Full Length Research Paper

Hydromagnetic forced flow between a rotating disc and a naturally permeable stationary porous disc saturated with fluid

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This paper considers the flow of a conducting viscous incompressible fluid between two non-conducting parallel discs, when the magnetic field was applied perpendicularly to the discs. The upper disc is in steady rotation, while the lower one is a stationary porous disc. The whole flow is divided into two regions: the free fluid region (between two parallel discs) and the porous region (this flow is of porous material). The approximate solutions are obtained by solving the Navier-Stokes equations in the free fluid region, and the Darcy's equations in the porous region with suitable boundary conditions at the interface. The effects of rotation, Hartmann number and forced parameter have been considered on the flow characteristics and are illustrated by graphs. The flow is essentially dominated by rotational effect as well as by the forced parameter.

Key words: Magneto-hydrodynamic (MHD), incompressible fluid, forced flow. MSC (2000): 76D10, 76W05, 80A20.

INTRODUCTION

Flow of a viscous fluid through and past a porous medium has been the subject of intensive studies in recent years because of its many engineering and scientific applications. The study of viscous flow near stationary or rotating discs has significant relevance to many applications for industrial devices. Many important applications have motivated studies involving complex geometries, often with through flow and heat transfer, cooling of gas turbines, turbo machinery, boundary layer control, cooling of turbine blades, cooling the skins of high speed aircraft designs, in extraction process of fluid from the porous ground and in lubrication of porous bearings. Probably for the first time, the flow due to an infinite plane disk, rotating with constant angular velocity was discussed by Karman (1921). Cochran (1934) integrated numerically the equations obtained by Karman and compared his results with that of Karman. Batchelor

(1951) and Stewartson (1953) applied these equations to the problem of steady flow between two infinite parallel plane discs, rotating at a finite distance apart. The flow due to a rotating disk of infinite radius with uniform suction at the disc has been discussed by Stuart (1954) and he obtained numerical solutions for small values of suction and asymptotic solutions for large values of suction. Rizvi (1962) examined the magneto hydro-dynamic flow over a single disk in the presence of weak magnetic field. The effects of an axial magnetic field on the flow about a rotating disk were studied by Kakutani (1962). Pande (1972) analysed a series solution for the effects of an axial magnetic field and suction (or injection) on the flow about an insulated rotating disk, when there is strong suction and a weak magnetic field. And Nath (1984) developed unsteady rotating flow over an infinite rotating disk with an applied magnetic field. Purohit and Bansal (1995) considered the flow of a viscous income-pressible electrically conducting fluid between a rotating and a stationary naturally permeable disk. Ariel (2002) discussed the numerical behavior of MHD flow near a

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rotating disk. Attia (2003) considered time varying rotating disk flow and heat transfer of a conducting fluid with suction or injection. Darcy (1937) initiated the theory of the flow through a porous medium. Joseph and Tao (1966) has analysed the coupled flow induced by the steady rotation of a naturally permeable disk saturated with fluid. The flow field is divided into two regions, namely (I) free fluid region, and (II) porous region, where the fluid flows through a porous medium. To link flows in the two regions, certain matching conditions are required at the interface of the two regions. This type of couple flows, with different geometries and with several kinds of matching conditions, has been examined by several authors. Khoo et al. (1998) discussed the flow between a rotating and a stationary disc. Steady flow between a rotating and a stationary naturally permeable disc had been studied by Verma and Bhatt (1975). Srivastava and Sharma (1992) studied the MHD flow and heat transfer of a porous medium of finite thickness. Steady viscous flow between two rotating naturally permeable discs had been discussed by Chauhan and Gupta (1999). Srivastava (1999) studied the flow in a porous medium induced by torsional oscillation of a disk near its surface. The flow of viscous incompressible fluid confined between a rotating disk and a porous medium was analyzed by Chaudhary et al. (2004). Sharma et.al (2007) studied forced flow of a conducting viscous fluid through a porous medium induced by a rotating disk with applied magnetic field. Recently, Dufour and Soret effects on unsteady MHD convective heat and mass transfer flow due to a rotating disk, has been investigated by Maleque (2010).

A few investigations have been reported in literature on the MHD flow of a viscous incompressible electrically conducting fluid between a rotating and a stationary naturally permeable disc. Hence, in the present analysis, it is proposed to study the flow of a conducting viscous incompressible fluid between two non-conducting parallel discs, when the magnetic field applied perpendicularly to the discs, is considered. The upper disc is in steady rotation while the lower one is a stationary porous disc.

MATHEMATICAL FORMULATION

We consider the motion of a viscous incompressible electrically conducting fluid confined between two parallel discs of infinite radius. They are placed at a distance h apart. The upper disc is rotating with uniform angular velocity Ω, while the lower disc is stationary and made up of a porous material upto a depth z = - h with an impermeable surface at the bottom. The whole region (- h $\leq$ z $\leq$ h) is divided into two regions namely free fluid region (0 $\leq$ z $\leq$ h) and the region made up of porous material (- h $\leq$ z $\leq$ 0). The cylindrical polar coordinates (r, θ, z) are being used with the origin at the centre of the lower disc and z-axis normal to the disc. A magnetic field of constant intensity $\vec{B}_0$ is applied perpendicular to the discs. The velocity components (u, v, w) in the free fluid region and (U_p, V_p, W_p), in the porous region, are taken to be in the

directions of (r, θ, z) respectively. The slip conditions suggested by Beavers and Joseph (1965) have been applied to the radial and transverse velocity components at the interface (z = 0).

The governing equations by Navier-Stokes equations for the steady magneto hydrodynamic flow in the free fluid region 0 $\leq$ z $\leq$ h are:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial P_1}{\partial r}+v\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right]-\frac{\sigma B_0^2 u}{\rho} \quad (1)$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=v\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}-\frac{v}{r^2}\right]-\frac{\sigma B_0^2 v}{\rho} \quad (2)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P_1}{\partial z}+v\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right] \quad (3)$$

and the equation of continuity is:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \quad (4)$$

The flow in porous region – h $\leq$ z $\leq$ 0 is governed by the Darcy's equations which are"

$$U_P=-\frac{K^*}{\mu}\frac{\partial P_2}{\partial r} \quad (5)$$

$$V_P=0 \quad (6)$$

$$W_P=-\frac{K^*}{\mu}\frac{\partial P_2}{\partial z} \quad (7)$$

and the equation of continuity is:

$$\frac{\partial U_P}{\partial r}+\frac{U_P}{r}+\frac{\partial W_P}{\partial z}=0 \quad (8)$$

where P_1, P_2 are the pressures in the free fluid and porous regions, respectively : ρ is the density; μ is the coefficient of viscosity; v is the kinematic viscosity and K^* is the permeability of the porous medium. The corresponding boundary conditions are:

$$\left.\begin{array}{l} z=h\,;u=ar\,,v=r\,\Omega\,,\;w=0\,,\dfrac{\partial P_1}{\partial z}=0 \\[2mm] z=0\,;P_1=P_2\,,w=W_P\,,e_{r\theta}=\gamma\,V_P\,,e_{rz}=\gamma\,(u-U_P) \\[2mm] z=-h\,;W_P=0\,,\dfrac{\partial P_2}{\partial z}=0 \end{array}\right\} \quad (9)$$

where

$$e_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{v}{r}\right), \ e_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}, \ \gamma = \frac{\alpha}{\sqrt{K^*}},$$

α is a constant depending upon the structure of porous material and a is a forced parameter. Following Batchelor (1951), we seek the solution of the equations (1) to (8) under the boundary conditions (9), in the following form:

$$\left.\begin{array}{l} u = r\,\Omega F(\zeta)\,, \ v = r\,\Omega G(\zeta)\,, \ w = (v\Omega)^{1/2}\,H(\zeta) \\[2mm] \dfrac{P_1}{\rho} = v\Omega P_{10}(\zeta) + \dfrac{1}{2}K\,\Omega^2\,r^2 \end{array}\right\} \quad (10)$$

and

$$\left.\begin{array}{l} U_P = r\,\Omega\,\overline{F}(\zeta)\,, \ W_P = (v\Omega)^{1/2}\,\overline{H}(\zeta) \\[2mm] \dfrac{P_2}{\rho} = v\Omega P_{20}(\zeta) + \dfrac{1}{2}\,\overline{K}\,\Omega^2\,r^2 \end{array}\right\} \quad (11)$$

On substituting equations (10) and (11) into the equations (1) to (8) of continuity and motion, we obtain the following set of equations, in non-dimensional form. In the free fluid region:

$$2R^{1/2}F + H' = 0 \tag{12}$$

$$F'' - FM^2 = R^{1/2}HF' + R(F^2 - G^2 + K) \tag{13}$$

$$G'' - GM^2 = R^{1/2}HG' + 2RFG \tag{14}$$

$$H'' = R^{1/2}(HH' + P_{10}') \tag{15}$$

and in the porous region:

$$\overline{F} = -\beta R\,\overline{K} \tag{16}$$

$$\overline{H} = -\beta R^{1/2}\,P_{20}' \tag{17}$$

$$2R^{1/2}\,\overline{F} + \overline{H}' = 0 \tag{18(}$$

where

$\zeta = \dfrac{z}{h}$; $R = \dfrac{\Omega h^2}{v}$, the rotational Reynold number; $\beta = \dfrac{K^*}{h^2}$, dimensionless permeability; $m = \dfrac{a}{\Omega R}$, dimensionless forced parameter; $M^2 = \dfrac{\sigma B_0^2 h^2}{\rho}$, the Hartmann number; and

primes denotes differentiation with respect to 'ζ'. The corresponding boundary conditions become:

$$\left.\begin{array}{l} \zeta = 1: F = mR\,, \ G = 1,\ H = 0 \\[2mm] \zeta = 0: H = \overline{H},\ P_{10} = P_{20},\ K = \overline{K}, G = 0 \\[2mm] \qquad F' = h\,\gamma\,(F - \overline{F}) \\[2mm] \zeta = -1: \overline{H} = 0 \end{array}\right\}$$

$$\tag{19}$$

SOLUTION

The Reynolds number R which is defined in terms of the angular velocity of the disc, is assumed to be small. Since the solution for a given ratio of angular velocities of the two discs is not unique for sufficiently high Reynolds number, therefore, the unknown functions can be expanded in ascending powers of R, in the following form:

$$[F(R,\zeta),\overline{F}(R,\zeta)] = R\,[F_1(\zeta),\overline{F}_1(\zeta)] + R^3[F_3(\zeta),\overline{F}_3(\zeta)] + \ldots$$

$$G(R,\zeta) = G_0(\zeta) + R^2 G_2(\zeta) + \ldots$$

$$[H(R,\zeta),\overline{H}(R,\zeta)] = R^{1/2}[R\{H_1(\zeta),\overline{H}_1(\zeta)\} + R^3\{H_3(\zeta),\overline{H}_3(\zeta)\} + \ldots$$

$$K = K_0 + R^2 K_2 + R^4 K_4 + \ldots$$

$$[P_{10}(R,\zeta),P_{20}(R,\zeta)] = R\,[P_{101}(\zeta),P_{201}(\zeta)] + R^3\,(P_{103}(\zeta),P_{203}(\zeta)] + \ldots \tag{20}$$

Substituting equation (20) into equations to (12) to (18) and collecting the coefficients of the like powers of R, we obtain the following set of equations, in the free fluid region : R^0 zeroth order term as:

$$\left.\begin{array}{l} F_1'' - F_1 M^2 = K_0 - G_0^2 \\[2mm] G_0'' - M^2 G_0 = 0 \end{array}\right\} \tag{21}$$

R^1 First order term as:

$$\left.\begin{array}{l} G_2'' - M^2 G_2 = 2F_1 G_0 + H_1 G_0' \\[2mm] P_{101}' = H_1'' \\[2mm] H_1' = -2F_1 \end{array}\right\} \tag{22}$$

R^2 second order term as:

$$F_3'' - F_3 M^2 = F_1^2 - 2G_0 G_2 + K_2 + H_1 F_1' \tag{23}$$

R^3 Third order term as:

$$\left.\begin{array}{l} H_3'' - H_1 H_1' = P_{103}' \\[2mm] \qquad H_3' = -2F_3 \end{array}\right\} \tag{24}$$

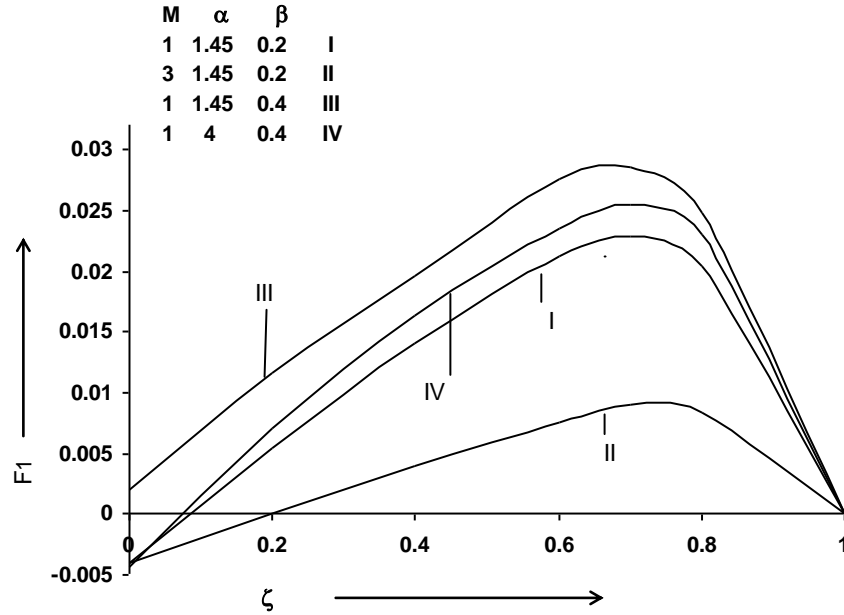| M	α	β	
1	1.45	0.2	I
3	1.45	0.2	II
1	1.45	0.4	III
1	4	0.4	IV

Figure 1. Radial velocity component $F_1(\zeta)$ vs. ζ for m=0.

and the equations in the porous region are

$$\left. \begin{array}{l} \overline{F}_1 = -\beta K_0 \\ \overline{H}_1 = -\beta P'_{201} \\ \overline{H}'_1 = -2\overline{F}_1 \end{array} \right\} \quad (25)$$

$$\left. \begin{array}{l} \overline{F}_3 = -\beta K_2 \\ \overline{H}_3 = -\beta P'_{203} \\ \overline{H}'_3 = -2\overline{F}_3 \end{array} \right\} \quad (26)$$

The corresponding boundary conditions are reduced to:

$$\left. \begin{array}{l} \zeta = 1 : F_1 = m, H_1 = 0, F_3 = H_3 = 0, G_0 = 1, G_2 = 0 \\ \zeta = 0 : G_2 = 0, H_1 = \overline{H}_1. H_3 = \overline{H}_3, P_{101} = P_{201}, P_{103} = P_{203}, \\ \quad\quad F'_1 = h\gamma(F_1 - \overline{F}_1), F'_3 = h\gamma(F_3 - \overline{F}_3) \end{array} \right\} \quad 27$$

$$\zeta = -1 : \overline{H}_1 = 0, \overline{H}_3 = 0$$

The solutions of the ordinary differential equations (21) to (27) are worked out subjected to the boundary conditions (28). We are not including solutions here for the sake of brevity of the paper. The important flow characteristics of the problem are further discussed.

RESULTS AND DISCUSSION

In the present paper, the forced flow of a viscous income-pressible electrically conducting fluid between a rotating and a stationary naturally permeable disc, under the application of a magnetic field acting perpendicular to the discs has been investigated. The whole flow field is divided into two regions; (i) free fluid region, and (ii) porous region. The flow in the free fluid region is governed by Navier-Stokes equations in the presence of magnetic field, while the flow in porous region is governed by Darcy's equations. The Reynolds number defined in terms of the angular velocities of the discs is assumed to be small. The effects of rotation, forced parameter and Hartmann number has been considered on the flow characteristics and illustrated by graphs.

The flow field behavior in the free fluid region and in the porous region under the presence of an applied magnetic field has been considered. The radial velocity component F_1 (zero[th] order term) versus distance ζ is shown in Figures 1 and 2 for m = 0, 0.1 and other parameters respectively. An examination of Figure 1 shows that the radial velocity component F_1 decreases in magnitude with the increase in α or Hartmann number M, whereas it increases by increasing β. The magnitude of the radial velocity component F_1 increases with increase in distance from lower disc, until it attains its maximum value, after which it decreases and it becomes zero at the upper disc. The radial velocity component F_1 takes its maximum value near the upper disc. Figure 2 shows that the magnitude of the radial velocity component F_1 decreases with the increasing α or M, where as it increases by increasing β. It increases when we move from lower disc to upper disc and it takes its maximum value at the upper disc. Transverse velocity is shown in Figure 3. The transverse velocity increases with increase in Hartmann number M, where as it decreases if we move towards the
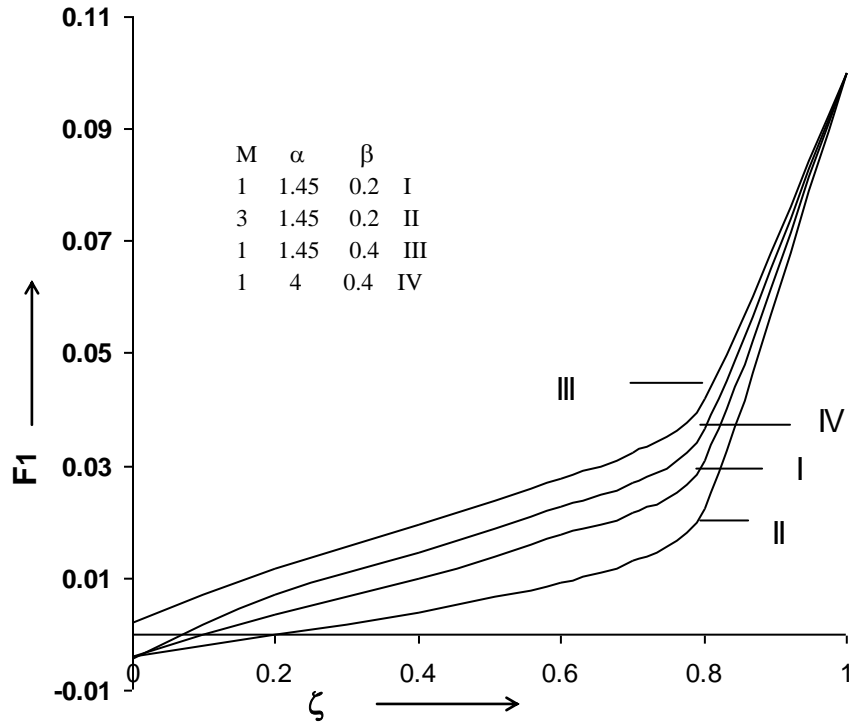
Figure 2. Radial velocity component F1(ζ) vs. ζ for m=0.1.
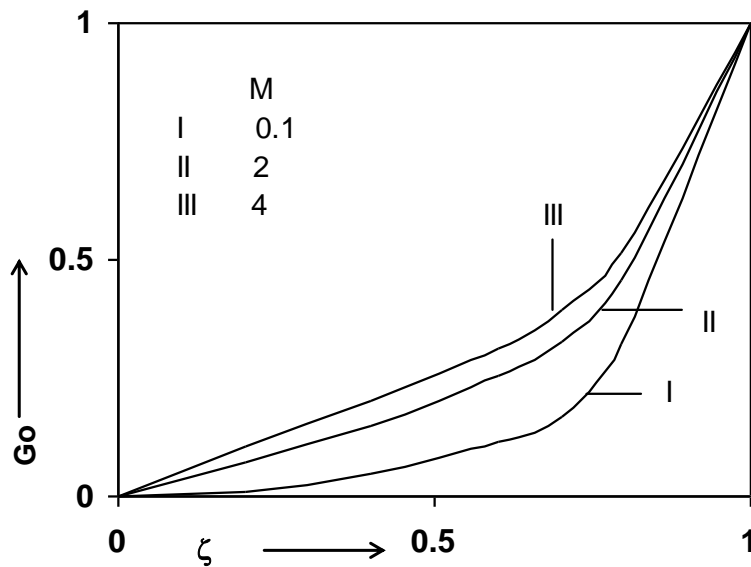

Figure 3. Transverse velocity component Go vs. ζ.

lower disc from upper disc and it becomes zero at the lower porous disc. The axial velocity component H_1 are drawn for different values of the parameters α, β, m and M in Figure 4. It is found that the axial velocity component H_1 decreases in magnitude with the increase in α or M where as it increases by increasing β or m. The magnitude of the velocity component H_1 increases with increase in distance in porous medium, until it attains its maximum value, after which it decreases and it becomes zero at upper disc. It is symmetrical about the axis $\zeta = 0$ (interface). As Reynolds number and forced parameter increase and Hartmann number decreases, more and
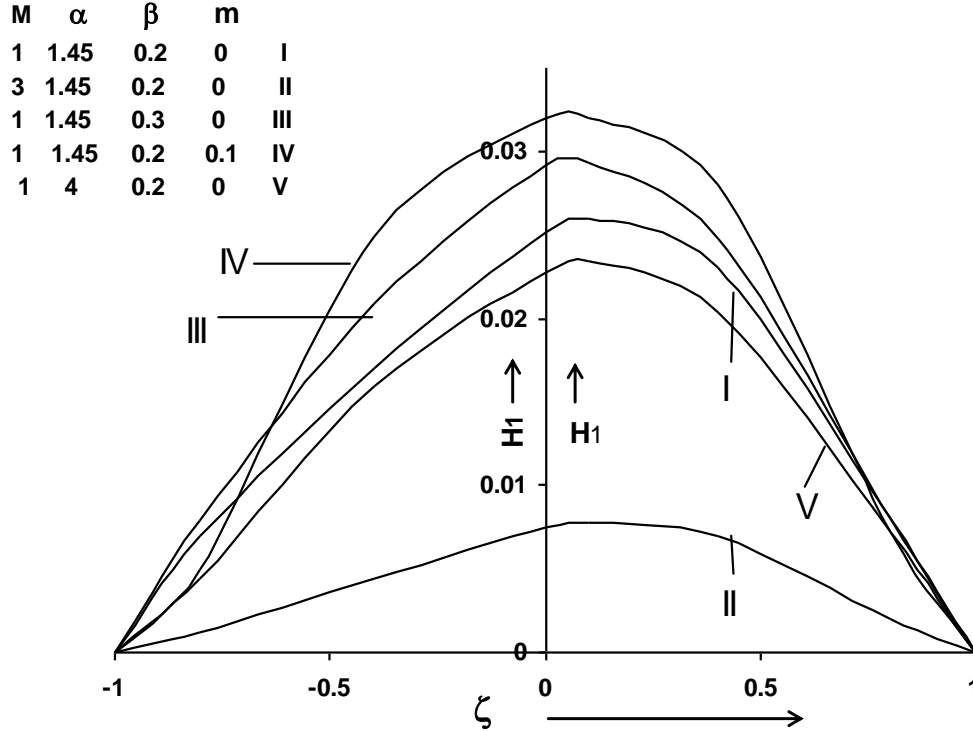
M	α	β	m	
1	1.45	0.2	0	I
3	1.45	0.2	0	II
1	1.45	0.3	0	III
1	1.45	0.2	0.1	IV
1	4	0.2	0	V

Figure 4. Axial velocity component $H_1(\zeta)$ vs. ζ.

more fluid is thrown out in the neighborhood of the upper disc and radial and axial velocity increases with the increasing of Reynolds number and forced parameter and decreasing of Hartmann number. Thus, magnetic field has a sobering effect on velocity distribution.

Stream functions of the flow

The stream functions ψ_1 and ψ_2 for the free fluid region and porous region respectively are given by:

$$\psi_1^*(\eta,\zeta) = \frac{\psi_1(\eta,\zeta)}{\frac{1}{2}h^2(\nu\Omega)^{1/2}} = \eta^2\, H(\zeta) \qquad (28)$$

$$\psi_2^*(\eta,\zeta) = \frac{\psi_2(\eta,\zeta)}{\frac{1}{2}h^2(\nu\Omega)^{1/2}} = \eta^2\, \overline{H}(\zeta) \qquad (29)$$

The streamlines are drawn in Figure 5, for R = 0.2, β = 0.2 and α = 1.45. We find that the fluid is thrown radially outwards due to the centrifugal forces, hence to fill the gap, the fluid rushes from infinity towards the axis in the stationary lower porous disc and comes out of the porous region to keep the continuity, consequently, as

compensation, the fluid is pumped out from the lower disc to maintain the flow. By introducing the forced flow, it is observed that the flux thrown radially outwards is more. As might be expected there is symmetry about the axis ζ = 0.

Skin-friction and torque on both disks

The coefficients of skin-friction are given by:

$$(C_f)_{\zeta=1} = \frac{\tau_1}{(\mu r_0\Omega/h)} = -\left(\frac{r}{r_0}\right)F'(1) \qquad (30)$$

and

$$(C_f)_{\zeta=0} = \frac{\tau_0}{(\mu r_0\Omega/h)} = \left(\frac{r}{r_0}\right)F'(0) \qquad (31)$$

Where $\tau_1 = -\left(\dfrac{\mu r\,\Omega}{h}\right)F'(1)$, is the shearing stress on

the upper disc, $\tau_0 = \left(\dfrac{\mu r\,\Omega}{h}\right)F'(0)$ is the shearing
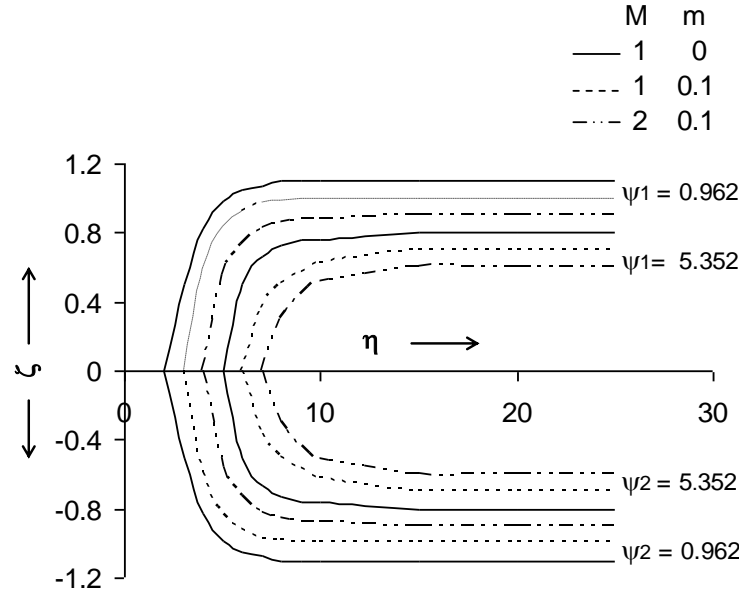
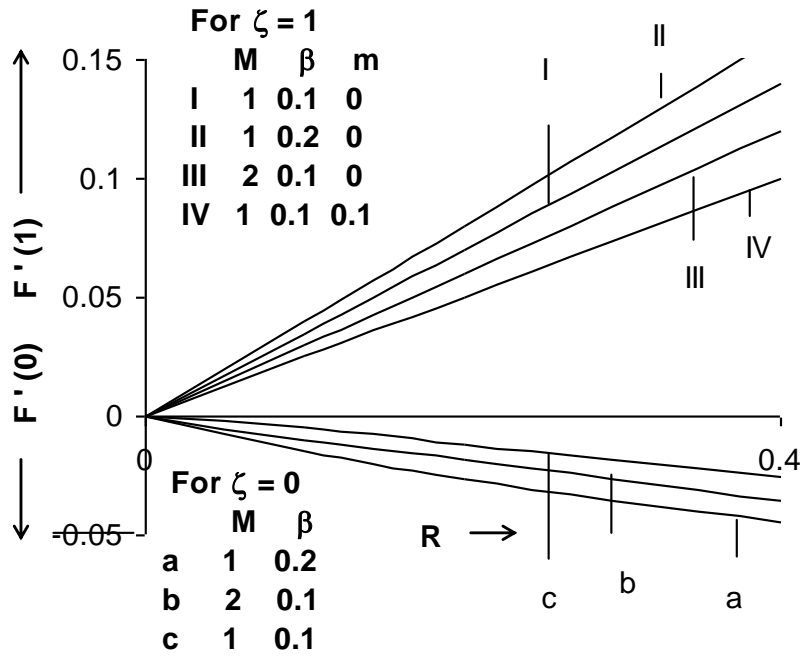**Figure 5.** Streamline pattern for R=0.2, α=1.45 and β=0.2.

Figure 6. Coefficients of skin friction F' (0) and F'(1) vs. R, for =1.45.

stress on the lower disc and r_0 is a certain distance in the radial direction from the centre of the disc.

Figure 6 shows the variations of coefficients of skin-friction. It is noted that the coefficient of skin-friction at the upper disc increases in magnitude with increase in R and decreases with increasing m or M. It increases with increasing β at the upper disc but it decreases with increasing β or R or M at the lower disc.

Conclusion

In this paper, the forced flow of a viscous incompressible electrically conducting fluid between a rotating and a stationary naturally permeable disc, under the application of a magnetic field acting perpendicular to the discs is studied. The following conclusions can be drawn as a result of the computations:

i. The flow is essentially dominated by rotational effect and as well as by the forced parameter.

ii. The radial velocity component F_1 decreases with the increasing α or M.

iii. The transverse velocity increases with increase in Hartmann number M.

iv. Axial velocity increases with the increasing of Reynolds number and forced parameter.

Nomenclatures: B_0, Uniform magnetic field; **K^*,** permeability parameter; **M,** magnetic field parameter (Hartmann number); **R,** rotational Reynold number; **m,** dimensionless forced parameter; **ν,** kinematic viscosity; **μ,** viscosity; **τ,** skin-friction (shearing stress); **σ,** scalar electrical conductivity; **ψ_1, ψ_2,** stream unctions; **C_f,** coefficient of skin-friction; **ρ,** density of the fluid; **P_1, P_2,** pressures in the free fluid and porous regions respectively; **α ,** a constant of structure porous material; **h,** distance; **u, v, w,** velocity components in the free fluid region in the **r, θ, z** directions; **U_P, V_P, W_P,** velocity components in the porous region in the r, θ, z-directions; **r, θ, z,** cylindrical polar coordinates; **Ω,** uniform angular velocity; **β,** non-dimensional permeability parameter; **ζ,** similar distance variable; **a,** forced parameter.

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