Full Length Research Paper

Investigation to improve hunting stability of railway carriage using semi-active longitudinal primary stiffness suspension

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Railway carriage model moving on tangent tracks is constructed by deriving the associated equations of motion where single-point and two-point wheel-rail contact is considered. The railway carriage is modeled by 31 degrees of freedom which govern vertical displacement, lateral displacement, roll angle and yaw angle of wheelset whereas vertical displacement, lateral displacement, roll angle, pitch angle and yaw angle of carbody and each of two bogies. Linear stiffness and damping parameters of primary and secondary suspensions are provided to the railway carriage model. Combination of linear Kalker’s theory and nonlinear heuristic model is adopted to calculate the creep forces in which introduced at wheel and rail contact area. Computer aided-simulation is constructed to solve the governing differential equations of motion using Runge-Kutta fourth order method. Principles of limit cycle and phase plane approach is applied to study the stability and evaluate critical hunting velocity of the system. The numerical simulation model is used to represent dynamic responses of the components of railway carriage subjected to specific parameters of wheel conicity and suspension characteristics. Longitudinal primary stiffness suspension is controlled using semi-active suspension with lateral displacement indicator. The controlled semi-active longitudinal primary suspension is examined to increase the critical hunting velocity and improve hunting stability of railway carriage.

Key words: Railway carriage, conventional bogies, tangent tracks, semi-active suspension, longitudinal suspension, critical hunting velocity.

INTRODUCTION

The safety of railway system requires that derailment or the wheel flange climb during railway carriage running is never be allowed while ride comfort requires that a self excited lateral railway oscillation or what called hunting phenomenon is eliminated. The classical hunting oscillation is a swaying motion of railway carriage caused by the forward speed of the vehicle and by wheel-rail interactive forces due to wheel-rail contact geometry and friction creep characteristics. Hunting is instability appears as an oscillation in the components of railway carriage at higher speeds known as the critical hunting velocities at which the railway carriage starts to hunt. The motion will be violent, damaging track and wheels, and may cause derailment when railway carriage running above the critical velocity. Hunting instability can be eliminated and improved by increasing the critical velocities of the railway carriage. In the present study a semi-active longitudinal primary suspension is used with lateral displacement indicator to improve the hunting stability of the railway carriage and increase the critical velocity. Lateral displacements occur due to imperfections and irregularities in the track which cause different undesirable motions like roll, yaw, and pitch. Lateral forces arise in the wheel-rail contact patch plane due to interactions between the wheel and the rail. These introduced forces called creep forces in which depend upon different creep coefficients. The magnitudes of creep coefficients depend upon the wheel-rail geometry, normal load, and material properties. Many investigations
used different magnitudes of creep coefficients and a combination of linear Kalker's theory (1979), and nonlinear heuristic is used in the present study. The study of railway carriage dynamic behavior should take into account the responses of railway carriage to these displacements and movements and more degrees of freedom should be considered to verify the accuracy of the system model. Different railway carriage models with different degrees of freedom are investigated and presented by many papers concerning railway carriage dynamic response. Dynamic stability of railway carriage wheelsets and bogies having profiled wheels was presented by Wickens (1969), in which two degrees of freedom model was suggested govern lateral and yaw angle of each wheelset. Nonlinear mathematical model of dynamic simulation has been established with 7 degrees of freedom by Jawahar and Gupta (1990), which govern lateral and yaw movements of wheelsets and lateral, yaw and roll movements for both conventional and unconventional bogies. Xu and Ding (2002, 2006), studied the dynamic analysis of coupled train-bridge systems under fluctuating wind and stated a railway carriage model that each 4-axle vehicle in a train is modeled by 27 degrees of freedom dynamic system. A vehicle model by Wang (1992), was developed to represent a 23 degrees of freedom conventional freight car, consisting of a carbody, two bolster and two truck assemblies where the carbody was assigned five degrees of freedom govern vertical lateral, yaw, pitch, and roll and each bolster was assigned three degrees of freedom vertical, lateral and roll motion. Nath and Jayadev (2005) studied the influence of yaw stiffness on the nonlinear dynamics of railway wheelset used two degrees of freedom model which govern the lateral and yaw motion. Nonlinear differential equations modeled by 8 and 10 degrees of freedom of railway carriage moving on curved tracks are presented by Sen-Yung Lee and Yung-Chang Cheng (2005, 2006). Train vehicle model considered by Kumaran et al. (2003), conforming to Indian railways consists of a vehicle body, two bogies with four wheelsets in which the system is modeled by 17 degrees of freedom. In the study of Mohan (2003), railway carriage model is suggested comprising carbody, two bogies with four wheelsets, in which the railway model assigned two degrees of freedom which govern lateral and yaw motions of each wheelset and bogies whereas the full vehicle assigned three degrees of freedom which govern lateral, yaw and roll motions. In additional a semi-active longitudinal primary suspension with yaw angle indicator used to increase the critical hunting velocity of the railway carriage. Study the effects of railway track imperfections on track dynamic behavior, and the effect of unsupported sleepers on the normal load of wheel-rail were investigated by Zhang et al. (2007), in which the system is modeled by 35° of freedom that consider the lateral and vertical displacement, roll, pitch and yaw angle for the carbody, front and rear bogie frames and the four wheelsets. An ideal truck model with full frame decoupling represented by Dukkipati and Narayana (2001a, b), which is modeled by 8 degrees of freedom. An investigation of dynamic interaction of long suspension bridges with running trains is presented by Xia et al. (2000), in which 27 degrees of freedom model is used. A new finite element model for three-dimensional analysis of high-speed train–bridge interactions is proposed by Song et al. (2003), in which the equations of motion of the vehicle-bridge were derived using Lagrange's equation where the carbody is considered with four degrees of freedom which govern bouncing, swaying, pitching, and yawing whereas bouncing, sliding, swaying, pitching, rolling and yawing motions are considered for the bogie. Li et al. (2007) investigated the problem of railway vehicle suspension estimation in which lateral and yaw modes are important and wheelsets and bogie have two degrees of freedom which govern lateral and yaw motions. A nonlinear model of a single wheelset moving with constant speed on a purely straight track is presented by De Pater (1980), and the equations of motion were written down either as six equations containing the normal forces, or as four equations which do not contain the normal forces. Yugat et al. (2009), presented an analytical model of wheel-rail contact force due to the passage of a railway vehicle on a curved track used equations of motion govern vertical and roll motion of right and left wheel while vertical motion of right and left rail. A nonlinear wagon-track model with 23 degrees of freedom is presented by Sun and Simson (2008), used to study rail corrugation formation due to the wheel stick-slip process. Rajib et al. (2008), presented equations of motion govern vertical motion of front and rear wheelset, bounce and pitch motion of bogie and bounce motion of carbody to study the dynamic analysis of railway vehicle-track interactions. Railway vehicle dynamics during motion along a curved track is examined by Zboinski (1998, 1999), in which the dynamic behavior of the system is studied using two different methods, the quasi-static and dynamical approach. In additional the research concerned the influence of vehicle suspension parameters as well as conditions of motion (speed, super-elevation, curve radius, transition curve existence) on limit cycle occurrence. The present study considers a railway carriage consists of carbody, two bogies and four conventional wheelsets modeled by 31 degrees of freedom which govern bounce, pitch, roll, lateral, and yaw motions of the system. The procedure done in this study is to derive the second order governing differential equations of motion of the full railway carriage and transformed these equations into a set of first order differential equations using a special technique to facilitate solving them with numerical methods. Computer-aided simulation is used to solve these equations with Runge Kutta fourth-order method and represent the dynamic behavior of the system by
Figure 1. Front view of railway carriage components equipped with sets of primary.

increasing the forward speeds of the system to reach the critical hunting velocity. Principle of limit cycle approach (Mohan, 2003), is used to obtain the critical hunting velocity of the system in which subjected to magnitudes of wheel conicity and suspension parameters. Semi-active longitudinal primary suspension with lateral displacement indicator is adopted as a controllable device in railway carriage to increase the critical hunting velocity. The railway carriage simulation model is used to examine the dynamic behavior of railway carriage with the semi-active longitudinal primary suspension.

Mathematical railway carriage model

The railway carriage is a combination of components and wheelsets joining together by a set of different primary and secondary suspension elements, in which the full railway carriage configuration model system consists of carbody, two conventional bogies, and four wheelsets as shown in Figure 1. A railway carriage model of 31 degrees of freedom is constructed in this research to study the dynamic responses of railway carriage components moving on tangent tracks. The differential equations of motion govern lateral displacement $Y_w$, $Y_b$, $Y_c$; roll angle $\phi_w$, $\phi_b$, $\phi_c$ and yaw angle $\psi_w$, $\psi_b$, $\psi_c$ of wheelset, bogie and carbody respectively while pitch angle $\theta_w$, $\theta_b$, $\theta_c$ of bogie and carbody. Railway carriage model is equipped with eight longitudinal, lateral and vertical primary suspensions of spring stiffness $K_{px}$, $K_{py}$, $K_{pz}$ respectively and viscous damping constant $C_{px}$, $C_{py}$, $C_{pz}$ respectively. Also the system is provided with eight longitudinal, lateral and vertical secondary suspensions of spring stiffness $K_{sx}$, $K_{sy}$, $K_{sz}$ respectively and viscous damping constant $C_{sx}$, $C_{sy}$, $C_{sz}$ respectively. Symbols and notations are illustrated in the nomenclature in Table 1. Dynamic behavior of railway carriage is caused by wheel-rail interactions in which creep forces are introduced at wheel-rail contact patch area. Non-conservative forces and elastic deformations at the contact patch introduce a phenomenon of creep and combination of linear Kalker's theory (Kalker, 1979), and nonlinear heuristic is considered to calculate the introduced creep forces. Vibrations are transmitted through connected suspensions to other railway carriage components and the dynamical behavior of the system is governed by the equations of motion of each component of railway carriage.

Wheelsets differential equations of motion

The railway carriage model is equipped with four conventional wheelsets in which consists of two wheels attached together by a solid axle. Wheelsets are used to steer and support the carriage. Wheelsets equations of motion are derived using Newton's laws with suspension, creep and normal forces in which some of these forces are calculated by (Sen-Yung Lee and Yung-Chang Cheng, 2005, 2006). The vertical, roll, lateral and yaw equations of motion of single wheelset are:

\begin{align*}
\dot{Y}_w &= 2\chi_{pc} \dot{Y}_w - 2\chi_{pc} \dot{Y}_b - 2\chi_{pc} \dot{\theta}_b + \frac{2f_{11}}{V} \lambda Y_\phi \\
\dot{\phi}_w &= \frac{2f_{12}}{V} \dot{Y}_w + \frac{2f_{12}}{V} \dot{\phi}_b + \frac{2f_{12}}{V} \dot{\theta}_b + 2K_{pc} \dot{Z}_w \\
\dot{\theta}_w &= -2K_{pc} \dot{\theta}_b - 2K_{pc} \dot{\theta}_b - \frac{2f_{12}}{r_o} \lambda = 0
\end{align*}
Bogies differential equations of motion

Railway carriage model consists of two bogies in which each bogie has two conventional unconnected front and rear wheelsets and two vertical secondary suspension elements are used to connect bogies with carbody in addition to the set of primary suspension elements connected each bogie with the wheelsets. The bogies differential equations of motion govern vertical, pitch, roll, lateral, and yaw degrees of freedom are:

\[ J_{wz} \phi_{wi} + \left( \frac{2f_{11}}{V} - 2C_{py} \right) Y_{wi} + 2f_{12} \frac{2f_{11}}{V} \phi_{wi} + 2f_{11} \psi_{wi} + \frac{2r_{o}f_{11}}{V} \phi_{wi} = 0 \]  

\[ J_{wz} \psi_{wi} + \left( \frac{2a^2 f_{33}}{V} + 2f_{22} \right) + 2C_{px} L_x^2 2C_{px} L_x^2 \psi_{bj} + \left[ 2K_{px} L_x^2 - (-2f_{12} + a) \right] \psi_{wi} - \frac{2f_{12} Y_{wi}}{V} + \left( - J_{wy} \frac{2r_{o}f_{12}}{V} \right) \phi_{wi} = 0 \]  

\[ \lambda W \psi_{wi} - \frac{2a^2 f_{33}}{r_{o}} Y_{wi} - 2K_{px} L_x^2 \psi_{bj} = 0 \]  

\[ m_b \phi_{bj} + (2C_{sz} + 4C_{pz}) \phi_{bj} - 2C_{sz} L_z^2 \dot{\theta}_{c} - (2K_{sz} + 4K_{pz}) \dot{Z}_{bi} - 2K_{sz} L_z \dot{\theta}_{c} - 2K_{s} \phi_{bj} \]  

\[ \dot{\theta}_{c} - 2C_{sz} \ddot{Z}_{c} - 2C_{pz} \dot{Z}_{wi} - 2C_{pz} \dot{Z}_{wi} = 0 \]  

\[ J_{by} \phi_{bj} + 4C_{pz} L_z^2 \phi_{bj} - 2C_{pz} L_z \ddot{Z}_{wi} - 2C_{pz} L_z \ddot{Z}_{wi} - 2K_{pz} L_z \dot{Z}_{wi} = 0 \]  

\[ J_{bx} \phi_{bj} + 4C_{pz} + 2C_{sz} \phi_{bj} \]  

\[ -2C_{sz} L_z^2 \dot{\phi}_{c} - 2C_{pz} L_z^2 \phi_{wi} - 2C_{pz} L_z^2 \phi_{wi} - 2C_{pz} L_z^2 \phi_{wi} \]  

\[ -2C_{sz} L_z \dot{\phi}_{c} - 2C_{pz} L_z \dot{\phi}_{wi} - 2K_{pz} \dot{L}_{cz} Y_{wi} - 2C_{pz} L_z \dot{L}_{cz} Y_{wi} - 2C_{pz} L_z \dot{L}_{cz} Y_{wi} \]  

\[ -2K_{pz} \dot{L}_{cz} Y_{wi} = 0 \]  

\[ m_y Y_{wi} + (2C_{sy} + 4C_{py}) Y_{wi} - 2C_{sy} \ddot{Y}_{c} - 2C_{py} \dot{Y}_{wi} = 0 \]  

\[ -2K_{py} Y_{wi} - 2K_{py} \ddot{Y}_{wi} = 0 \]  

\[ K_{sy} Y_{wi} - 2C_{py} Y_{wi} + (2K_{sy} + 4K_{py}) Y_{bj} - 2K_{sy} \ddot{Y}_{c} \]
Carbody differential equations of motion

Carbody is the heaviest component in railway carriage makes crush between wheel and track and elastic deformation is introduced at contact patch area to produce creep forces and moments. Carbody differential equations of motion govern bounce, pitch, roll, lateral, and yaw degrees of freedom are derived applying Newton’s law. The derived equations of motion of carbody with mass $m_c$, and moment of inertia about longitudinal axis $J_{cx}$, about lateral axis $J_{cy}$, and about bounce axis $J_{cz}$ are

$$m_c \ddot{Z}_c + 4C_{cx} \dot{Z}_c \dot{Z}_c - 2C_{cx} \dot{Z}_c \dot{Z}_b + 4K_{cx} Z_c - 2K_{cx} Z_b = 0$$  \hspace{1cm} (10)

$$J_{cy} \dot{\theta}_c + 4C_{cy} \dot{\theta}_c \dot{Z}_c - 2C_{cy} \dot{\theta}_c \dot{Z}_b + 4K_{cy} \dot{\theta}_c - 2K_{cy} \dot{Z}_b = 0$$  \hspace{1cm} (11)

$$J_{cz} \dot{\phi}_c + 4C_{cz} \dot{\phi}_c \dot{Z}_c - 2C_{cz} \dot{\phi}_c \dot{Z}_b + 4K_{cz} \dot{\phi}_c - 2K_{cz} \dot{Z}_b + 4K_{cz} \dot{\theta}_c \dot{Z}_c - 2C_{cz} \dot{\theta}_c \dot{Z}_b + 4K_{cz} \dot{\theta}_c \dot{Z}_b = 0$$  \hspace{1cm} (12)

$$m_c \ddot{Y}_c + 4C_{cy} \dot{Y}_c \dot{Y}_c - 2C_{cy} \dot{Y}_c \dot{Y}_b - 2C_{cy} \dot{Y}_c \dot{Z}_b + 4K_{cy} Y_c - 2K_{cy} Y_b - 4K_{cy} \dot{Z}_c \dot{Y}_b = 0$$  \hspace{1cm} (13)

$$J_{cz} \ddot{\phi}_c + 2(2l_c^2 C_{cy} + C_{xz}) \dot{\phi}_c - C_{xz} \dot{\phi}_c - C_{xz} \dot{\phi}_b + 2l_c K_{cx} Y_c \dot{\phi}_c - 2l_c K_{cx} Y_b \dot{\phi}_c - 2K_{cx} Y_c \dot{\phi}_b = 0$$  \hspace{1cm} (14)

Numerical simulation

Railway carriage runs on tangent tracks is modeled by the second order differential equations of motion (1) - (14). A simple and important technique used to transform the governing equations of motion into first order differential equations in suitable form known as state space equations. This technique is used to facilitate solving the equations with numerical integration methods. The transformed equations of motion are simulated with computer-aided simulation to solve by Runge-Kutta fourth order numerical method. The data which used in numerical simulation from (Mohan, 2003), also initial conditions are assumed for the dynamic motions of the system. Simulation is executed to represent the dynamic responses of railway carriage components subjected to different parameters. Procedure is achieved by increasing the speeds to reach the critical velocity and principle of limit cycle and phase plane approach is utilized to represent the critical hunting velocity of the system. Figures 2 - 4 show the dynamical response of the carriage components to lateral displacement at critical hunting velocity (76 Km/h) and it can be visualized that wheelset is more sensitive to lateral dynamic response at critical hunting velocity than other railway carriage component.

The numerical simulation model also presents the dynamic response of railway carriage components to yaw displacement at critical hunting velocity (76 Km/h) as shown in Figures 5 - 7. It can be noticed that railway carriage components all are sensitive to dynamic yaw displacement and it is difficult to predict the more sensitive component so principle of limit cycle and phase plane approach is utilized to represent railway carriage in which more sensitive to dynamic yaw displacement. Figures 8 - 10 represent the phase portrait trajectories of railway components which show that wheelset also is more sensitive to dynamic yaw response than other components.

Conclusion

It is well known that hunting instability is eliminated by increasing the critical hunting velocity and semi-active
Figure 2. Dynamic response of railway carriage wheelset to lateral displacement at critical hunting velocity (76 Km/h).

Figure 3. Dynamic response of railway carriage bogie frame to lateral displacement at critical hunting velocity (76 Km/h).

Figure 4. Dynamic response of railway carriage carbody to lateral displacement at critical hunting velocity (76 Km/h).

Figure 5. Dynamic response of railway carriage wheelset to Yaw displacement at critical hunting velocity (76 Km/h).

Figure 6. Dynamic response of railway carriage bogie frame to Yaw displacement at critical hunting velocity (76 Km/h).

Figure 7. Dynamic response of railway carriage carbody to Yaw displacement at critical hunting velocity (76 Km/h).
Figure 8. Phase portrait trajectories of wheelset dynamic response to yaw displacement at critical hunting velocity (76 km/h).

Figure 9. Phase portrait trajectories of bogie frame dynamic response to yaw displacement at critical hunting velocity (76 km/h).

Figure 10. Phase portrait trajectories of carbody dynamic response to yaw displacement at critical hunting velocity (76 km/h).

Figure 11. Dynamic response of railway carriage wheelset to lateral displacement at forward railway carriage velocity (76 Km/h).

Figure 12. Dynamic response of railway carriage bogie frame to lateral displacement at forward railway carriage velocity (76 Km/h).

Figure 13. Dynamic response of railway carriage carbody to lateral displacement at forward railway carriage velocity (76 Km/h).
suspension is used to improve the hunting phenomenon of railway carriage. The simulation model represents the dynamic response of railway carriage components due to lateral and yaw displacement in which lateral dynamic response of wheelset is more sensitive to critical hunting velocity than other components and displacements. So longitudinal primary semi-active suspension is adopted and the magnitudes of primary longitudinal suspension stiffness Kpx is considered as a function of lateral displacement in limited range and the magnitudes of Kpx are made to increase when lateral displacement of wheelset reaches a certain magnitude. Figures 11-16 show the dynamic response of railway carriage components using longitudinal primary semi-active suspension with lateral displacement indicator due to lateral and yaw displacement at forward railway carriage speed (76 Km/h). The Figures show that using semi-active longitudinal stiffness suspension as a function of lateral displacement is able to increase critical hunting velocity with good results whereas (Mohan, 2003) presented good results with yaw angle indicator but it is observed that the railway model using lateral threshold as an indicator for longitudinal primary semi-active suspension will improve the critical hunting velocity and gives good results than the model with yaw threshold indicator because of the difference in degrees of freedom for the two models. It is concluded that hunting velocity of railway carriage is increased and hunting phenomenon is improved by using semi-active longitudinal primary suspension as a function of lateral displacement indicator but with limited range since high magnitudes of suspension stiffness makes the suspension rigid and hunting phenomenon is not improved then. Finally the satisfied percentage gained according to use longitudinal semi-active suspension with lateral threshold indicator can be deduced by running the constructed simulation model and increase the railway forward velocity starting from 76 Km/h until reaches the new critical hunting velocity which is 93 Km/h with 0.005 m lateral threshold so the improvement percentage gained about 19%.

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