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Transfer matrix technique for determining the resonance conditions in retrieving stuck drill pipes with a top vibratory suspended drive

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A stuck drill pipe has been recognized as one of the most costly and non-productive challenges in drilling operations. Fishing jars are routinely used to un-lock or loosen the stuck (jammed) pipes which in many cases are expensive and the time taken to complete the job can reach several days of continuous jarring. The use of surface mounted vibratory systems has offered an alternative cost effective means to free the stuck pipes. Almost all of these systems are based on eccentric-weight oscillators which impart simple harmonic vertical forces that are transmitted down the pipe via elastic standing waves through the pipe material. A more recent development also uses a suspended oscillator but imparts a sinusoidal oscillatory displacement (rather than force) to the drill pipe at the top surface end, which again is transmitted down the pipe via elastic standing waves. This paper provides a generalized technique for solving the governing equations describing this top oscillatory system and the transmission of the elastic waves along the drill pipe. The transfer matrix technique is used to describe the travelling/standing waves along the pipe, the connecting couplings and the top suspended drive system. Effects of damping are introduced in the complex wave number and at the coupling locations. Examples of drill pipe scenarios are presented to elucidate the usefulness of the technique to determine the resonance condition, that is, the excitation frequencies for maximum retrieving forces at the stuck end, for any given drill pipe geometry. The resulting force amplitudes at the top driver end and the resulting retrieving forces imparted at the stuck end are quantified for any given imposed displacement amplitude at the drive end. A more complex system involving a drill pipe, spear and an elastic liner is also described where the transfer matrix technique is demonstrated to be an effective means to determine the overall system dynamics and resonance conditions.

Key words: Drilling, drill pipe, spear, liners, solid elastic dynamic, elastic waves.

INTRODUCTION

Drilling or fishing jars have been known and used almost since the start of the drilling industry (Gonzalez, 1987). They are classified as mechanical or hydraulic jars, the operations of which are similar in that they both deliver

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approximately the same impact blow. Jars are designed to be reset by simple string manipulation and are capable of repeated operation or firing before being recovered from the well. As such, they require downhole tool intervention such as spears, over-shots, taper taps, wash-over pipes, etc. Jarring operations may require anywhere from a few to sometimes thousands of impacts to release a fish, and the total time involved for a successful jarring operation can reach over 50 h of continuous impacts (Gonzalez et al., 2007, 2009). Therefore, operations involving jarring usually last days, sometimes weeks, resulting in a considerable loss of productive rig time (Scolfield et al., 1992).

An alternative method of freeing a stuck drill pipe is by means of surface mounted vibratory systems, which perhaps originated in the 1940s, and probably stemmed from the use of vibration to drive pilings. The early use of vibration for driving and extracting piles was confined to low frequency operation; that is, frequencies less than the fundamental resonant frequency of the system and consequently, although effective, the process was only an improvement on conventional hammer equipment. Early patents of this concept are by Bodine (1961, 1987, 1993) which introduced the concept of resonant vibration that effectively eliminated the reactance portion of mechanical impedance, thus leading to the means of efficient sonic power transmission. Another patent along the same concept is by Vogen (1986). The first published work on this technique was outlined by Gonzalez (1987) and was demonstrated by Baker Oil Tools (1994). It is based on surface mounted vibratory systems, whereby eccentric-weight oscillators impart simple harmonic vertical forces that are transmitted down the pipe via elastic standing waves through the pipe material. A derivative of this concept is a suspended oscillator but imparts a sinusoidal oscillatory displacement (rather than force) to the drill pipe at the top surface end, which again is transmitted down the pipe via elastic standing waves.

The present paper provides a generalized technique in solving the governing equations describing these top oscillatory systems and the transmission of the elastic waves down the drill pipe. The transfer matrix technique is used to describe the travelling/standing waves along the pipe, the connecting couplings and the top suspended drive system. Effects of damping are introduced in the complex wave number and at the coupling locations. Examples of drill pipe scenarios are presented to demonstrate the usefulness of the technique to determine the resonance condition, that is, the excitation frequencies for any given drill pipe geometry.

DESCRIPTION OF ELASTIC WAVE MOTION IN LONG RODS

The governing equation for the elastic wave motion in a long, thin rod and the basic propagation characteristics will be described. Consider a straight, prismatic rod of a cross–sectional areas *S* as shown in Figure 1. The coordinate *x* refers to an axial distance along the rod, while *u*(*x,t*) represents the longitudinal displacement at location *x* and time *t*.

The equation of motion applied to the differential element (*dx*) can be written as (Graff, 1975):

$$\rho\frac{\partial^2 u}{\partial t^2} = -\frac{\partial \sigma}{\partial x} + q - \tau\, D \tag{1}$$

where: *D* = pipe (or rod) outside diameter; *q* = body force per unit volume of the pipe material; *t* = time; *x* = axial distance; *u* = displacement; ρ = density of the rod material; $\sigma = $ stress (positive when compressive); τ = external shear force

Assuming the material behaves elastically and follows a simple linear Hooke’s law (Mead, 1975 and Gei, 2010):

$$\sigma = -E\,\varepsilon = -E\frac{\partial u}{\partial x} \tag{2}$$

Where: ε = strain (positive in the x-direction); σ = stress (positive when compressive); *E* = elastic modulus of the rod material.

The negative sign in Equation (2) is imposed because the stress (σ) is defined as positive when compressive. Substituting Equation (2) in Equation (1), we get:

$$\frac{\partial^2 u}{\partial t^2} = c_o^2\frac{\partial^2 u}{\partial x^2} + q - \tau\, D \tag{3}$$

Where, c_o is the speed of elastic wave in the rod material defined as:

$$c_o = \sqrt{\frac{E}{\rho}} \tag{4}$$

TRANSFER MATRIX TECHNIQUE

Now, we will introduce the concept of the transfer matrix [T.M.] technique to facilitate the solution of the above wave equation for elastic rods, and extend it to mechanical systems involving mass-spring-damping.

Transfer matrix for a uniform section of elastic rod

If we neglect for a moment the body force (*q*) and the external damping force (τ) in Equation (3), it reduces to the fundamental wave equation, namely:

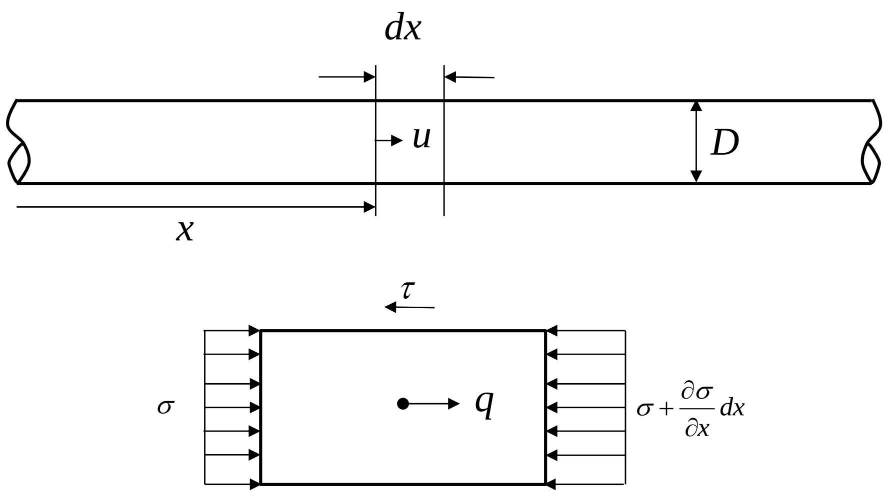

Figure 1. Definition of parameters along an elastic thin rod, and compressive stresses acting on a differential element (*dx*).

$$\frac{\partial^2 u}{\partial t^2} = c_o^2 \frac{\partial^2 u}{\partial x^2} \tag{5}$$

or

$$\frac{\partial^2 \sigma}{\partial t^2} = c_o^2 \frac{\partial^2 \sigma}{\partial x^2} \tag{6}$$

The form of solution of Equation 5 for the displacement *u*can be obtained by the method of separation of variables (Arfken, 2005) in that it can described as a product of a function *X(x)* which depends only on the distance (*x*) and a harmonic function $e^{i\omega t}$ which depends on time and frequency ω (where ω = *2πf*; *f* is the frequency in Hz), that is,

$$u(x,t) = [a\, e^{-ik_o x} + b\, e^{+ik_o x}]e^{iwt} \tag{7}$$

Likewise, the stress (*s*) and the velocity $v(x,t) = \frac{\partial u}{\partial t}$ can also be expressed by similar functions in the form:

$$\sigma(x,t) = [a\, e^{-ik_o x} + b\, e^{+ik_o x}]e^{iwt} \tag{8}$$

and

$$v(x,t) = \frac{1}{\rho_o c_o}[a\, e^{-ik_o x} - b\, e^{+ik_o x}]e^{iwt} \tag{9}$$

Where: $k_o = \omega / c_o$ is the wave number.

Introducing the compressive force (f = *σS*), where *S* is the cross-sectional area of the rod, Equations (8) and (9) can be written as:

$$\mathrm{f}(x,t) = [a\, e^{-ik_o x} + b\, e^{+ik_o x}]e^{iwt} \tag{10}$$

And

$$v(x,t) = \frac{1}{\rho_o c_o S}[a\, e^{-ik_o x} - b\, e^{+ik_o x}]e^{iwt} \tag{11}$$

The term $\rho_o c_o S = Z_o$ is known as the mechanical characteristic impedance of the elastic rod.

Now, we shall introduce the concept of the transfer matrix [T.M.]. The transfer matrix relates the force and velocity amplitudes at two stations (1) and (2) along a straight rod as shown in Figure 2, in the form of a 2x2 matrix:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{12}$$

Where the 2x2 matric on the R.H.S. of the above equation is called the transfer matrix whose elements (*A, B, C* and *D*) are all complex numbers. They can be obtained by writing Equations (10) and 11 at the two stations (1) and (2), and substituting *x=0* at station (1) and *x=L* at station (2), and solving for the constants a and b in terms of (F_1, V_1), and (F_2, V_2). Here (F_1, V_1), and (F_2, V_2) are the amplitude of compressive force and velocity oscillations at stations (1) and (2), respectively, which are also complex numbers, that is,

Figure 2. Relationship between amplitudes of force and velocity at different stations along elastic rod.

$$F_1(x=0,t)=F_1\,e^{iwt};\;F_2(x=L,t)=F_2\,e^{iwt} \qquad (13)$$

and

$$v_1(x=0,t)=V_1\,e^{iwt};\;v_2(x=L,t)=V_2\,e^{iwt} \qquad (14)$$

After some arithmetic manipulation, and replacing the exponential terms by trigonometric functions, we obtain:

$$\begin{bmatrix}F_1\\V_1\end{bmatrix}=\begin{bmatrix}\cosh(kL) & Z_o\sinh(kL)\\ \frac{1}{Z_o}\sinh(kL) & \cosh(kL)\end{bmatrix}\begin{bmatrix}F_2\\V_2\end{bmatrix} \qquad (15)$$

Where: $k=ik_{\circ}=i\dfrac{\omega}{c_o}$

The 2x2 matrix on the RHS of Equation (15) is the [T.M.] of a straight section of elastic rod (drill pipe), which is expressed as:

$$[T.M.]_{rod}=\begin{bmatrix}\cosh(kL) & Z_o\sinh(kL)\\ \frac{1}{Z_o}\sinh(kL) & \cosh(kL)\end{bmatrix} \qquad (16)$$

The above $[T.M.]_{rod}$ is very useful in facilitating analysis of the dynamic response of a drill pipe subjected to oscillatory force or displacement at the top end, while the bottom end is stuck as depicted in the simple drill pipe schematic shown in Figure 3. Here, station (1) is the top end of the drill pipe while station 2 is the bottom (stuck) end. In this case, the boundary condition at station (2) is V_2=0. Hence, from Equations (12) and (15), the mechanical impedance at station (1) is:

$$Z_1=\frac{F_1}{V_1}=(\frac{A}{C})_{from\,Eq.11}=[Z_o\coth(kL)]_{from\,Eq.14} \qquad (17)$$

Equation (17) is plotted as function of frequency of the driving oscillatory system on top for drill pipe parameters shown in the Figure. With the aid of the [T.M.] expression of Equation (15), it can be shown that when the top end of the drill pipe is excited by an oscillatory force of amplitude F_1, the force amplitude exerted at the bottom (stuck) end, F_2, is maximum when impedance Z_1 is at minima, e.g. at frequencies = 0.41, 1.23 Hz, etc. in the example of Figure 4. That is when the length of the drill pipe (L) is equal to odd multiples of $\lambda/4$ (where λ is the wave length = c_o/f). Conversely, when the top is excited by an oscillatory displacement (or velocity of amplitude V_1, the force amplitude exerted at the bottom (stuck) end, F_2, is maximum when impedance Z_1 is at maxima, e.g. at frequencies = 0.82, 1.64 Hz, etc. also in the example of Figure 4. That is when the length of the drill pipe (L) is equal to even multiples of $\lambda/2$.

Figure 5 shows the force amplitudes (F_1 and F_2) resulting from exciting the top end of a drill pipe with an oscillatory displacement of amplitude X_1 = 0.0254 m (1 in), that is, $V_1 = i\omega X_1$. Since the bottom end is assumed stuck, V_2 = 0, and hence Equation (15) can be solved for the amplitudes of forces (F_1 and F_2) shown in Figure 5. The maximum force amplitude at the stuck end of the drill pipe is realized when the displacement excitation frequency at the drive end is 0.81 Hz, that is, when Z_1 is at maximum. Conversely, if the top end is excited with an oscillatory force (as in the case of an eccentric rotating weight), the maximum ratio of force amplitudes (F_2/F_1) is realized at f = 0.41 Hz, that is, when Z_1 is at minimum. Note that the magnitude of F_2 at this frequency is simply equal to $i\omega X_1 Z_0$, according to Equation (15) since *sinh(kL)* = 1. Note also the overall trend of increasing the amplitude of the force F_2 with frequency which is due to the fact that V_1 is linearly increasing with frequency for the same amplitude of displacement X_1.

Finally, the effect of damping due the shear force (τ) in Equation (1) can be accounted for in the transfer matrix solution of the wave equation via introducing a real parameter (α) in the complex wave number as a damping parameter, or a damping coefficient (ξ) in the form:

$$k=(\alpha+ik_{\circ})=\frac{\omega}{c_o}(\xi+i) \qquad (18)$$

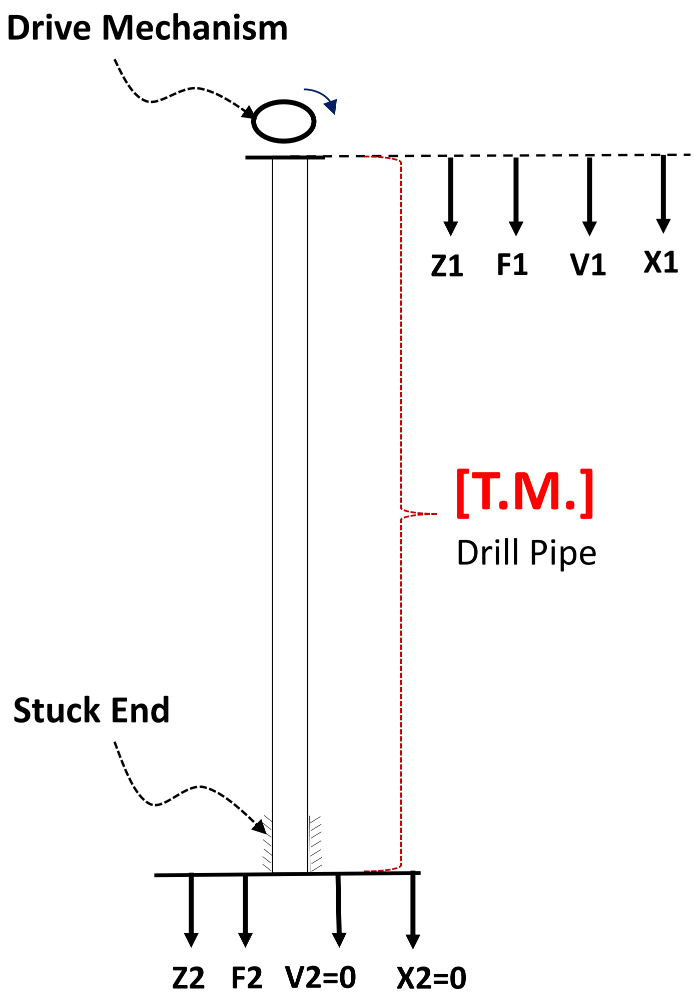

Figure 3. Simple drill pipe stuck at the bottom end and a vibrator at the top end.

Drill Pipe Parameters:		
Drill Pipe O.D.	5.5 in	0.1397 m
Drill Pipe W.T.	0.415 in	0.010541 m
Drill Pipe I.D.	4.67 in	0.118618 m
Cross-Sectional Area	6.62962 in^2	0.004277 m^2
Length of Drill Pipe	10000 ft	3048 m
Elastic Modulus	28275 kpsi	195 GPa
Density	485.917 lb/ft^3	7800 kg/m^3
Speed of Elastic Wave	16374.8 ft/s	5000 m/s

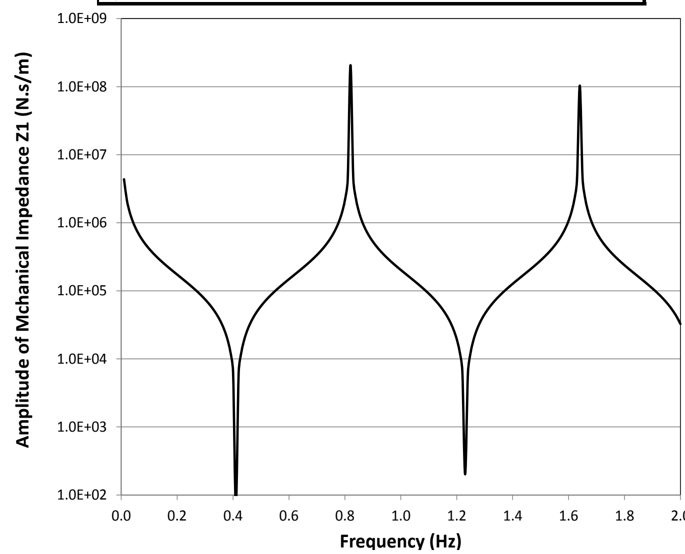

Figure 4. Example of amplitudes of mechanical impedance at the top end of a stuck drill pipe (Ideal System, that is, no damping, no coupling).

Transfer matrix for a coupling with inherent stiffness and damping

Typically, drill pipe segments are connected via couplings which affect the transmission and reflection characteristics of the elastic wave motion in the connected segments. Therefore it is necessary to derive a [T.M.] for these couplings to be combined with the [T.M.] of the respective connecting drill pipe segments (Lin, 1962). Figure 6 shows one type of these coupling where it can generally be represented by a mass-spring-damping system. The quest here is to develop a [T.M.] relating the forces and velocities at stations (2) and (3) in the form:

$$\begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (19)$$

The equation of motion for the system depicted in Figure 6 can be written as (Harris and Crede, 1976):

$$F_2 - F_3 = (i\omega m)\, V_1 \quad (20)$$

Where;

$$F_3 = -i\frac{k}{\omega}(V_2 - V_3) + c(V_2 - V_3) \quad (21)$$

And with some mathematical manipulation we get:

$$\begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{i\omega m}{(c - \dfrac{ik}{\omega})} & (i\omega m) \\ \dfrac{1}{(c - \dfrac{ik}{\omega})} & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (22)$$

Hence the transfer matrix for the coupling element of this type is:

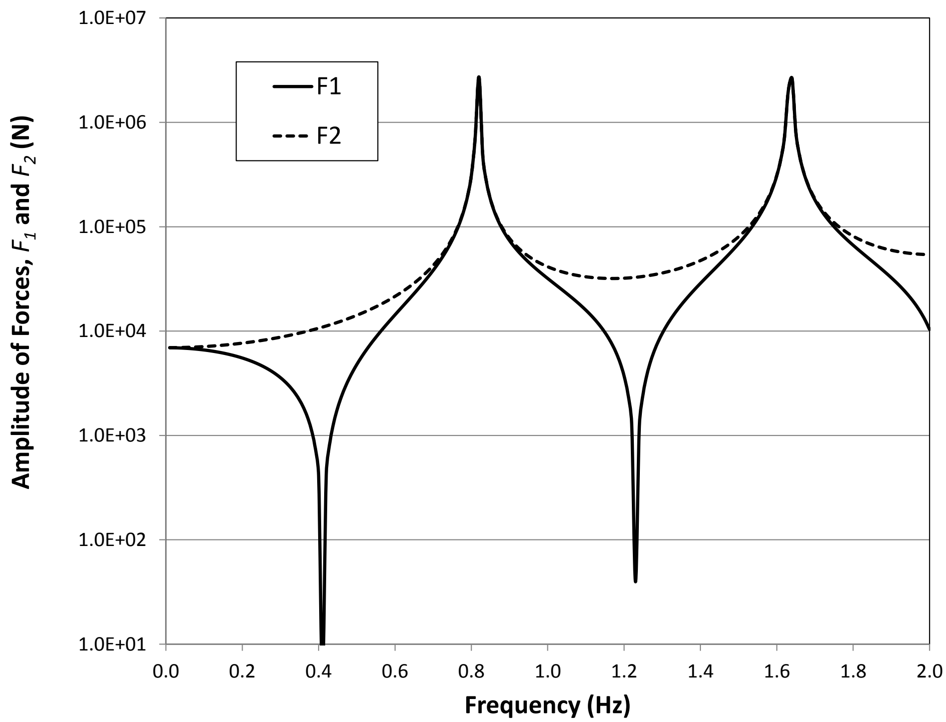

Figure 5. Amplitude of forces at the top and the stuck ends of the ideal drill pipe of Figure 3 (Top end is subjected to oscillatory displacement of amplitude = 0.0254 m).

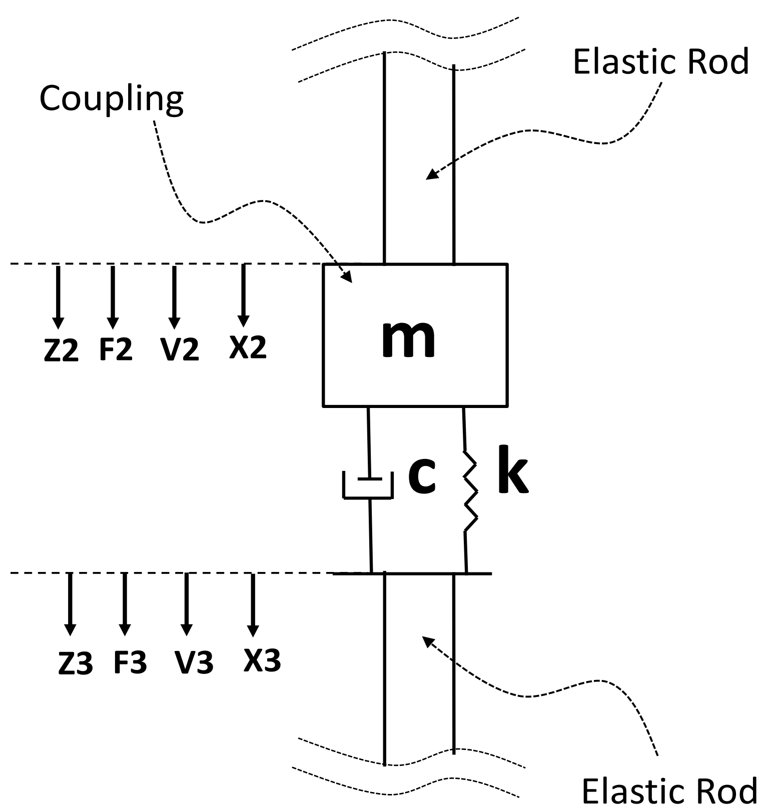

Figure 6. Mass-spring-damping system representing a coupling between two segments of a drill pipe (Type I: Coupling has inherent damping and stiffness).

$$[T.M.]_{coup-1} = \begin{bmatrix} 1+\dfrac{i\omega m}{(c-\dfrac{ik}{\omega})} & (i\omega m) \\ \dfrac{1}{(c-\dfrac{ik}{\omega})} & 1 \end{bmatrix} \quad (23)$$

The above [T.M.] is also applied to elements of the top drive as will be shown later.

It is interesting to note two special cases that can be deduced from the above expression for $[T.M.]_{coup-1}$. The first is the case when station (3) is fixed, that is, $V_3 = 0$. In this case, the impedance Z_2 is reduced to:

$$Z_2 = \frac{A_2}{C_2} = (c + i\omega m - \frac{ik}{\omega}) \quad (24)$$

Which is the well-known expression for the impedance of a mass-spring-damping system mounted on a fixed foundation. The other case is when station (3) is free, that is $F_3 = 0$, hence the impedance Z_2 is reduced to:

$$Z_2 = \frac{B_2}{D_2} = i\omega m \quad (25)$$

Which is the simple equation of motion of the mass element alone.

Transfer matrix for a coupling subjected to external stiffness and damping

Another type of couplings is shown schematically in Figure 7, where the coupling is actually in contact with the borehole which will introduce damping and stiffness between the coupling mass and the surrounding ground. The relating forces and velocities at stations (2) and (3) can be written as follows (Harris and Crede, 1976):

$$F_2 - F_3 = (c + i\omega m - \frac{ik}{\omega})\, V_1 \quad (26)$$

With the condition that:

$$V_2 = V_3 \quad (27)$$

The above two equations can be put in the [T.M.] format as follows:

$$\begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & (c + i\omega m - \dfrac{ik}{\omega}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (28)$$

Hence the transfer matrix for the coupling element of this type is:

$$[T.M.]_{coup-2} = \begin{bmatrix} 1 & (c + i\omega m - \dfrac{ik}{\omega}) \\ 0 & 1 \end{bmatrix} \quad (29)$$

Overall transfer matrix

Having established the transfer matrices for the fundamental elements of a drill pipe, namely: $[T.M.]_{rod}$, $[T.M.]_{coup-1}$ and $[T.M.]_{coup-2}$, it is now possible to determine the overall transfer matrix of a drill pipe system comprising all of these elements. Consider the drill pipe shown in Figure 8 where the drill pipe is composed of three segments connected with couplings of the two types (I and II) as shown.

It can be shown that the relationship between the force and velocity amplitudes at the top end (1) and the bottom end (2) can be expressed via the following overall transfer matrix expression:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{overall} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = [T.M.]_{overall} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (30)$$

Where, the overall transfer matrix, $[T.M.]_{overall}$ is determined from multiplication of the individual transfer matrices corresponding to the elements in the same order, that is:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = [T.M.]_{rod1}[T.M.]_{coup1}[T.M.]_{rod2}[T.M.]_{coup2}[T.M.]_{rod3}[T.M.]_{coup3} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (31)$$

That is:

$$[T.M.]_{overall} = [T.M.]_{rod1}[T.M.]_{coup1}[T.M.]_{rod2}[T.M.]_{coup2}[T.M.]_{rod3}[T.M.]_{coup3} \quad (32)$$

In the case of a stuck end (that is, at station 2), the impedance at the top end:

$$Z_1 = \frac{A_{overall}}{C_{overall}} \quad (33)$$

The example in the following section will illustrate the application of this overall transfer matrix technique in determining the resonance condition of a stuck drill pipe.

STUCK DRILL PIPE PROBLEM

Now let us consider a stuck drill composed of many

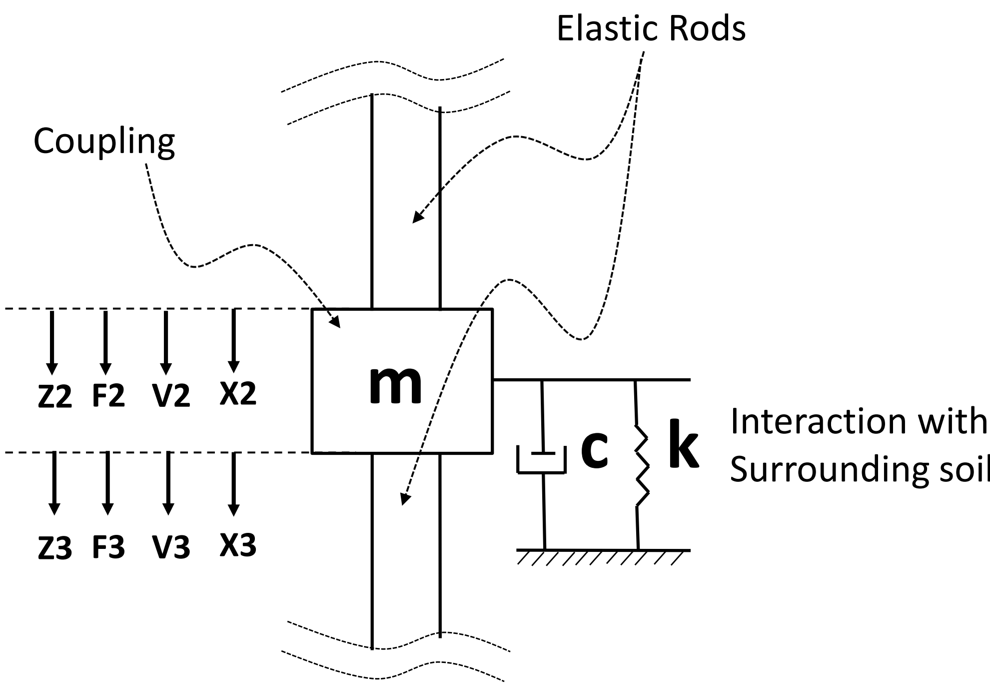

Figure 7. Mass-spring-damping system representing a coupling between two segments of a drill pipe (Type II: Coupling subjected to external damping and compliance.

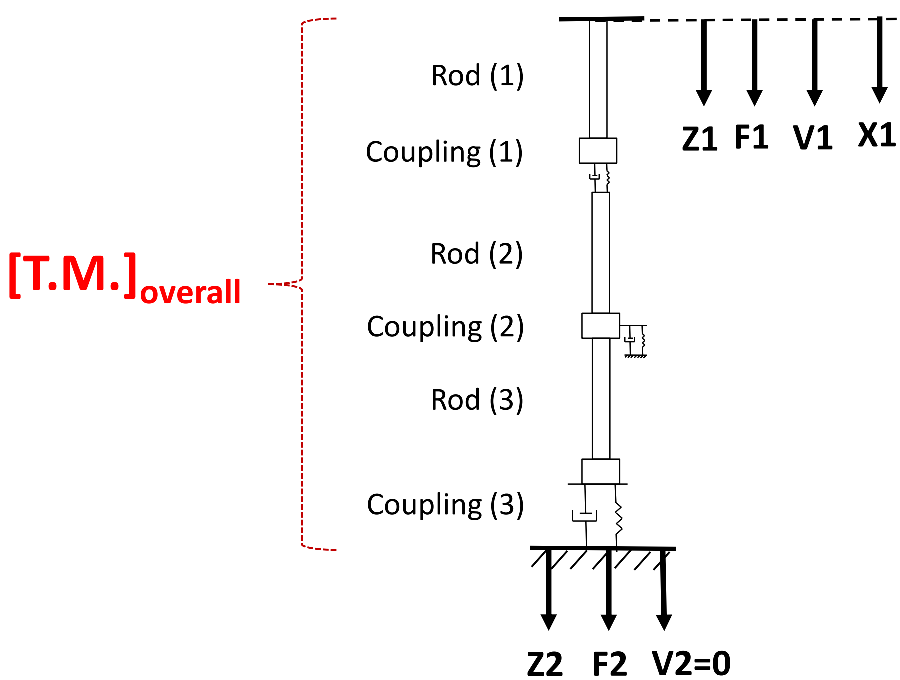

Figure 8. Example of drill pipe segments connected by Type I and Type II couplings.

segments of pipes (could be of different geometries and materials), and couplings of type I or type II connecting these segments as shown on the L.H.S. of Figure 9. Again, the drill pipe is assumed stuck at the bottom end. In order to free this end, a cable-suspended drive is used to generate an oscillatory displacement in a manner such that:

$$X_1 + X_2 = Amp \tag{34}$$

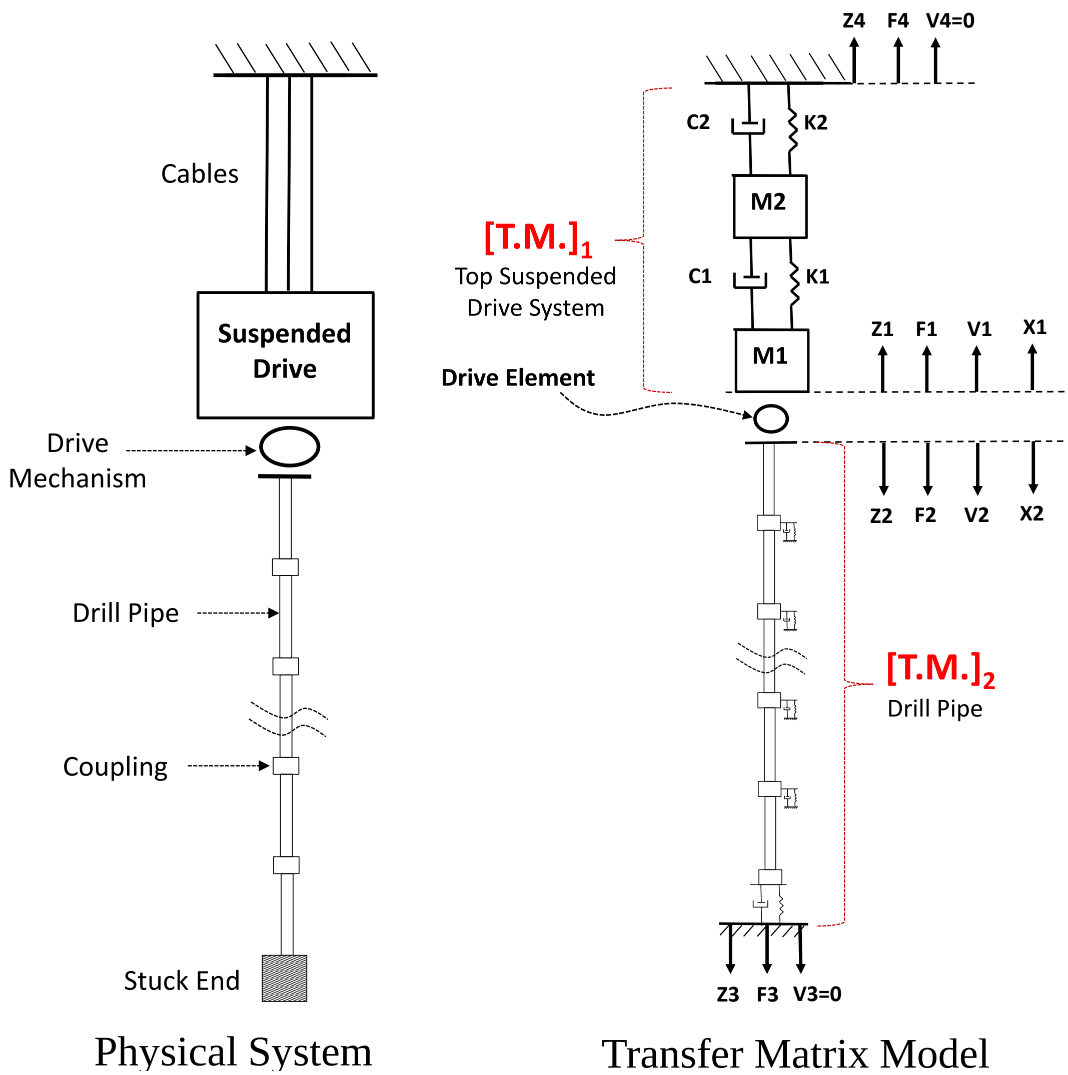

Figure 9. Schematic of the physical system and transfer matrix model depiction of a stuck drill pipe.

Where: *Amp* is a specified amplitude of displacement, X_1 is the amplitude of upward displacement of the suspended drive, while X_2 is the amplitude of displacement of the top end of the drill pipe. The sign convention depicted in Figure 9. The suspended drive can be considered as a two-degree of freedom system as shown on the model schematic on the R.H.S. of Figure 9.

Following the above formulation of the overall transfer matrix, it is possible to obtain both $[T.M.]_1$ and $[T.M.]_2$ by a simple multiplications of the [T.M.]'s corresponding to all of the sub-elements in each, and again, in the correct order, that is,

$$[T.M.]_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \; ; \; [T.M.]_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \tag{35}$$

The corresponding impedances (Z_1 and Z_2) can also be determined since $V_3 = V_4 = 0$. That is:

$$Z_1 = \frac{A_1}{C_1} \; ; \; Z_2 = \frac{A_2}{C_2} \tag{36}$$

Now, substituting $X=V/i\omega$ in Equation (34), we get:

$$\frac{V_1}{i\omega} + \frac{V_2}{i\omega} = \frac{1}{i\omega}\left(\frac{F_1}{Z_1} + \frac{F_2}{Z_2}\right) = Amp \tag{37}$$

And since $F_1=F_2$, Equation (37) becomes:

$$F_1 = F_2 = \frac{i\omega \, Amp}{\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)} \tag{38}$$

Once F_1 is determined, all other parameters (F_3, V_1, V_2, X_1 and X_2) are determined at any given excitation

Table 1. Parameters of the stuck drill pipe system and suspended drive shown in Figure 9.

System parameters	Values
Suspended mass (M1)	27273 kg
Damping (C1)	160000 N.s/m
Stiffness (K1)	7022190 N/m
Suspended mass (M2)	9091 kg
Damping (C2)	16000 N.s/m
Stiffness (K2)	98310666 N/m
Drill pipe O.D.	0.1397 m
Drill pipe I.D.	0.118618 m
Drill pipe X-Area	0.004277 m^2
Drill pipe overall length	3048 m
Weigh per unit length (including Coulings)	37 kg/m
Drill pipe damping coefficient (ξ)	0.05
Damping at coupling (C)	1600 N.s/m
Stiffness at coupling (K)	0 N/m
Amplitude of *X1 + X2 =Amp*	0.0254 m

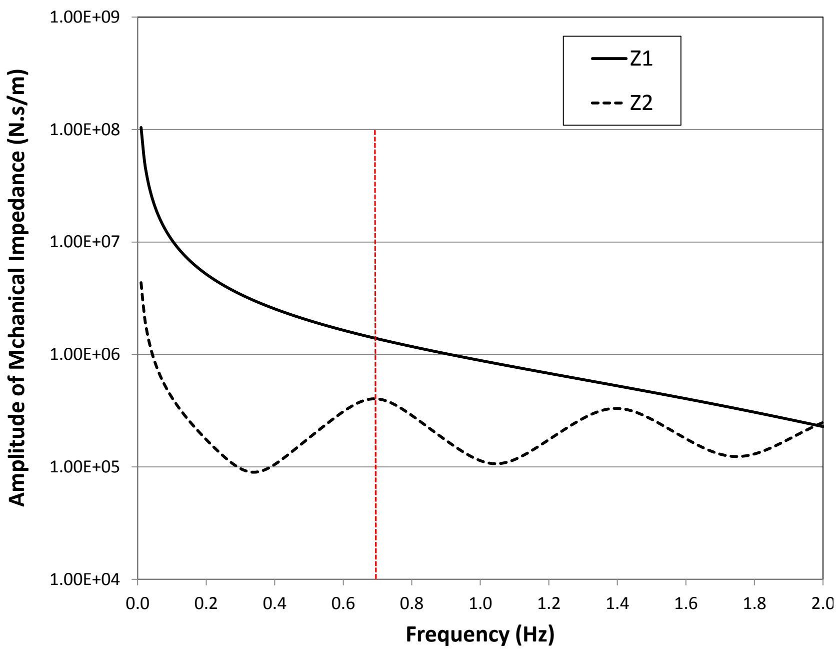

Figure 10. Results of the mechanical Impedances (Z_1 and Z_2) of the drill pipe system shown in Figure 9.

frequency. Table 1 gives values for the relevant parameters for an example drill pipe stuck at a depth of 3048 m (10,000 ft). The drill pipe is divided into 100 segments and a coupling of Type II is placed between each consecutive segments whose mass is combined with 1/3 of the mass of the preceding segment. The resulting impedances Z_1 and Z_2 are shown in Figure 10, the amplitudes of displacements X_1 and X_2 are shown in Figure 11, and the amplitude of the driving force, F_1, and the retrieving force at the stuck end, F_3, in Figure 12.

The magnitude of Z_1 is generally higher than that of Z_2 (Figure 10), which is desirable as it indicates that it is '*easy'* to displace down the top end of the drill pipe than to push up the massive suspended drive. In other words, the mobility (which is the inverse of the impedance) of the drill pipe top end is much greater than the suspended drive. This is manifested in the resulting displacement amplitudes X_1 and X_2 in Figure 11. Note also that at the

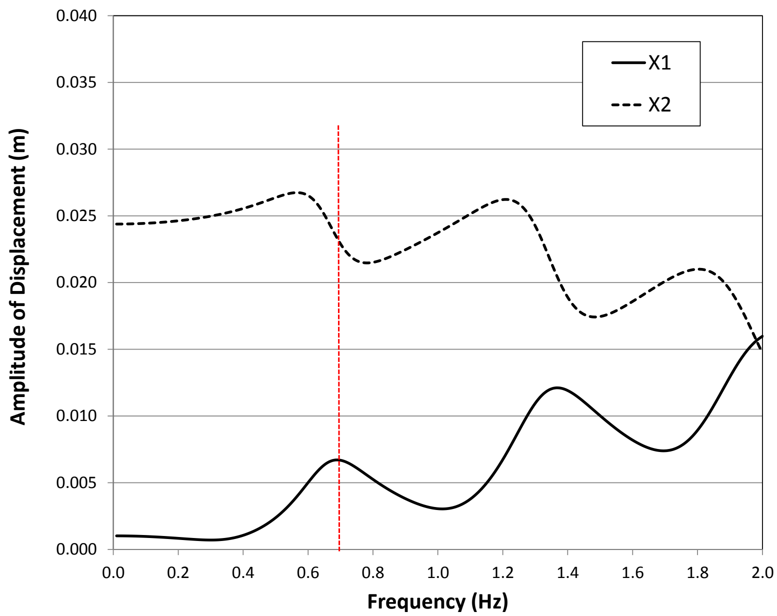

Figure 11. Results of the displacements (X_1 and X_2) of the drill pipe system shown in Figure 9.

desired drive frequency of 0.69 Hz (corresponding to the first maximum impedance in Figure 10), the amplitude of X_1 = 0.0067 m while the amplitude of X_2 = 0.0242 m.

It probably begs the question as to why the frequency at maximum impedance Z_2 is now lower (0.69 Hz) than that in the case of Figure 4 (0.82 Hz) despite the fact that the drill pipe length is the same in both cases (that is, 3048 m). The main reason is that in the present case damping was introduced along the drill pipe (ξ = 0.05) as well as a damping parameter, c, to all of the the couplings. Additionally, the weight of the drill pipe segments and couplings were accounted for in the analysis of this example, while in the example of Figure 4, it was clearly stated that the drill pipe was clear of any damping (that is, ideal). Damping is also manifested in decreasing the force amplitude F_3 than F_2 as shown in Figure 12.

A MORE COMPLEX PROBLEM OF A STUCK LINER

Let us now consider a more complex geometry of a case involving a drill pipe in a liner, where the liner is stuck at the bottom end as depicted in the schematic of Figure 13. The corresponding transfer matrix model system is also shown on the R.H.S of Figure 13. The drill pipe is assumed stuck at the bottom end, while it is rigidly supported at the top surface. To free the liner, the same cable-suspended drive is used to generate an oscillatory displacement at the top end of the drill pipe in a manner similar to the last example, that is, Equation 34. Similarly, it is possible to obtain the overall transfer matrices $[T.M.]_1$, $[T.M.]_2$, $[T.M.]_4$ and $[T.M.]_5$ via simple multiplications of the respective [T.M.]'s corresponding to each of the sub-elements in each sub-system shown in Figure 13, that is,

$$[T.M.]_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \; ; \; [T.M.]_2 = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$
$$[T.M.]_4 = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix} \; ; \; [T.M.]_5 = \begin{bmatrix} A_5 & B_5 \\ C_5 & D_5 \end{bmatrix} \qquad (39)$$

The corresponding impedances (Z_1 and Z_4 and Z_5) can also be determined from:

$$Z_1 = \frac{A_1}{C_1} \; ; \; Z_4 = \frac{A_4}{C_4} \; ; \; Z_5 = \frac{A_5}{C_5} \qquad (40)$$

The condition at the spear is such that:

$$F_3 = F_4 - F_5 \;\; and \;\; V_3 = V_4 = -V_5 \qquad (41)$$

It follows that:

$$Z_3 = Z_4 + Z_5 \qquad (42)$$

Hence:

$$Z_2 = \frac{A_2 Z_3 + B_2}{C_2 Z_3 + D_2} \qquad (43)$$

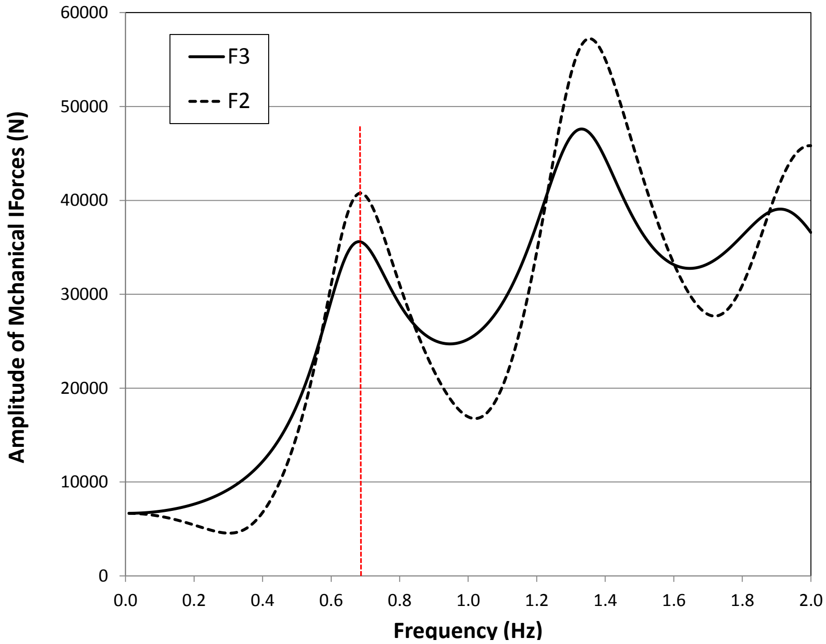

Figure 12. Results of the displacements (F_1 and F_3) of the drill pipe system shown in Figure 9.

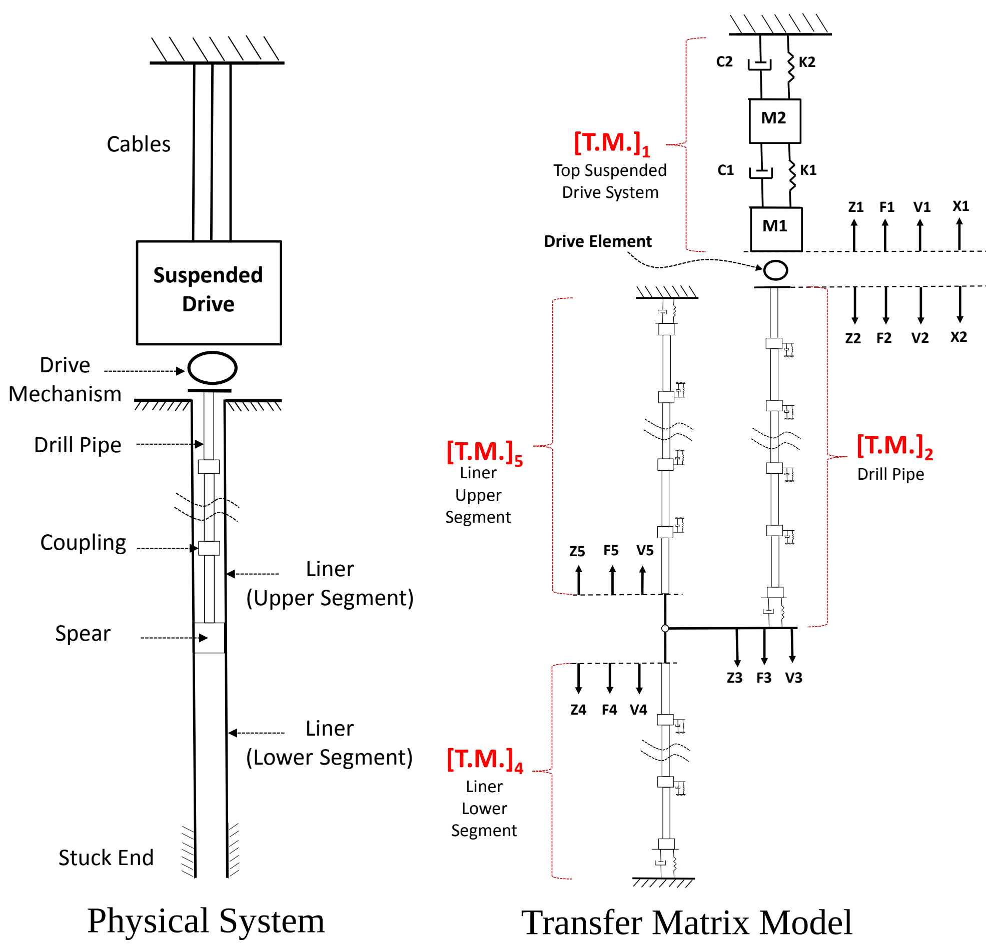

Figure 13. Schematic of the physical system and transfer matrix model depiction of a stuck liner.

Equation 38 is applied to determine the force F_1 and all other amplitudes of impedances, forces, velocity and displacements. It should be remembered that the parameters described in Equations 33 through 43 are all complex numbers.

DISCUSSION

The transfer matrix technique [T.M.] is demonstrated to be a very powerful and useful technique to describe any complex drill pipe/liner dynamic response to a top surface oscillatory drive to retrieve a stuck bottom end of the drilling assembly. The general form of the [T.M.] is a 2×2 matrix whose elements are generally complex numbers. For a drill pipe, the [T.M.] accounts for the length of the drill pipe, its cross sectional area, material properties and effective damping. The [T.M.] for couplings and surface drive elements are derived from the dynamic response of generally mass-spring-damping system. The elements of these 2×2 [T.M.] are also expressed in complex numbers.

Once the [T.M.] corresponding to each element in the system is formulated, the overall system [T.M.] can be obtained. For example elements connected in series (such as drill pipe segments connected via couplings), the overall [T.M.] of the string of drill pipe will be a simple multiplication of the individual [T.M.]'s in the same order as connected. Sign conventions of amplitudes of impedance, force, velocity and displacement, which all are also complex numbers, should be observed in formulating the problem.

It was shown that damping affects the resonance frequency as well as the amplitude of the retrieving forces at the stuck end. Therefore, it is important to accurately quantify all possibilities of damping imposed on the system, whether it is from the ground soil contacting with the mechanical system, or inherent within the actual design of the element.

Finally, it was shown if the vibrator drive at the surface end is imparting an oscillatory force on the drill pipe at the top, the resonance condition for maximum force at the stuck end is corresponding to the minimum impedance at top end. Conversely, if the vibrator is imparting an oscillatory displacement at the top, the resonance condition for maximum force at the stuck end is corresponding to the maximum impedance at top end. Therefore, it is necessary to determine the length of the drill pipe to the stuck end so as to drive the top vibrator close to the resonance frequency for best results.

Nomenclatures: ***A, B, C, D***, elements of the 2x2 transfer matrix; ***Amp,*** amplitude of excitation displacement; ***c,*** damping parameter; ***c_o,*** speed of elastic wave in the drip pipe; ***D,*** pipe (or rod) outside diameter; ***E,*** elastic modulus; ***F,*** force amplitude; ***I,*** $i=\sqrt{-1}$; ***k,*** complex wave number or spring stiffness; ***k_o,*** wave number; ***L,*** length of drill pipe; ***m,M,*** mass; ***q,*** body force per unit volume of the pipe material; ***S,*** drill pipe cross-sectional area; ***t,*** time; **[T.M.,],** transfer matrix; ***u,*** displacement; ***v,*** velocity; ***V,*** velocity amplitude; ***X,*** axial distance; ***X,*** displacement amplitude; ***Z,*** impedance (=*F/V*); **A,** damping parameter; **ε,** strain (positive in the x-direction); ***F*,** excitation frequency (Hz); ρ, density of the rod material; **σ,** stress (positive when compressive); **τ,** external shear force; ω, excitation frequency (rad/s); **ξ,** damping coefficient.

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