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Identifying the flow dimension in fractured rock using an interference test

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The actual flow dimension in fractured rock may be between 2-D cylindrical flow and 1-D linear flow. This paper demonstrated the importance of the method used to identify the flow dimension in fractured rock correctly. A case study of a long-duration interference test was presented to illustrate the application of Barker's type curves and to identify a flow dimension in fractured rock. It was difficult to judge the flow dimension based solely on the type-curve matching. Both geological and production data could also be taken into account to help select the appropriate flow dimension. To investigate the sensitivities to pumping time for each observation well, the σ function was defined for an observation well equals the sum of the square relative-errors between the observed and predicted drawdowns. For a long-duration interference test with sufficient pumping time, the σ function of an observation well is sensitive to the flow dimension. An empirical rule, θ (dimensionless time) > 100, was developed for estimating the minimum pumping time required in an interference test to identify the flow dimension between the production well and an observation well in fractured rock.

Key words: Interference test, fractured rock, flow dimension.

INTRODUCTION

Analytical solutions and type curves for 2-D cylindrical flow and 1-D linear flow dimensions developed by Theis (1935) and Jenkins and Prentice (1982), respectively, are widely used for analyzing interference test data for estimating the aquifer parameters (Fan et al., 2005). To determine the aquifer parameters, it is usually assumed that the flow dimensions are predefined along with assumptions of homogeneity and isotropy before analyzing drawdown data. However, it is usually the circumstance that no presumption about the dimension of the flow system can be made (Chakrabarty, 1994). In addition, the calculated aquifer parameters would be varied with flow dimensions. Thus, hydrologically based approaches for determining the flow dimensions and aquifer parameters are greatly needed. Walker and Roberts (2003) indicated that the flow dimension is not necessarily a simple function of radial distance; the flow dimension and heterogeneity are inter-changeable when interpreting the flow dimension based on the assumption that hydro-geologic properties are function of radial distance. Chen and Liu (2007) pointed out the determination of apparent flow dimensions should consider all other knowledge of the system to construct a meaningful conceptual model of the system when commenting on the article by Walker and Roberts (2003).

While 2-D cylindrical flow dimension is most common, linear flow has been recognized in some fractured aquifers (Muskat, 1937; Jenkins and Prentice, 1982). Many studies recognized linear flow in the vicinity of a production well with buildup or drawdown data collected at the production well during short-duration tests (Gringarten et al., 1975). However, very little information is currently available for interference tests with sufficient pumping

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(a) Flow dimension (n) = 1

(Barker, 1988)

(b) 1 < Flow dimension (n) < 2

(c) Flow dimension (n) = 2

Figure 1. Flow dimensions, source boundary conditions and analytical solutions for fractured rock.

time to identify the flow dimension in fractured rock at an observation well. Jenkins and Prentice (1982) reported an interference test to identify the linear flow dimension in fractured limestone. However, Sen (1986) noted that the pumping time of Jenkins and Prentice's test was too short to identify the linear flow dimension at observation wells.

Barker (1988) developed a generalized radial flow model for hydraulic tests in fractured aquifers by regarding the dimension of the flow as a parameter. The objective of this paper was to demonstrate the application of Barker's generalized solution to identify the flow dimension correctly in fractured rock using a long-duration interference test in Chingshui geothermal field.

For exploring the flow dimension, it is important to determine the hydrogeological parameters with the flow dimension simultaneously. When analyzing drawdown data from the interference test, it is difficult to choose an appropriate flow dimension in a fractured formation system. The flow geometry may be considered as a 3-D spherical flow, 2-D radial flow, or 1-D linear flow according to the fracture density and isotropic/ anisotropic distribution. And the fractional flow dimension would exist with the variations of connectivity of the fracture system. spatial and temporal changes of flow dimension (Leveinen et al., 1998; Leveinen, 2000). To identify the flow dimension between the production well and an observation well, the σ function was utilized to further examine the curve-fitting through the sum of the square relative-errors between the observed and predicted drawdowns. This study focuses on 2-D areal flow; both geological and production data could also be taken into account to help select the appropriate flow dimension. The case study was further investigated to develop an empirical rule for determining the minimum pumping time required for an interference test using observation wells to identify the flow dimension in fractured rock at an observation well.

MATERIALS AND METHODS

The two flow dimensions of practical interest for fractured aquifers are radial flow and linear flow. In radial flow dimension, the flow lines are straight and converge in two dimensions (n = 2) toward a common center, for example, a well (Figure 1c). In linear flow dimension, the flow lines are parallel in one dimension (n = 1) and the cross section exposed to flow is constant (Figure 1a). Barker (1988) recognized that the actual flow dimension may be between radial and linear in fractured rock (1< n < 2; Figure 1b) and developed generalized Theis solution.

Theis solution - line source solution

Theis (1935) investigated unsteady radial flow problems in a confined aquifer with the following equation in terms of drawdown, *s* :

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t}$$
(1)

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subject to the following boundary and initial conditions:

$$s(r,t) = 0 \qquad \text{for } r \to \infty$$
$$\lim_{r \to 0} \left(r \frac{\partial s}{\partial r} \right) = -\frac{Q}{2\pi T} \quad \text{for } r \to 0$$

and

$$s(r,o) = 0 \qquad for \quad t = 0$$

The well-known Theis solution written in terms of the aquifer drawdown, ${\it S}$, is

$$s = \frac{Q}{4\pi T} \left[-Ei(-u) \right] = \frac{Q}{4\pi T} W(u)$$
⁽²⁾

where Q is the constant well production rate; T is the aquifer transmissivity; -Ei(-u) is the exponential integral; and W(u) is the well function; and $u = (r^2S)/4Tt$; where r is the distance between the pumping well and observation well, S is the storage coefficient, and t is the pumping time. Both u and W(u) are dimensionless. Well function, $_{W(u)}$, represents an exponential integral, -Ei(-u), that is,

$$W(u) = -Ei(-u) = \int_{u}^{\infty} \frac{e^{-\theta}}{\theta} d\theta$$
(3)

Generalized Theis solution

In order to extend Theis solution to a general flow dimension $(1 \le n \le 2)$, Barker (1988) defined an area factor of equipotential surfaces, α_n , for any flow dimension, n. Using Theis assumptions, Barker (1988) derived a generalized flow equation in term of drawdown, s, expressed as follows:

$$S_{s}\frac{\partial s}{\partial t} = \frac{K}{r^{n-1}}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial s}{\partial r}\right)$$
(4)

where S_s is the specific storage of the fracture system; K is the hydraulic conductivity; n is the dimension of the fracture flow system; r is the radial distance from the centre of the source; t is the well production time. For the constant-rate condition, equation (4) could be subjected to the following boundary and initial conditions:

$$s(r,t) = 0 \qquad \text{for } r \to \infty$$
$$\lim_{r \to 0} (Kb^{3-n}r^{n-1}\frac{\partial s}{\partial r}) = -\frac{Q}{\alpha_n} \quad \text{for } r \to 0$$

and

$$s(r,o) = 0 \qquad for \quad t = 0$$

$$s(r,o) = 0 \qquad \qquad for \quad t = 0$$

where Q is the constant well production rate; $\alpha_n = 2\pi^{n/2} / \Gamma(n/2)$; b is the extent of the flow region; $\Gamma(x)$ is the gamma function.

The generalized Theis solution of drawdown, S, at any radial distance, r, from the center of the source and at any time, t, was solved using Laplace transform by Barker (1988) as follows:

$$s(r,t) = \frac{Qr^{2\nu}}{4\pi^{1-\nu}Kb^{3-n}} \Gamma(-\nu,u)$$
 (5)

or

$$s(r,t) = \frac{Qr^{2\nu}}{4\pi^{1-\nu}Tb^{2-n}} \Gamma(-\nu,u)$$
 (5a)

$$\nu = 1 - \frac{n}{2} \tag{6}$$

$$u = \frac{S_s r^2}{4Kt} \tag{7}$$

or

$$u = \frac{S r^2}{4 T t}$$
(7a)

where S is storage coefficient; T is aquifer transimissivity; $\Gamma(-\nu, u)$ is the incomplete gamma function and can be calculated as follows:

$$\Gamma(a,u) = \Gamma(a) - \sum_{m=0}^{\infty} \frac{(-1)^m u^{a+m}}{m!(a+m)} \qquad \text{for} \quad a \neq 0, -1, -2...$$
(8)

In metric units, equations (5a) and (7a) can be written as follows:

$$\Gamma(-v,u) = W(\frac{n}{2}-1,u) = \frac{4\pi^{1-\nu}Tb^{2-n}}{Qr^{2\nu}} s$$
(9)

$$u = \frac{Sr^2}{4(60)Tt} \tag{10}$$

where Q is in m³/min; T is in m²/min; s is in meter; r is in meter; b is in meter; S is dimensionless; and t is in hour. $\Gamma(-v,u)$ was regarded as generalized well function,



Figure 2. Barker's type curves for various flow dimensions (n = 1, 1.25, 1.5, 1.75, and 2).

$$W(\frac{n}{2}-1,u)$$
 (Barker, 1988).

Using the generalized Theis solution, equations (8), (9), and (10), Barker (1988) also constructed a family of type curves on a log-log plot for various flow dimensions using *n* as a parameter. The flow dimension, *n*, is not necessarily an integer and must be determined empirically (Barker, 1988). Figure 2 shows five type curves with various n values (that is, n = 1, 1.25, 1.5, 1.75, 2). The type curves of n = 2 and n = 1 are line source solution for radial flow dimension (Theis, 1935) and planar source solution for linear flow dimension (Jenkins and Prentice, 1982), respectively.

RESULTS

Case study

Taiwan is located at the western rim of the Circum-Pacific margin, a convergent and compression boundary between the Philippine Sea and Eurasian Plates. The Chingshui geothermal field is located in the Northeast portion of Taiwan. It can be imaged that Chingshui geothermal area is a fracture-dominated aquifer. Production in the liquid-dominated Chingshui geothermal field is largely from a fractured formation. An interference test was conducted for the initial assessment of Chingshui geothermal reservoir to determine the transmissivity and storage coefficient for estimating deliverability and reserves (Chang and Ramey, 1979). Figure 3 is a scale

map showing both surface and bottom-hole locations of these wells. Also, it is evident that the subsurface fracture dominated the movement (N-S direction) of borehole while drilling. Furthermore, The Chingshui geothermal area is situated on a monocline structure, which is cut internally by numerous thrust faults that essentially trend parallel to the bedding (NE-SW) and are lightly curved; the most important ones are the Tashi, Hsiaonanao and Hanhsi faults. To identify the flow dimension, the geological data shows a possible linear flow dimension that the fractured formation dominated the flow system in subsurface.

During the aquifer test, the well 16T was produced, and pressure responses were observed in wells 4T, 5T, 9T, 12 T, 13T, and 14T. A complete set of aquifer data is presented in Table 1. Table 2 summarizes the well capacity and completion for wells 4 T, 9 T, 12 T, 14 T, and 16 T. The hot-water production rate of the well 16T measured by weir ranged from 80 to 84 tons/hour during the eleven-day aquifer test. Equivalent to 80 tons/hour of hot-water, the total production rate of well stream was 1.89 m³/min. The test was conducted by observing wellhead pressures at the observation wells. The aquifer data for wells 5 T and 13 T did not appear to be reliable because of some malfunction of equipment.

The test data can be analyzed by means of curve fitting using a family of Barker's type curves for various flow dimensions (n = 1, 1.25, 1.5, 1.75, and 2). Table 3 shows



Figure 3. Location of the wells and the isotherms of presumed formation temperature for 1500 m depth in the Chingshui geothermal area (Chang and Ramey, 1979).

that the calculated values of aquifer transmissivity and storage coefficient (T and S) obtained for each observation well depended on the flow dimension selected for type-curve matching. However, it is difficult to judge the flow dimension based solely on the type-curve matching. Both geological and production data must also be considered to help select the flow dimension.

The geological data also strongly indicate a possible linear flow dimension on a field-wide scale. Since the porosity and permeability of slates (aquifer rock) are low, faults, joints, and other extensive fractures provide the conduits for the fluid flow. Figure 4 shows the rose diagram for 67 joints measured at an outcrop of the aquifer rock located near the Chingshui geothermal field (Tseng, 1978). The most predominant set of joints strikes a linear direction of Northwest.

In addition, the capacity of a well depends on a number of factors such as transmissivity, well skin, and well completion. According to Darcy's law, under similar conditions of well completion and skin, well capacity is proportional to the transmissivity. The transmissivity estimated for various flow dimensions was compared and correlated to well capacities to select the appropriate flow dimension. An idea of linear correlation between well capacity and transmissivity was presented in this work to demonstrate the positive relationship. A production test was strongly suggested for each well to identify the well capacity. The well capacities could be further utilized to verify the flow dimension.

Figure 5 illustrates regression lines of well capacity versus transmissivity using data from wells 4T, 9T, 12T, and 14T for five various flow dimensions (n = 1, 1.25, 1.5, 1.75, and 2). The smallest value of the sample correlation squared regression coefficient (that is, $R^2 = -0.4319$) corresponding to the radial dimension (n = 2) indicates that the two parameters are not well correlated. On the other hand, coefficient is the highest (i.e., $R^2 = 0.9013$) for the linear flow dimension (n = 1). According to the results



Figure 4. Rose diagram of 67 joints in the Chingshui geothermal area (Tseng1978).

shown in Figure 4, the transmissivity estimated using the linear flow dimension (n = 1) appeared to best correlate with well capacity, whereas the correlation was the worst for the radial flow dimension (n = 2).

DISCUSSION

Method to identify the flow dimension in fractured rock

To identify the flow dimension and investigate the sensitivities to pumping time for each observation well, the σ function for an observation well was proposed as follows:

$$\sigma = \sum_{i=1}^{M} \left[\left(s_i - \frac{Qr^{2\nu}W(\frac{n}{2} - 1, u_i)}{4\pi^{1-\nu}Tb^{2-n}} \right) / s_i \right]^2$$
(11)

where s_i is the observed drawdown; $\frac{Qr^{2\nu}W(\frac{n}{2}-1,u_i)}{4\pi^{1-\nu}Tb^{2-n}}$

is the predicted drawdown; $u_i = Sr^2/4Tt_i$ for the *i* th data point; and *M* is the total number of data points at an observation well.

Figure 2 shows that the type curves of various flow dimensions are not discernible for a large value of u. Therefore, the σ function is not sensitive to the flow dimension for an observation well located too far away from the production well, or, for a short-duration interference test with insufficient pumping time. However, for a small value of u, the type curves of various flow dimensions separate from each other. For a long-duration interference test with sufficient pumping time, the σ function of an observation well is sensitive to the flow dimension. The correct flow dimension can be selected at the minimum value of the σ function.

Figure 6 shows the variations of the σ function with



Figure 5. Regression lines of well capacity versus aquifer transmissivity for various flow dimensions.

the flow dimension, n, for four observation wells (4T, 9T, 12T, and 14T) in Chingshui field. For the observation wells 9T and 14T which are 300 m and 330 m from the pumping well 16T, the σ function is not sensitive to the flow dimension. The total pumping time of the well 16T in Chingshui interference test was 258.5 h and was not long enough to determine the flow dimension in fractured rock at observation wells 9T and 14T because the distance from both observation wells is too far away from the pumping well 16T.

For the observation well 4T which is 175 m from the pumping well 16T, the σ function shows a little sensitivity to the flow dimension. For the observation well 4T, the pumping time of 258.5 h starts to reveal the flow dimension.

For the observation well 12T, which is the closest observation well and only 90 m from the pumping well 16T, the σ function is very sensitive to the flow dimension. The pumping time of 258.5 h is sufficient to identify the flow dimension between the observation well 12T and the



Figure 6. Variations of σ function with flow dimension for four observation wells (12T, 4T, 14T, and 9T).

Time (h)	Observation wells										Flowing well				
	4T			9T		12T		14T		16T					
	WHP*		S **	WHP		S	WH	Р	S	WH	Р	S	WH	Р	Weir water rate
	kg/cm ²	psi	m-H₂O	kg/cm ²	psi	m-H₂O	kg/cm ²	psi	m-H₂O	kg/cm ²	psi	m-H₂O	kg/cm ²	psi	(tons/h)
0	12.09	172	0.00	9.70	138	0.00	13.15	187	0.00	9.35	133	0.00	18.14	258	0
18.5	12.02	171	0.73	9.63	137	0.70	13.01	185	1.41	9.35	133	0.00	4.85	69	24
42.5	11.81	168	2.93	9.49	135	2.11	11.39	162	3.52	9.14	130	2.11	4.08	58	83.5
66.5	11.67	166	4.41	9.35	133	3.52	12.80	182	3.52	8.79	125	5.63	3.94	56	83.1
90.5	11.67	166	4.41	9.14	130	5.63	12.66	180	4.92	8.79	125	5.63	3.94	56	83.1
114.5	11.60	165	5.14	9.14	130	5.63	12.59	179	5.63	8.65	123	7.03	3.94	56	82
138.5	11.53	164	5.87	9.14	130	5.63	12.52	178	6.33	8.51	121	8.44	3.94	56	82.4
162.5	11.53	164	5.87	9.07	129	6.33	12.44	177	7.03	8.44	120	9.14	3.80	54	82.4
186.5	11.46	163	6.61	9.00	128	7.03	12.37	176	7.74	8.37	119	9.85	3.80	54	81
210.5	11.39	162	7.35	8.93	127	7.74	12.30	175	8.44	8.37	119	10.55	3.73	53	80
234.5	11.39	162	7.35	8.93	127	7.74	12.30	175	8.44	8.23	117	11.25	3.66	52	80
258.5	11.32	161	8.08	8.86	126	8.44	12.30	175	8.44	8.09	115	12.66	3.66	52	80***

 Table 1. Aquifer test in Chingshui geothermal field (Chang and Ramey, 1979).

* WHP: Wellhead pressure. ** Δp : Pressure interference. *** Equivalent of well stream total production rate of 105 tons/h, or, 1.89 m³/min.

pumping well 16T in fractured rock.

The variation of the $\,\sigma\,$ function at the observation well 12T as shown in Figure 6 can be used to judge the $\,$ flow

dimension. The values of the σ function at the observation well 12T are 0.434, 0.208, 0.099, 0.071, and 0.056 for the flow dimension, n = 2, 1.75, 1.5, 1.25,

Table 2. Well capacity and completion data for wells in Chingshui geothermal field.

Well	Well capacity (tons/h)	Total depth TD (m)	Completion interval (m)	Temperature at TD (ºC)	Distance from well 16T (m)
4T	126.7	1505	498 - 1503	201	175
9T	74.0	2079	490 - 2074	205	300
12T	46.9	2003	1048 - 1998	223	90
14T	88.0	2003	947 - 1995	215	330
16T	116.2	3000	830 - 2990	225	-

Table 3. Aquifer transmissivity (T) and storage coefficient (S) estimated for various flow dimensions using type-curve matching.

	Observation wells						
Flow dimension —	4T	9T	12T	14T			
Transmissivity (m ² /min)							
1	280 × 10 ⁻³	195 × 10 ⁻³	136 × 10 ⁻³	235 × 10 ⁻³			
1.25	154 × 10 ⁻³	116 × 10 ⁻³	103 × 10 ⁻³	124 × 10 ⁻³			
1.5	826 × 10 ⁻³	102 × 10 ⁻³	65.8 × 10 ⁻³	65.1 × 10 ⁻³			
1.75	47.0 × 10 ⁻³	52.1 × 10 ⁻³	38.5 × 10 ⁻³	26.7 × 10 ⁻³			
2	36.1 × 10 ⁻³	33.1 × 10 ⁻³	34.6 × 10 ⁻³	18.0 × 10 ⁻³			
Storage coefficient							
1	6.72 × 10 ⁻³	5.14 × 10 ⁻³	9.79 × 10 ⁻³	3.21 × 10 ⁻³			
1.25	7.40 × 10 ⁻³	3.88 × 10⁻³	7.17 × 10 ⁻³	2.98 × 10 ⁻³			
1.5	5.95 × 10 ⁻³	2.45 × 10 ⁻³	13.0 × 10 ⁻³	1.88 × 10 ⁻³			
1.75	5.87 × 10 ⁻³	2.50 × 10 ⁻³	18.5 × 10 ⁻³	1.47 × 10 ⁻³			
2	5.02 × 10 ⁻³	1.75 × 10 ⁻³	1.99 × 10 ⁻³	1.65 × 10 ⁻³			

and 1, respectively. The value of the σ function attains a minimum with a linear flow dimension (n = 1). Therefore, the flow dimension between the production well 16T and the observation well 12T can be identified as linear (n = 1). The validity of identifying the linear flow dimension between wells 16T and 12T is strongly supported by the geological data.

An empirical rule for estimating the minimum pumping time

It is also of practical interest to be able to estimate how long the test duration is required to determine the flow dimension between the production well and an observation well in fractured rock. The field data of Chingshui aquifer test could provide opportunities to identify the flow dimension in fractured rock, and be investigated further to develop an empirical rule for determining the minimum pumping time required to identify the flow dimension in fractured rock.

For a long-duration interference test with sufficient

pumping time, the type curves of the generalized well function become discernible among various flow dimensions when the value of dimensionless time $(\theta = \frac{4Tt}{Sr^2})$ gets large enough, or, $u = \frac{Sr^2}{4Tt}$ gets small enough (Figure 2). Table 4 lists the calculated values of dimensionless time (θ) for observation wells 12T, 4T, 9T, and 14T using a pumping time of 258.5 h and a linear flow dimension. Based on Table 4, we tried to develop the following empirical rule for estimating the minimum pumping time required in aquifer tests to identify the flow dimension.

$$\theta_{\min} = \frac{4Tt_{\min}}{Sr^2} > 100 \tag{12}$$

where θ_{\min} is the minimum dimensionless pumping time; t_{\min} is the minimum pumping time required; r is the distance between the observation well and pumping well; T is the aquifer transmissivity; and S is the storage

Observation wells	Transmissivity, T*, (m²/min)	Storage coefficient, S*	Distance from pumping well 16 T r, (m)	Dimensionless time $\theta = \frac{4Tt}{Sr^2}$
12T	0.136	0.00979	90	106
4T	0.280	0.00672	175	84
9T	0.195	0.00514	300	26
14T	0.235	0.00321	330	41

Table 4. Calculated values of u for various observation wells in Chingshui aquifer test.

*Estimated aquifer parameters for flow dimension, n = 1. **production time of well 16T, t = 258.5 h.

coefficient. The proposed empirical rule would be useful for planning an interference test. Very little information is currently available for interference tests with sufficient pumping time to identify the flow dimension in fractured rock at an observation well. It is recommended to improve the concept of equation (12) when additional data sets become available.

Conclusions

1. The actual flow dimension in fractured rock may be between radial flow and linear flow. If the flow dimension in fractured rock is not identified correctly, there would be significant error in aquifer parameters estimated from the analyses of aquifer tests.

2. Barker's type-curves of generalized Theis solution can be employed to analyze drawdown data from an interference test in fractured rock. However, it is difficult to judge the flow dimension based solely on the type-curve matching. Both geological and production data must also be taken into account for selecting an appropriate flow dimension.

3. Very little information is currently available for interference tests with sufficient pumping time to identify the flow dimension in fractured rock at an observation well. The case study of Chingshui demonstrates that a long-duration interference test with sufficient pumping time can provide important information concerning the flow dimension between the production well and an observation well in fractured rock.

4. To identify the flow dimension in fractured rock, the σ function for an observation well to investigate the sensitivities to pumping time for each observation well was proposed. For a long-duration interference test with sufficient pumping time, the σ function of an observation well is sensitive to the flow dimension. The σ function of an observation well is not sensitive to the flow dimension for a short-duration interference test with insufficient pumping time.

5. An empirical rule was developed for estimating the minimum pumping time required for an interference test to identify the flow dimension in fractured rock at an observation well. The proposed empirical rule would be useful for planning an interference test. It is recommended to improve the concept of equation (12) when additional data sets become available.

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