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Review

In the road of assessing the validity of logarithmic law in wake flows: A review

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In any application of wall-bounded or shear fluid flows, near-wall boundary layer and shear layer are the places of struggle between viscous and inertial forces. After development and spread of using wall functions for modeling near-wall region of boundary layer in wall-bounded turbulent flows, the importance of accurate prediction and modeling of different layers of boundary layer, particularly the so-called logarithmic layer becomes more crucial in aerodynamics. Due to the intrinsic characteristics of flow structure in log-layer and wide-spread applications of wake flows, the presence of these characteristics in wake flows rather than wall-bounded flows opens a new window to the researchers to investigate the possibility of using log-law within wake regions, particularly on wake centerline for modeling purposes. On the applications of wake modeling and studying, we can point out to wing and blade trailing edge design, exhaust flow of pipes and ducts (e.g. nozzle exhaust or internal flow of polymers in dimpled pipes), and vortex generator design which are just a few examples of the areas with great interest in both fundamental scientific research, that is, developing optimum and accurate Computational Fluid Dynamics (CFD) tools and their industrial applications. In this article, a brief description about different approaches, previous efforts and case studies, and similar analogous problems is presented to give a better perception to the future researchers.

Key words: Logarithmic law, turbulent boundary layer, wake centerline, vortical structure.

INTRODUCTION

In viscous flows, shear force with wall due to normal velocity gradient create a complex shear layer region with interesting behavior near wall called (hydrodynamic) boundary layer. The reason of simultaneous use of words "complex" and "interesting" is simultaneous existence of viscous-dominant and inertial-dominant effects in boundary layer from which several complex phenomenon originate, that is, separation/reattachment, near-wall convection/conduction, transition, etc. The extent of boundary layer begins from wall surface upward in normal direction to distance where the streamwise component of velocity roughly equals to 0.99 of the free stream velocity magnitude (Wang et al., 2019; Pähtz et al., 2018). As shown in Figure 1, boundary layer comprises three sublayers; viscous, buffer and log layers, from surface of the wall to the free stream. Within the boundary layer, as we get farther from surface in normal direction, viscous dissipative effects become less dominant and give up their dominancy against inertial sustaining effects. Keeping the general shape of velocity profile in boundary layer, in practice, we can find such profiles in areas rather than on solid surfaces. For

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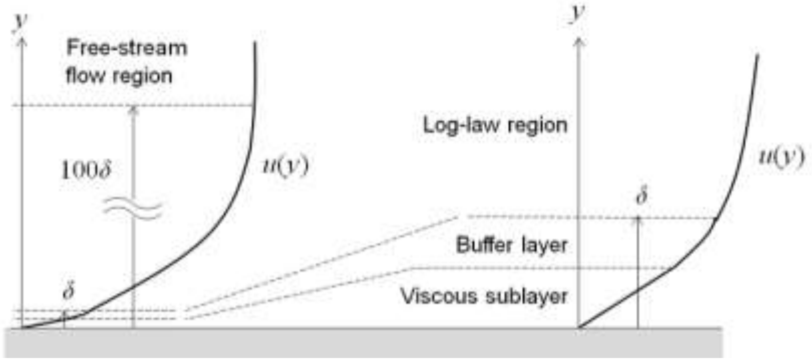


Figure 1. Schematic of boundary layer main regions (Comsol©).

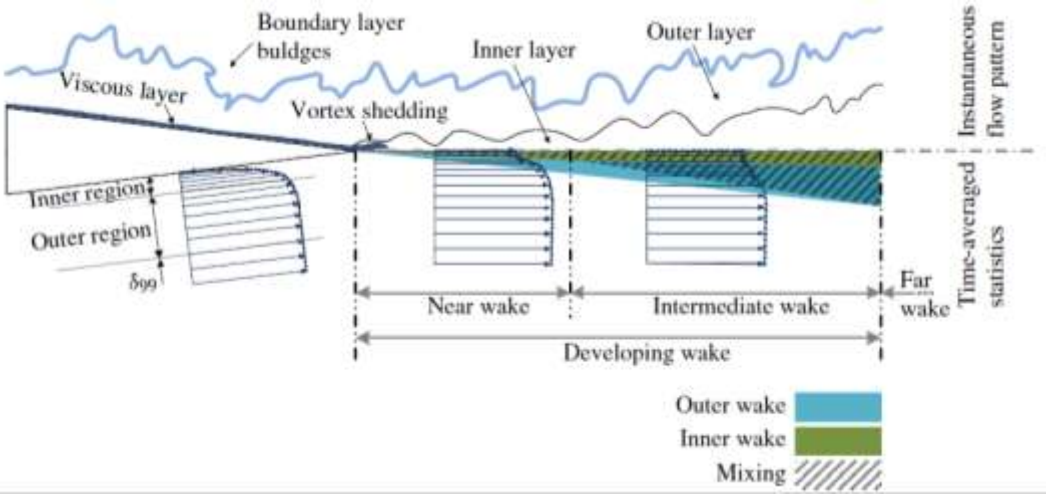
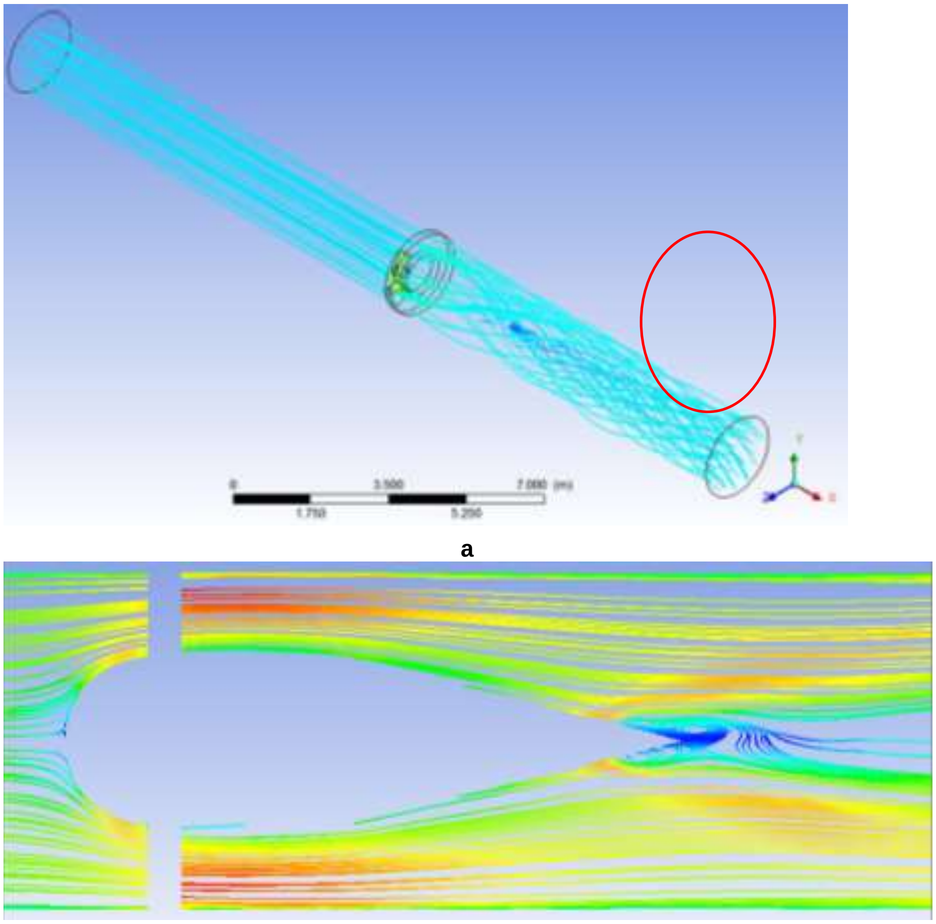


Figure 2. Graphical illustration of wake regions.
Source: Ghaemi and Scarano (2011).

instance, as Figure 2 finely depicts, shear layers occur in wake region, that is, downstream airfoil trailing edge of a wing or wind turbine blade (Ghaemi and Scarano, 2011; Momeni et al., 2019). Assuming quasi-symmetry pattern of such wakes, rotational velocity of vortices deficit streamwise mean velocity profile that causes velocity gradient and consequently shear layer in radial direction across the wake. As another example, such wake flows also exist downstream rotary devices, that is, in a fan rotor disk downstream blades and hub cone (Sabzpoushan et al., 2016) or shear-driven flow mixing in numerous industrial applications. Figure 3a and b shows two views of a time strand of an axially-shed 3D wake downstream an axial fan hub cone that is numerically visualized. It should be noted that although these wakes are timely transient, but in most cases due to repetitive oscillatory behavior can be treated as axisymmetric 3D wake structure (Sabzpoushan et al., 2016). About wake regions formed in shear-driven mixing process, supersonic wake region in ejectors has been of great interest among researchers. Actually, shear layer caused by considerable velocity difference between motive and suction flows in an ejector leads to an annular 3D wake structure with conical shape-centerline downstream the motive nozzle discharge tip (Darbandi et al., 2018a, b; Sabzpoushan et al., 2018; Sabzpoushan and Darbandi, 2018). Figure 3a shows a 2D side-view of the location of the centerline of such wakes. Having these applicable examples of wake flow, in this article attempt is made to take a concise look over the probable similarity between wake region (particularly wake centerline) and the logarithmic region of viscous boundary layer and its future perspective.

LITERATURE

Studying wake structures and turbulent wake modeling has been an interesting topic for researchers in both experimental and numerical parties for several years.



a

b

Figure 3. Wake region downstream a ducted axial fan hub cone formed by axially-shed vortices; a) isometric view and b) side-view.
Source: Sabzpoushan et al. (2016)

Indeed, the basis of such studies can be found in those of more abundant devoted to turbulent boundary layer. Here, a concise but in-a-long-period-of-time literature review is proposed to get familiar with the key notes revealed by researchers. In fact, here proposes a logical timeline-oriented roadmap of supporting and main research efforts to study the existence/validity of log-law in wake flows.

Indeed, experimental study of time-evolving wakes is far more complicated than spatial study. The main challenges include providing desired precision for the instruments, repeatability measurements resolution, etc. Explaining test procedure and method of experimental investigations on wake flows can be elaborative to the future researchers. As the most common infrastructure for experimental aerodynamic studies, wind tunnels have been widely used to experimentally investigate wakes structure. Cook (1971) experimentally performed series of measurements on boundary layer and wake of high Re high-subsonic flow over two cambered airfoils in wind tunnel in order to extract skin friction (friction coefficients). Reynolds number in Cook's tests was about 15×10^6 and15.6×10^6 with Mach number 0.725 and 0.664, respectively. They utilized a wind tunnel with test section of 2.4 m by 2.4 m test section in which two sample wings were mounted invertly (weight and lift in downward direction). Using roughness bands, the position of transition point of the boundary layer was fixed and wake region was swept by means of a 4-tube pitot rake and a static tube. Traversing the rake in normal direction to the free stream was made by tertiary rotating the rake in order to sweep the wake region, while all the pressure measurements were made by means of a 70 kN compression/tension pressure transducer. The pitot tubes were manufactured by hypodermic tubing method having inner and outer diameters of 0.3 and 0.5 mm, respectively. Besides, the spacing between tubes of the rake in this study was 2.5 mm.

In addition to high Re experiments, another experimental investigation was proposed by Nakayama and Liu (1990) on the effects of low Re between 812 and 5494 in near wake region of a flat plate. They found that the mean velocity near the wake centerline depends on Re and log-layer slope in velocity profile increases as Re increases. This is while large eddies of outer wake surprisingly influence the growth of the inner wake. They studied turbulence near wake of 3 Aluminium flat plate with identical shapes of leading edges and wedge trailing edges but different lengths of 25, 51 and 102 cm at low Reynolds numbers in a low-speed wind tunnel of 70 cm by 55 cm test section. In this study, free stream velocity magnitudes of 12, 24, 37 and 49 m/s were provided with turbulence level of at most 0.1%. In addition, boundary layer on the wings was thickened using sandpaper strips. It was also reported that although at low Re some problems may arise from laminar-to-turbulent transition and laminar flow instabilities, but even at the lowest Re, turbulent boundary layer at the trailing edge is fully developed. For measurements in this experiment, a small static pressure probe integrated with an electronic manometer was used in order to quantifying streamwise variations of static pressure along the wake edge. Moreover, a pitot tube and x-wire probes were utilized for measuring mean velocity and turbulence parameters, respectively. Measurement points were chosen at 6 sections downstream the trailing edge among which the most upstream one was close enough to the trailing edge where the obtained results, except on a few points near the wake centerline could be analogized to the boundary layer at the trailing edge. This is a promising evidence for verifying the validity of logarithmic law in wake flows, particularly at the wake centerline.

Extensive research efforts can be found in literature devoted to proposing unified (universal) flow -and thermal-law of the wall, that is, a paper published by Wang et al. (2019). As the main result in this paper, sublayers partitions were defined in a Zero Pressure Gradient (ZPG) turbulent boundary layer on a semi-infinite flat plate based on the boundary layer thickness. In fact, the presented data by Direct Numerical Simulation (DNS) were investigated and formulated as Reynolds-independent velocity formulations. In other words, the governing equations were derived under the ensemble-averaged (the mean value of a quantity as a function of the microstate of the system) scales. These analytical formulations, called as the complete law-of-the-wall were developed for entire thickness of the boundary layer, including laminar, buffer, (semi-)log and wake sublayers and validated against the existing experimental and DNS data. However, this integrated approach is still incomplete and needs to be kept under detailed investigations.

In a research proposed by Hwang and Sung (2019), by addressing insufficient structural understanding of the logarithmic law, mean velocity logarithmic law has been implemented to predict skin friction. In such approaches, height-weighted averaging is performed on wall-normal distributions of the streamwise velocity in the wall-attached turbulent structures. Although, researchers mentioned that there is a lack of understanding of the structure of the logarithmic layer, they have tried to extract the turbulent structures of streamwise velocity fluctuations through DNS of pipe flow. It was found that the presence of the logarithmic region can be revealed by employing the statistical properties of identified structures in flow field. As an application, it is suggested that the dynamics of so-called U-structures would be the key factor to provide turbulence control and have insight into the sustaining mechanism(s) of wall turbulence at high Re. Similarly, as another numerical effort for internal flows presented by Laadhar (2019), it is validated the DNS results of turbulent pipe and so-called ZPG boundary layer flows against experimental data. The investigations show a flow-independent logarithmic law for scaled mean streamwise velocity within a certain range of non-dimensional radial distance from inner wall. Conversely, it is shown that the mean velocity deficit follows a flow-dependent logarithmic law.

Despite much more investigations on turbulent wake flows in subsonic regime, some scholars have paid attention toward supersonic regime. Nakagawa and Dahm (2006) presented their experiments to visualize vortical structure and flow growth rate of a supersonic planar turbulent bluff-body wakes using Laser Mie scattering and Schlieren methods. They found that the interaction of large-scale flow structures with reflected expansion waves (reflected from the recompression regions) creates forcing mechanisms in case of existence of subsonic upstream paths, e.g. in the Mach=2 wake. In their study, pressure measurements (for determining velocity field) were performed to understand compressibility and confinement effects. In this regard, measurements results for M=2 and 3 were compared with those of forced and unforced (perturbed/unperturbed) incompressible wakes. For this purpose, they used a specific supersonic facility with test section of 34.6 mm by 38.4 mm. This test setup mainly consists of a subsonic slot nozzle centered in the surrounding supersonic nozzle acting as a bluff-body that generates supersonic turbulent wake. In addition, difference in velocity magnitude of flows of the two jets downstream the subsonic nozzle discharge creates as shear layer between the two flows. This could be another clue to direct researchers to investigate the possible validity of the logarithmic law on wakes centerlines. For static pressure measurements, they improvised 8 pressure taps on each sidewall of the test section. As an important point, it was reported that wake displacement effect (temporal behavior of the wake) may cause fluctuations in wall static pressure which is negligible compared to those caused by the sequential reflections of the expansion and compression waves along the supersonic wake from the tunnel walls. Therefore, such wakes could be treated as spatial-evolving wake.

Transport phenomenon in boundary layer on a finite wall and its following wake development, e.g. downstream of an airfoil trailing edge shall be corresponded by some 3D vertical structures like hairpin and counter-hairpin vortices. In this regard, Gheami and Scarano (2011) studied the disappearance of the viscous sublayer after a sharp trailing edge with focus on wake centerline. They reported an interesting chain of findings. Their results imply that transporting high speed flow into wake centerline and neighbouring low and high speed flow layers result in shear layer along the centerline. In other words, just downstream the tailing edge, interaction of low and high speed streaks coming from pressure and suction sides forms a high shear region on the wake centerline. This suggests that wake centerline can be considered as a slip-wall and consequently, because laminar sublayer no longer exists, the possibility of log-layer analogy on the wake centerline increases. It should be noted that due to turbulence-driven fluctuating of axial velocity component which leads to existence of Reynolds shear stress at wake centerline, it cannot be treated as non-slip wall.

Giving some examples of the real applications of studying wake flow regions as well as flow regions dominated by logarithmic law does make sense to the researchers in order to better understand the importance and applicability of the subject. In this regard, statistical approach in studying wake region can be seen in the literature less or more. As an example, Rai (2016) proposed a statistical comparison of near wake velocity, normal intensity and fluctuating shear stress distributions of a flat plate with circular and elliptical trailing edges using DNS. He found out that using elliptical trailing edge weakens vortex shedding, shrinks separation region and lowers intensity level, a finding that would be practically and beneficially applied to wind turbine or compressor blade design, etc. On the other hand, among the researches on turbulent internal flows, Monkewitz (2017) tried to give some interpretation for two logarithmic flow regions in pipes and channels (two of the three canonical flows); an interior region with same Karman constant as in the ZPG turbulent boundary layer flow, and an exterior one with behavior totally dependent on the pipe/duct axial pressure gradient. As mentioned by Luchini (2018) many documents in the literature point out to the discrepancies between the analytical solutions come from the logarithmic law and experimental data. Therefore, one debate is measuring/calculating Karman constant in logarithmic formulation. Introduced as "law of the wake", by taking wake components perturbations into account in asymptotic modeling of the log-layer, the discrepancies can be interpreted by implementing higher order corrections proportional to pressure gradient intensity along the wake.

Indeed, wake modeling has several applied outcomes in different applications, that is, in design of wings and blades. Among the applied research in this area, Wang and Ghaemi (2019) published a parametric study on the effect of sweep angle on wake flow structure of different vortex generator geometries. Using Particle Image Velocimetry (PIV) method for measurements in this research, it was found that the strength of the vortices is inversely proportional to the sweep angle of the Vortex Generator (VG). Moreover, rectangular VG showed the best mixing performance. Additionally, this research suggests that from efficiency point of view, trapezoidal VG is the most efficient one with the largest mixing enhancement to drag ratio, while large sweep angles potentially decrease VG performance in simultaneously reducing drag, enhancing mixing and preventing separation.

As another application-oriented research on log layer in turbulent flow, Shaban et al. (2018) experimentally investigated channel flow of polyacrylamide solution to extract study flow structure using time-resolved planar PIV. It was shown that addition of certain amount of polymers in order of parts per million (ppm) efficiently reduces drag. In a similar research, Ebrahimian et al. (2019) studied motion of suspended inertial particles near wall in a non-Newtonian turbulent channel flow. It is revealed that 90 ppm solution of polyacrylamide in water reduces drag by 66%. In general, the polymer solution reduces wall-normal fluctuations of the glass beads and parallelizes their trajectories with the channel wall. In practice, these can reduce wall erosion in the particle-laden pipes and channels.

In order to better investigate the possibility of governance of the logarithmic law in wake flows, the first step could be making some analogy between the logarithmic law-governed phenomenon and the problems or phenomenon under study. In this regard, the more famous and more applicable the analogy is, the more interesting and more sensible it is to study the logarithmic law possibility. A very useful analogy, for instance, is to compare velocity profile perpendicular to the wake centerline to that of on a wall. As other examples, Figure 5 shows four selected samples among the results proposed in the literature. Figure 5a represents mean velocity profile downstream a cylinder on its wake centerline having sign changes just upstream and downstream the twin vortices, wherein horizontal axis represents a streamwise coordinate centered at the saddle point of the wake nondimensionalized using vortex formation length and the streamwise location corresponding to the maximum mean velocity of reversed flow at wake centerline, and vertical axis represents nondimensional velocity as the ratio of mean velocity of reversed flow along the wake centerline over its maximum value. Therefore, in this figure, zero value at the horizontal axis is the streamwise location of saddle point in the wake pattern (Kuo et al., 2007). In Figure 5b, inverse effect of Re is depicted on the slope of velocity profile in log-layer in turbulent boundary layer on a flat plate. Figure 5c shows that log-law is observed along the

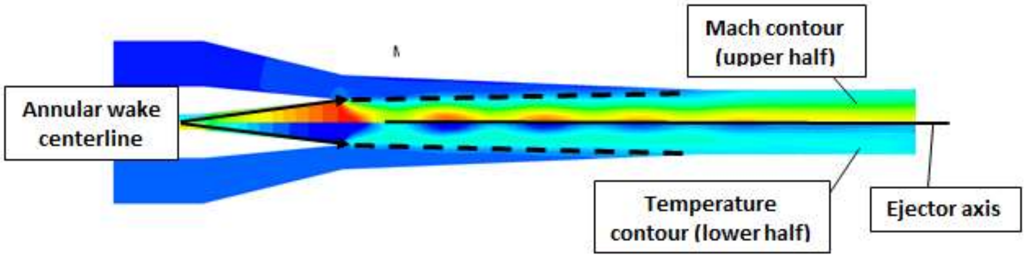
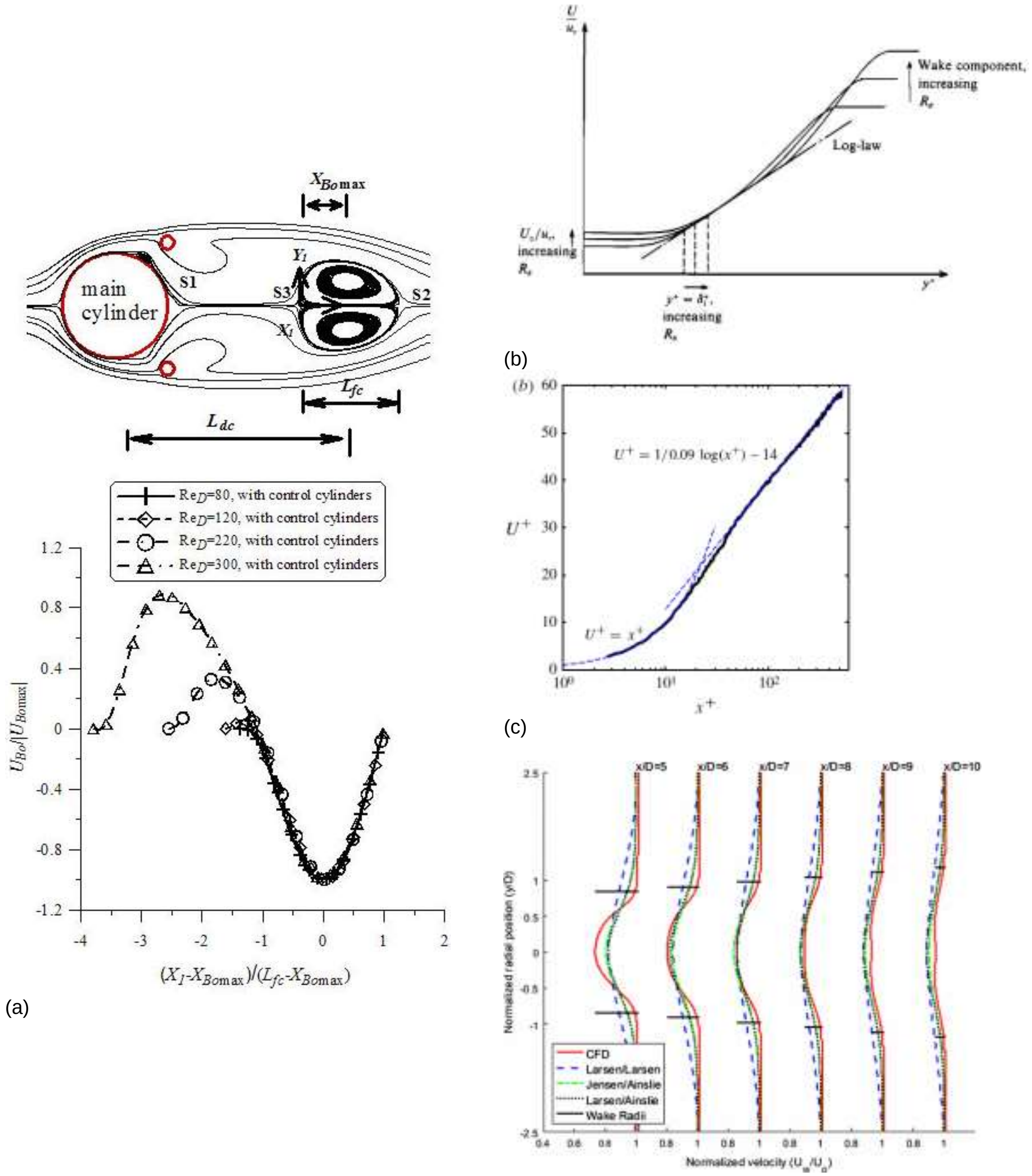


Figure 4. Side-view of the position of centerline of a conical (annular) wake downstream the primary nozzle discharge inside a steam ejector.
Source: Darbandi et al. (2018)

centerline of a wake flow downstream a sharp trailing edge which is a proof of existence of logarithmic region on the wake centerline. Finally, Figure 5d shows velocity deficit along a wake deteriorates as we move downstream. By moving downstream into far wake, although wake region radially grows, but velocity deficit is to be deteriorated and damped out (Pyakurel et al., 2017). The accuracy of capturing this axial deterioration with respect to available experimental data is a good factor by which researchers can examine their analytical and numerical models and simulations. In other words, as the first step, one should develop an analytical or numerical tool and examine its accuracy in order to be able to study and assess the validity of governing laws like logarithmic law. For example, in an analytical study proposed by Wang et al. (2019), the maximum error in calculated velocity using derived equations under ensemble-averaged scales is reported about 2% with respect to verified DNS data.

The common type of study on wake structures and wake modeling as shown in Figure 5a and d is studying the so-called spatial wake. In such studies, despite unsteady nature of wakes, they are treated as quasi-steady or in other words they are treated as time-averaged structures and flow field in wake region are to be studied in different spatial directions in a 2D or 3D space. In addition to spatial wake flows, temporal or time-evolving wake structures are more complex to be modeled and studied but most of the times, not always, more realistic ones. Experimental study of time-evolving (sometimes called spatiotemporal) wakes is quite complex and costly with essential technical limitations. This is why in the related literature we mostly find numerical efforts by using previously-validated codes with spatial experimental data. As a recent study, Jacob et al. (2020) investigated 3D time-evolving incompressible plane wake through DNS and Large Eddy Simulation (LES) methods. By implementing periodic boundary condition in all the boundaries of computational domain they studied different behaviors of stability, laminar-to-turbulence transition as well as onset of 3D vortical structures and nonlinear regions under the effect of perturbations with different amplitudes. In another study on time-evolving wakes by Moser et al. (1998), 3 turbulent planar wakes were perturbed with planar disturbances. Reynolds number based on momentum thickness is reported to be 670 which is equivalent of mass flux-based Reynolds number in spatial wakes. About the effect of forcing the wake structure with disturbances, it was concluded that in highly forced flows large-scale structures (similar to those in transitional wakes) are dominant and more organized. In addition to temporally treating the wakes, compressibility is another feature that may be taken into account in some cases, absolutely with high Mach numbers. Chen et al. (1990) studied the effect of Mach number on the stability of a compressible (supersonic) wake flow using DNS method. By investigating possible means for controlling the temporal evolution of perturbed (forced) planar wake, it was found that at a higher Mach number wake has more stabilizing behavior and reduced growth rate which slows down the rollup motion of the spanwise vortices. These consequences like dominancy of large-scale structures in highly forced (perturbed) wake flows and slower 3D vortex rollup in supersonic wakes with higher Mach numbers are crucial points that should be taken into account in case of using simplifying assumptions for investigating the validity of logarithmic law in wake flows.

After reviewing several references, as proposed, due to many real-world applications of wake flows, researchers intentionally or unintentionally are supposed to study the physics and structure of wake flows. This leads to a need to propose and combine simplified and even reduced-order models to better model complex wake flows. Such modeling procedure could be achieved by using of simplifying assumptions (like incompressibility) and simpler shear flows, that is, free jet flow, mixing jets, flow on a flat plate, incompressible and so on and trying to make analogies and implement a combination of their governing rules, models and relations on modeling the main flow under study (Darbandi et al., 2018a, b; Sabzpoushan et al., 2018; Sabzpoushan and Darbandi, 2018; Nakayama, and Liu, 1990; Pyakurel et al., 2017). In this regard, logarithmic law particularly seems to be a candidate modeling tool to better understand and describe the flow behavior at wakes centerline.



(a)

(b)

(c)

(d)

Figure 5. Literature sample results; a) Mean velocity of the flow over a cylinder along the wake centerline at different Reynolds numbers (Kuo et al. 2007), b) Mean velocity profile as function of wall-normal direction at different Reynolds numbers Nakayama and Liu (1990), c) Semi-log profile of mean streamwise velocity along the wake centerline and the dashed lines curve fits to the linear and logarithmic layers (Ghaemi et al. 2011), and d) Different solutions for velocity profiles in radial direction at different streamwise locations across wake for turbulence intensity of 9 percent (Pyakurel et al. 2017).

FUTURE CHALLENGES

The main objectives of the present article are to present different approaches of studying wake flow structures, particularly those related to logarithmic law governance in different problems as well as addressing the possible connections and analogies between such research efforts and investigating the validity of logarithmic law in wake flows, in particular at wakes centerline. In general, small-scale and accurate experimental measurements and highly robust numerical simulations are required to investigate if velocity (total, mean or perturbed) shows logarithmic trend as function of streamwise or radial distance along the wake centerline. For instance, even in case of using wall functions in CFD simulations, only the two near-wall sublayers of boundary layer can be modeled. The third sublayer (log-layer) should be numerically solved aside with the free stream. Therefore, accurately studying log-layer still remains critical.

In the following, some of the most important challenges in wake simulation, observing and modeling logarithmic regions in wake flows are listed to which researchers will get involved in their future studies.

(1) Developing reliable experimental methods and tools to accurately measure flow details particularly those related to small 3D vortical structures and thermal mechanisms in wakes. The measuring equipment should note unwanted effects on flow field while being highly durable in operation.
(2) Numerical model development for proposing universal laws of the wake that couple aerodynamic and thermal mechanisms.
(3) Continuously implement the developed tools, methods and models into practice while simultaneously dig for new applications and more complex wake conditions to be investigated.
(4) Investigating far wake structures to which much less attention has been paid by the scholars. This can be highly practical in applications like wind farms, aircrafts with canard surfaces, state-of-the-art concepts like commercial airplanes flying in group, etc.

CONCLUSION

Focusing on a fixed point on wake centerline, the adjacent streamlines at that point behave like slip walls for each other due to almost equal (streamwise) velocity. Therefore, the centerline could be considered as a slip wall locally and temporally.

Unlike viscous laminar sublayer, in log layer known as the third layer of boundary layer from wall (adjacent to the free stream), inertial effects are dominant on viscous effects under the affection of free stream. In other words, in the wake region, we face with two sub-regions in radial direction; inner wake that encompasses wake centerline and outer wake. There is always an interesting struggle between inertial (sustaining) and viscous (dissipative) effects inside these regions, for which turbulent mixing and local temporal shear layers can be mentioned as representatives, respectively. Particularly, along the wake centerline, because adjacent quasi-symmetry vortices assimilate slip wall condition for each other, inertial effects are dominant on the wake centerline in absence of strong dissipative viscous effects. Therefore, considering the convective nature of log-layer of a turbulent boundary layer, it is logically possible implementing logarithmic laws on wake centerline. However, moving downstream the wake, randomness and unsteadiness in wake region rise. This may lead to larger streamwise relative velocity between adjacent vortices on the wake centerline causing local slip wall condition to be vanished. Moreover, the longer the distance moving downstream, the more dissipation of turbulence kinetic energy as sustainer of the inertial effects occurs, causing less dominancy of convective terms in far wake. The rate of dissipation depends on the intensity of the velocity deficit and vortex shedding unsteadiness in near wake.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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