_s^T(\xi,\eta)\boldsymbol{C}_s^{(k)}(\theta_k)\boldsymbol{B}_s(\xi,\eta)\det(\boldsymbol{J})d\eta\, d\xi \quad (13)$$

In Equations (11) to (13) $\det(\boldsymbol{J})$ indicates the determinant of the Jacobian of the transformation from the in-plane physical variables (x,y) to the natural variables (ξ,η), and matrices $\boldsymbol{C}_b^{(k)}(\theta_k)$ and $\boldsymbol{C}_s^{(k)}(\theta_k)$ represent, respectively, the orthotropic bending and shear elastic matrices of the kth layer, which are constructed according to the Classical Laminate Theory (CLT) as follows:

$$\boldsymbol{C}_b^{(k)}(\theta_k) = \boldsymbol{T}_b(\theta_k)\bar{\boldsymbol{C}}_b^{(k)}\boldsymbol{T}_b^T(\theta_k) \quad (14)$$

$$\boldsymbol{C}_s^{(k)}(\theta_k) = \boldsymbol{T}_s(\theta_k)\bar{\boldsymbol{C}}_s^{(k)}\boldsymbol{T}_s^T(\theta_k) \quad (15)$$

Where $\bar{\boldsymbol{C}}_b^{(k)}$ and $\bar{\boldsymbol{C}}_s^{(k)}$ are, respectively, the bending and shear elastic property matrices of the k^{th} layer, referred to its principal orthotropic axis, and $\boldsymbol{T}_b(\theta_k)$ and $\boldsymbol{T}_s(\theta_k)$ are the associated rotation matrices. From the elementary matrices computed for each element of the finite element mesh, the global equations of motion are constructed, accounting for the node connectivity, using standard finite element assembling procedures, Huebner et al. (1982). After assembling, the global equations of motion in the time domain can be written as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{f}(t) \quad (16)$$

Where $\boldsymbol{M} = \bigcup_{e=1}^{nelem} \boldsymbol{M}^{(e)}$ and $\boldsymbol{K} = \bigcup_{e=1}^{nelem} \boldsymbol{K}^{(e)}$ are the global FE mass and stiffness matrices. Symbol $\bigcup$ indicates matrix assembling and $\boldsymbol{q}(t)$ is the vector of global d.o.f's. $\boldsymbol{f}(t)$ is the vector of generalized external loads.

The equations of motion in the time domain (16) can be used to perform various dynamic analyzes such as the calculation of response time, eigenvalues and eigen vectors, and frequency responses. In the frequency domain the above equation in (16) takes the form:

$$\left[\boldsymbol{K}(\omega,T)-\omega^2\boldsymbol{M}\right]\boldsymbol{Q}(\omega)=\boldsymbol{F}(\omega) \tag{18}$$

$$\boldsymbol{F}(\omega)=\boldsymbol{bU}(\omega),\boldsymbol{Y}(\omega)=\boldsymbol{cQ}(\omega) \tag{19}$$

which $\boldsymbol{M},\boldsymbol{K}(\omega,T)\in R^{NxN}$ respectively represent the mass matrix (symmetric, positive-definite) and the stiffness matrix (symmetric and nonnegative definite.) $\boldsymbol{Q}(\omega)\in R^N$ and $\boldsymbol{F}(\omega)\in R^N$ represent, respectively, the displacement vector and the vector of external forces. $\boldsymbol{Y}(\omega)\in R^c$ is the vector of responses, and the vector $\boldsymbol{U}(\omega)\in R^f$ is reduced from external forces. And the matrices $b\in R^{Nxf}$ and $c\in R^{cxN}$ are the matrices that allow to choose among degree-of-freedom of the finite element model the degrees of freedom where the forces are applied excitation, and the degrees of freedom which are calculated system response, respectively. Expression (17) yields the following expression for the complex dynamic stiffness matrix:

$$\boldsymbol{Z}(\omega,T)=\boldsymbol{K_e}+G(\omega,T)\overline{\boldsymbol{K}}_{\boldsymbol{v}}-\omega^2\boldsymbol{M} \tag{20}$$

Defined the complex stiffness, the next step is to solve the system in the frequential domain, which can be done by building dynamic flexibility matrix or array of Frequency Response Functions (FRFs):

$$\boldsymbol{H}(\omega,T)=\boldsymbol{cZ}(\omega,T)^{\boldsymbol{-1}}\boldsymbol{b} \tag{21}$$

SENSITIVITY ANALYSIS FOR DYNAMIC RESPONSES OF FINITE DIFFERENCES

The global finite element matrices appearing in finite element modeling establish the dependence of the response of the system with respect to a set of design parameters. Such functional dependence can be expressed as follows: (Lima et al., 2009):

$$\boldsymbol{r}=\boldsymbol{r}[\boldsymbol{M}(\boldsymbol{p}),\boldsymbol{K}(\boldsymbol{p})] \tag{22}$$

Where ***r*** and ***p*** designate vectors of structural responses and design parameters, respectively.

The sensitivity of the responses with respect to a given parameter p_i, evaluated for a given set of values of the design parameter $\boldsymbol{p}^0$ can be estimated by finite differences and then defined as the following partial derivative:

$$\left.\frac{\partial\boldsymbol{r}}{\partial\,\mathrm{p_i}}\right|_{\boldsymbol{p}^0}=\lim_{\Delta\mathrm{p_i}\to 0}\left\{\frac{\boldsymbol{r}\left[\boldsymbol{M}(\mathrm{p_i^0}+\Delta\mathrm{p_i}),\boldsymbol{K}(\mathrm{p_i^0}+\Delta\mathrm{p_i})\right]}{\Delta\mathrm{p_i}}-\frac{\boldsymbol{r}\left[\boldsymbol{M}(\mathrm{p_i^0}),\boldsymbol{K}(\mathrm{p_i^0})\right]}{\Delta\mathrm{p_i}}\right\} \tag{23}$$

Where Δp_i is an arbitrary variation tending to zero, applied to the current parameter value p_i^0 while all other parameters are kept unchanged. The sensitivity of the response with respect to p_i can be estimated numerically by finite differences by calculating successive responses corresponding to $p_i=p_i^0$ e $p_i=p_i^0+\Delta p_i$, as follow:

$$\left.\frac{\partial\boldsymbol{r}}{\partial\,\mathrm{p_i}}\right|_{\boldsymbol{p}^0}\approx\left\{\frac{\boldsymbol{r}\left[\boldsymbol{M}(\mathrm{p_i^0}+\Delta\mathrm{p_i}),\boldsymbol{K}(\mathrm{p_i^0}+\Delta\mathrm{p_i})\right]}{\Delta\mathrm{p_i}}-\frac{\boldsymbol{r}\left[\boldsymbol{M}(\mathrm{p_i^0}),\boldsymbol{K}(\mathrm{p_i^0})\right]}{\Delta\mathrm{p_i}}\right\} \tag{25}$$

This procedure is efficient when it comes to small structures and their use qualitatively describes the degree of influence of different design parameters on the dynamic response: the larger the amplitude of the sensitivity functions with respect to a given design parameter, the greater the influence of this parameter on the dynamic responses.

NUMERICAL SIMULATIONS

Here, we present two different numerical applications were implemented in the programming environment Matlab ®. The first attempt to evaluate the FSDT accuracy of the theory in obtaining the natural frequencies of composite structures. The second and third numerical applications show the effect of orientation and stacking sequence of layers in the loss factors and natural frequencies vibration of laminated composite structures.

In the two numerical applications it is considered a flat plate composite laminate, using the FE model of a simple supported square L_x = L_y = 0.16 m, composite plate as shown in Figure 2 illustrate the model composed by a total number of 64 finite elements and 225 nodes. The following simply supported boundary conditions are applied on the square composite plate (Correia et al., 2000): $u_0 = w_0 = \psi_z = \varsigma_x = \varsigma_z = 0$ in y = 0 and y = Ly, e $u_0 = w_0 = \psi_z = \varsigma_y = \varsigma_z = 0$ in x = 0 and x = Lx. The composite plate consists of 5 layers of the same thickness *h/5* (*h=a/128m*).The real values of the material properties characteristics of each layer are $\overline{E}_1=172{,}4GPa$, $\overline{E}_2=\overline{E}_3=6{,}89GPa$, $\overline{G}_{12}=\overline{G}_{13}=3{,}45GPa$, $\overline{G}_{23}=1{,}38GPa$, $\nu_{12}=\nu_{13}=0{,}25$, $\nu_{23}=0{,}30$, $\rho=1566\,kg/m^3$.

First application

In this first application will be considered five layers laminated oriented (0 ° / 90 ° / 0 °/ 90 ° / 0 °) and (90 ° / 0

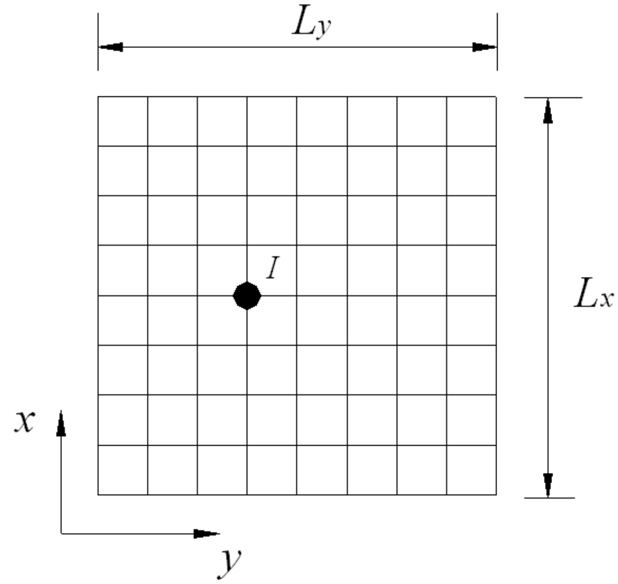

Figure 2. Mash of finite elements.

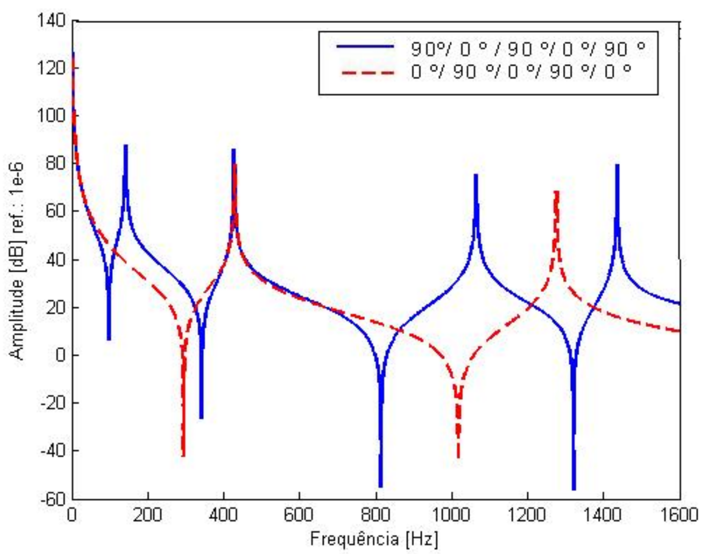

Figure 3. Vibration amplitudes for different stacking sequences.

Table 1. Natural frequencies for the first three modes of vibration to (0°/ 90°/0°/90°/0°).

Mode	Natural frequency (Hz)
1	293.09
2	429.49
3	1273.7

Table 2. Natural frequencies for the first three vibration modes of the (90°/0°/90°/0°/90°).

Mode	Natural frequency (Hz)
1	97.696
2	141.94
3	341.01

° / 90 ° / 0 ° / 90 °) with the same thickness. Figure 3 shows vibration amplitudes of the laminated structure. The results shown in Tables 1 and 2 clearly show the influence of orientation of the fibers on the dynamic behavior, since the symmetry of the variation layers alters significantly the frequencies of vibration.

Second application

The discretized by finite elements, the geometrical

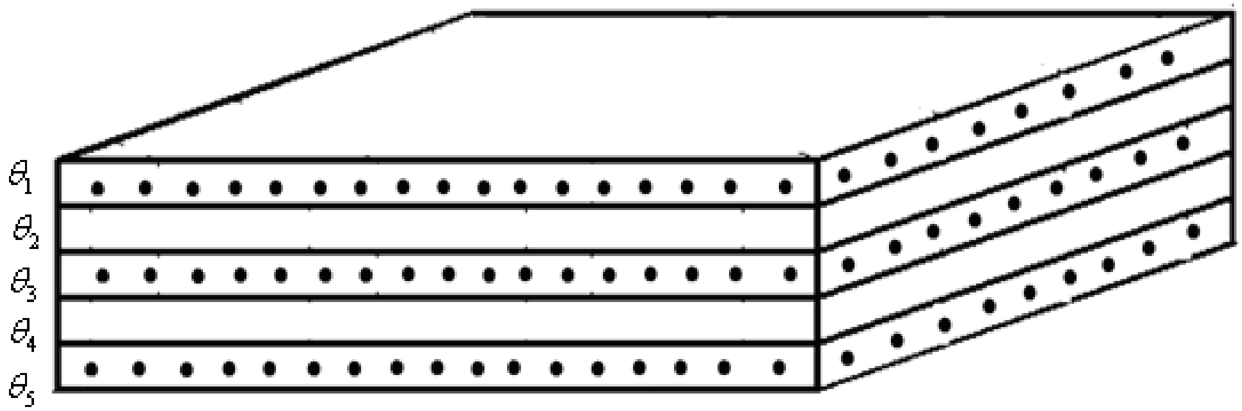

Figure 4. Composite laminate illustrating the design parameters.

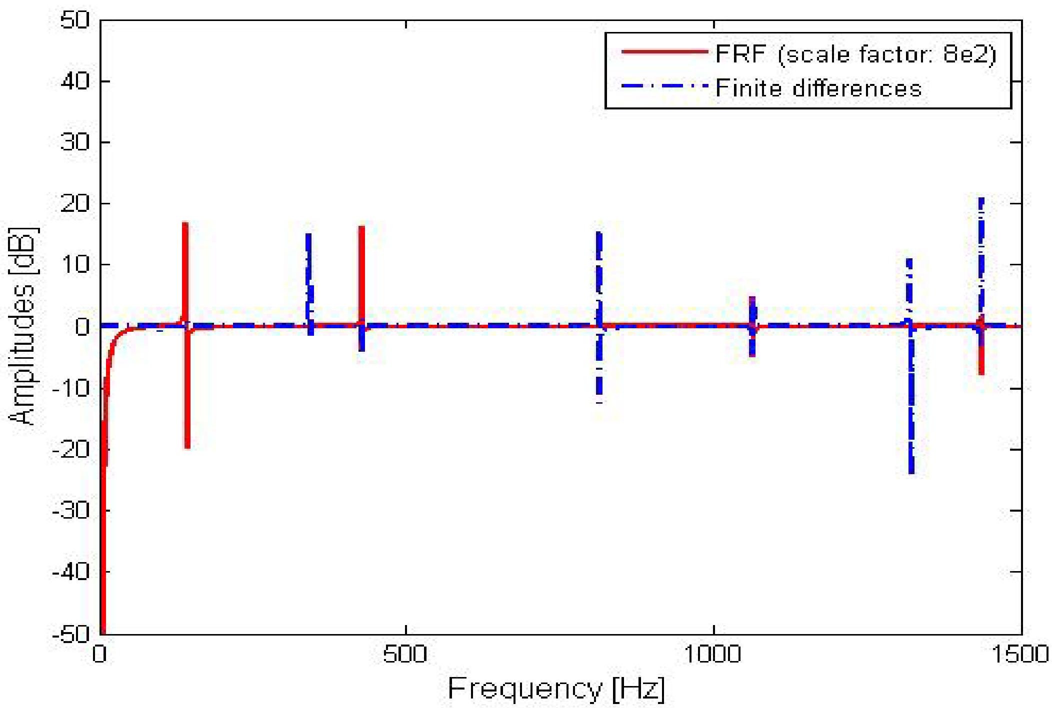

Figure 5. Sensitivity of the FRF obtained with respect to θ_1 for $\Delta\theta_1/\theta_1 = 1\%$.

characteristics and conditions of laminated composite contour of the plate are the same as those in the example section earlier. It present the calculation of sensitivities of the FRFs of the composite plate laminated according to Equation (25) using successive variations of fiber orientation, corresponding to 1% of the nominal configuration (90°/0°/90°/0°/90°). Figure 4 illustrates the composite laminate. Figures 5 to 7 show the response functions frequency calculated by finite differences. The larger the amplitude of the sensitivity function with respect to a given design parameter, the greater the influence of this parameter on dynamic responses. In this case, it is observed that the orientation of the three five layers show a significant influence on the dynamic behavior of composite plates.

Conclusions

The numerous numerical simulations conducted to evaluate the performance of modeling procedures

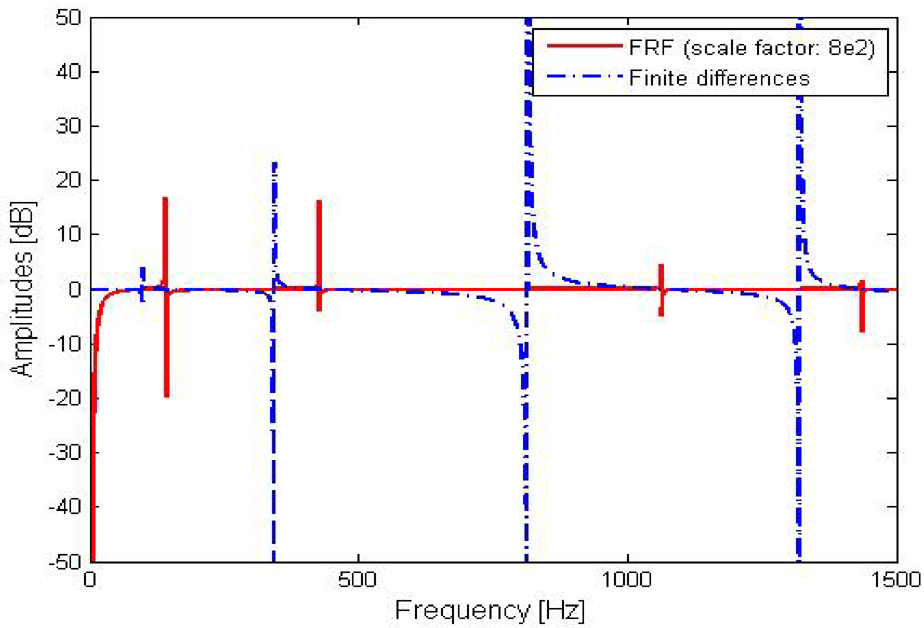

Figure 6. Sensitivity of the FRF obtained with respect to θ_3 for $\Delta\theta_3/\theta_3 = 1\%$.

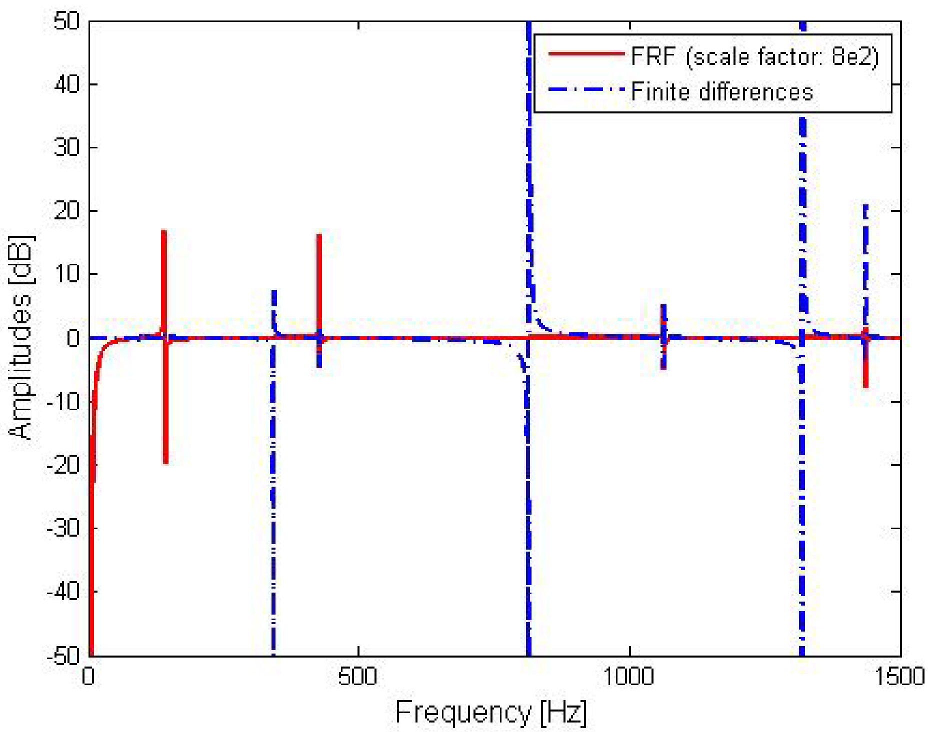

Figure 7. Sensitivity of the FRF obtained with respect to θ_5 for $\Delta\theta_5/\theta_5 = 1\%$.

developed as a tool for analysis and design laminated composite structures to show important aspects of dynamic behavior of the same in terms of the parameter sensitivities orientation of the fibers in the layers.

Conflict of Interest

The authors have not declared any conflict of interest.

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REFERENCES

Correia VMF, Gomes MAA, Suleman A, Soares CMM, Soares CAM (2000). Modelling and Design of Adaptive Composite Structures. Comput. Methods Appl. Mech. Eng. 185:325-346.

de Lima AMG, Diacenco AA, Côrrea EO (2009). Finite Element Modeling of Composite Sandwich Plates with Viscoelastic Layers. In: 20th International Congress of Mechanical Engineering (COBEM), Gramado, RS.

Diacenco AA, Lima AMGD, Corrêa EO (2013). Finite element reduction strategy for composite sandwich plates with viscoelastic layers. Mater. Res. 16(2):473-480.

Faria AW (2006). Modelagem por Elementos Finitos de Placas Compostas dotadas de Sensores e Atuadores Piezoelétricos: implementação computacional e avaliação numérica. 2006. 152f. Dissertação (Engenharia Mecânica) – Faculdade de Engenharia Mecânica, Universidade Federal de Uberlândia.

Huebner KH, Thornton EA (1982). The finite element method for engineers. New York: John Wiley & Sons.

Mendonça PTR (2005). Materiais Compostos and Estruturas - Sanduíche: Projeto e Análise 1.ed. Manole.

Reddy JN (1997). Mechanics of laminated composite plates: theory and analysis. 2nd ed. CRC Press.