Journal of Soil Science and Environmental Management Vol. 1 (4), pp. 77-85, June 2010
Available online at http://www.academicjournals.org/JSSEM
ISSN 2141-2391 ©2010 Academic Journals

Full Length Research Paper

Application of queuing theory to the container terminal at Alexandria seaport

M. E. El-Naggar

Civil Engineering Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt.
E-mail: naggarmo@gmail.com.

Accepted 29 April, 2010

This paper describes a methodology designed to support the decision-making process by developing seaport infrastructure to meet future demand. In order to determine an optimum number of berths at a sea port, the queuing theory is applied in the light of port facilities and activities. The aim is to avoid inadvertent over and under-building. Within this methodology, the movements in port should firstly be analyzed. The waiting time of vessels outside the port and in queue is calculated in accordance with the considered queuing model. The theoretical functions representing the actual vessel arrival and service time distributions are determined. For the economic considerations, cost estimate studies including cost of port and waiting vessels are carried out. Finally, the optimum number of berths that minimizes the total port costs can be decided. Both proposed mathematical and economical models are applied to Alexandria port in Egypt.

Key words: Port capacity, port economy, port modeling, queuing theory.

INTRODUCTION

The port transportation system includes different physical elements, e.g. berths, handling equipments, storage and traffic facilities. Although the capacity of any single element may be expressed as an absolute figure, such as the number of containers loaded per hour by a certain crane, the aggregate capacity of the whole port cannot be so simply described. Each element can limit the overall port productivity.

Port productivity can be viewed from two standpoints. To ship operators, productivity implies the time needed at the port to serve ships, while at national level, port productivity can be defined as the amount of cargo transported through the port during a certain time period.

Port development is often affected by operating policies as well as by the traffic demand imposed in the port in terms of the volume of cargo expected to be accommodated, the service time at the available berths within which this volume should be handled, and the frequency of ships arrivals.

It would be possible to develop the port facilities so that its capacity is fully utilized at all times. In this manner, changes in demand have to be accommodated by forcing ships to wait (at anchorage) until ships that arrived previously had been serviced. This policy would be inefficient and uneconomic due to the delay costs of waiting ships. Conversely, developing the port so that ships are never forced to wait also represents an uneconomic use of port resources.

The ideal situation is one in which all berths are occupied at all times and no ship is ever kept waiting. This situation is impossible to achieve in practice because of the random arrivals of cargo ships and the variations in service time of ships of different sizes. Therefore, decisions concerning port development can be made by trading-off the cost of increasing the port capacity and the costs of both waiting and service times.

The purpose of this paper is to introduce a methodology which can be used to facilitate the decision-making process of port development. The proposed methodology covers two principal areas:

a) Investigation of the pattern of ship traffic at a seaport from the standpoint of queuing theory, and to use the findings to draw some hypotheses regarding its application to the overall operation in sea ports.
b) Determination of the optimum number of berths needed in a sea port that will minimize the total port usage costs.

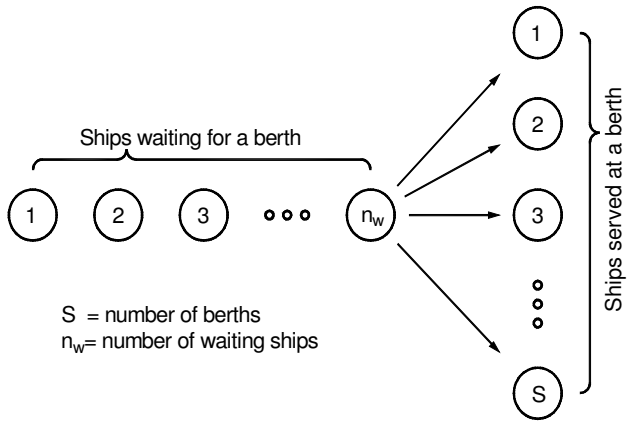

Figure 1. Ship queue at a seaport.

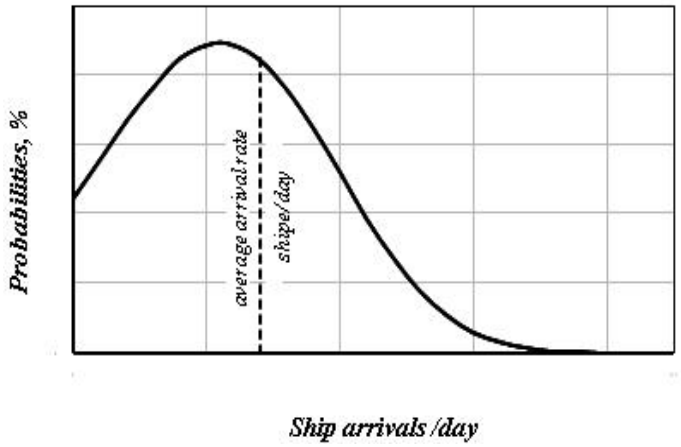

Figure 2. Ships arrival distribution as poisson function, hypothetical port.

ANALYSIS OF SHIPS' MOVEMENT IN A SEAPORT

An important parameter measuring the performance of a seaport is the delays that ships experience while waiting to be processed. Two factors affect these delays: (a) the pattern of ships arrival, and (b) the berth time requirement for cargo handling.

The arrival of a cargo ship in a port is often irregular, and when it arrives, it may be able to move directly onto a berth or has to wait until a berth becomes empty, if all berths are occupied. The berth time needed to serve a ship is also variable, as it depends on the amount of cargo which the ship carries and the capacity of the present facilities for handling and storing cargo (Gokkup, 1995). Figure 1 shows ship behavior at a seaport.

The investigation of such random occurrences requires a complex and detailed analysis. The concept of "Queuing theory-waiting line problem" can successfully be applied. Queuing Theory is one of the most useful tools for analyzing the behavior of waiting units (ships in this case), for investigating the components of a multiple operation system (Branislav and Nam, 2006). Thus, queuing theory may be adequate for studying ship movement in sea ports.

Two basic elements are necessary for the application of queuing theory to a waiting line problem: an arrival function and a service function. These functions should first be modeled. Once the validity of these models is tested, the different characteristics of the theoretical models, which describe the actual system with the accuracy that may be realized in estimating future traffic, can then be determined.

To analyze the movement of ships in a sea port using the queuing theory, the following conditions are assumed:

i) Ships arrivals and service times conform to the pattern of random occurrences.
ii) Ships are processed on the "first-come first-served" queue discipline.
iii) The queue length is unlimited, that is, if a ship arrives and finds a long queue, it joins the waiting ships and does not leave the port.

Modeling ship arrival

Probably the two most commonly encountered arrival patterns of ships in a sea port are the random and scheduled arrivals with considerable delays. Thus, to predict the number of ships present in a port in a certain time period (usually a day), the arrival pattern of ships may be approximated by a Poisson function (Tadashi, 2003). In this way, the probability P_n of the arrival of n ships in the port in a given time can be expressed as in Figure 2:

$$P_n = \frac{(\lambda)^n}{n!}.e^{-\lambda}$$

Where
λ = average arrival rate of ships during the given time (one day, for example),
e = base of the natural logarithm (e = 2.71828...),
λ_n = the average arrival rate of n ships, and;
n! = the factorial of the ship number.

The distribution of ships arrivals with Poisson function can be calculated, only if the average arrival rate during an entire period is known. The expected frequency F_n of n ships in port in a given time T is:

$$F_n = T.P_n$$

T is the considered time period of the port operation (often expressed on an annual basis as 365 days).

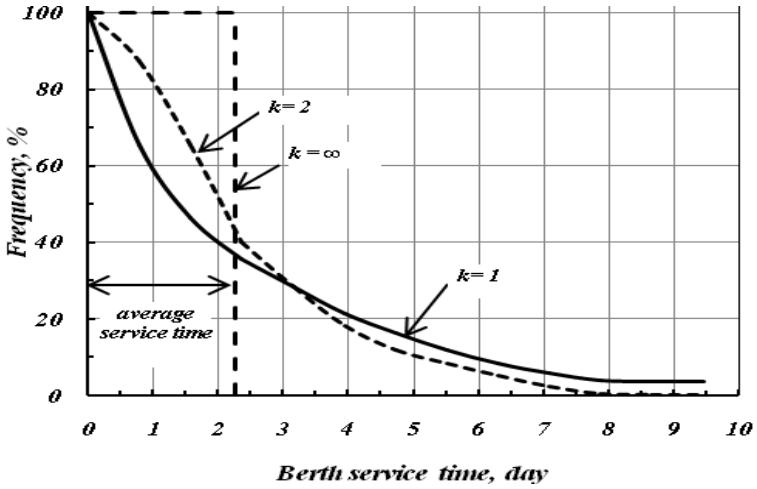

Figure 3. Service time distribution as Erlang Function, hypothetical port.

Modeling of service time

The duration of ships at a berth for handling cargo may be described as an Erlang-function (Son and Kim, 2004) which is usually used to present service times that are more regularly spaced in time than those represented by the Poisson distribution.

There are purely theoretical curves (Erlang-functions), each of which is based on the assumption that the service time is split into two or more operating phases following one another, and that the ship does not leave the berth until all phases are completed. “k” is the number of “Erlang Phases” of ships service time distribution at a berth. Each function has a negative exponential distribution. As “k” increases, the total service times become more uniform, until finally with k = ∞ all service times are identical. In the general case the total service time probability P_0 is given in Figure 3.

$$P_o = e^{kb.\sum_{n=0}^{n=k-1}(kb).\frac{n}{n!}}$$

Where
b = Average berth service time (in days),
k = Erlang number (k = 1, 2, 3, …., ∞), and;
n = Counter.

for k = 1, $P_o = e^{-b}$.

for k = 2, $P_o = e^{-2b.(1+2b)}$.

for k = 3, $P_o = e^{-3b.(1+3b+9b^2/2)}$.

Through the choice of k, a service time function may be described as anything from the purely random exponential type (k = 1) to the completely regular constant service time type (k = ∞), the value of k should be selected and tested to provide the best fit to the observed data.

QUEUING PHENOMENON

As the nature of the problem is defined, in this paper, as multi-channels (berths), with exponential arrivals (Poisson), and multiple exponential services (Erlang), no feasible mathematical solution is possible (Zoran and Branislav, 2005). The theoretical models available in the literature for multi-channel systems are inflexible for other than expo-nential distribution of arrivals and multiple exponential service time distribution. For investigating queuing situa-tions of multi-channel systems, models are accessible only for the following two cases:

Case I: Exponential distributions for both arrivals and service times.
Case II: Exponential arrivals and a constant service time.

An approximate method has recently been proposed regarding the queuing model of case II (Wen-Chih et al., 2007). The essential parameters are derived as follows:

λ = Average arrival rate in ships/day (Poisson-distribution),
μ = Average service rate in ships/day (Erlang-distribution) = 1/ average berth service time = 1/ b, and;
S = Number of berths.

The ratio of the arrival rate to the service rate σ is usually known as the traffic intensity, thus:

$$\sigma = \frac{\lambda}{\mu}$$

In this case, it can be noted that the average waiting time before service w_k is given by (Erlang function) (Wen-Chih

et al., 2007):

$$w_k = w_1 \frac{1}{2(k=1)}$$

Where,
w_1 is the correction of the average constant service time obtained by selecting an Erlang-function with constant k number. w_1 can be calculated from the following function, (Wen-Chih et al., 2007):

$$w_1 = \frac{\lambda^S}{(S-1)!\mu^{(S-1)}.(S.\mu-\lambda^2)\left[\sum_{n=1}^{S-1}\frac{1}{n!}.\left(\frac{\lambda}{\mu}\right)^n\right]+\lambda^S.(S.\mu-\lambda)}$$

Where, n = actual number of ships present in a port in a certain time period.
Thus the average time that a ship spends in seaport t_s can be determined as follows:

$$t_s = b + w_k$$

From the above analysis of delays in the queue, computation can readily be made of the average length of queue, that is,for average number of ships waiting for a berth n_w, the appropriate expression is:

$$\eta_w = \lambda . w_k$$

The average number of ships n_s present in port with *S* berths in a certain time period can be determined using the following formula:

$$\eta_s = \eta_w + \eta_b$$

Where,
n_b = average number of *n* ships served at *S* berths
= *S* × berth utilization factor
= $S. (\lambda/(\mu.S) = \sigma.$

Thus, it is seen that the traffic intensity, σ defined in the queuing theory equals the average number of ships served at berths n_b.

ANALYSIS OF PORT CAPACITY

Minimum capacity

The minimum number of berths S_{min} needed in a seaport to handle a certain amount of cargo can be calculated using the following procedure:

Let Q = the total amount of cargo (in tons) handled in a port section in a time period T (for example, T = one year = 8760 h), and R = average rate of cargo transfer between ship and berth (in tons per hour). Then,

$$S_{\min} = \frac{Q}{(R.T)}$$

Thus, the gross berth time available is "S_{min}. T".

Then, let β (berth utilization) equal the % of berth usage throughout the period T.
β = (berth time required)/ (berth time available), or
$= \frac{Q}{S_{\min}.(R.T)}$.
In this manner, the calculated number of berths is based on average values; regardless of the random arrivals of ships and the variation in berth service times.

Optimum capacity

If the number of berths in a port is S, the total cost spent in the port during a certain period, C equals the sum of two different types of costs: cost related to berths and cost related to ships present (Jan and Robert, 2002). Thus, it can be expressed as (Figure 4):

$$C = C_{b.} T.S + C_s \eta_s$$

In which,
C = total cost of a port with S berths during the period T, usually one year = 365 days, (in L.E.),
C_b = average cost of a berth; that is, construction and maintenance costs (L.E./day/berth),
c_s = average delay cost of a waiting ship (L.E./day/ship), and;
n_s = average number of ships present in port.

Accordingly, if the amount of cargo that must be dealt with at a port during the period T is given as time planning target, then such number of berths S becomes the optimum that minimizes the total cost C. Therefore, C is a proper measure to examine the optimality of a port system.

Now, both sides of the above equation are divided by "C_s.t" in order to decrease the number of the parameters involved. Thus,

$$R_s = C / (C_S.T) =) (c_b/c_S).S + \eta_s = (r_{bs}. S) + \eta_s$$

In which,
r_s = ratio of the total annual cost for port to annual ship cost, and
r_{bs} = berth-ship cost ratio.
Assuming that S is optimum, then the following optimization condition must be held:
$r_s < r_s + 1$, and $r_s < r_{s-1}$

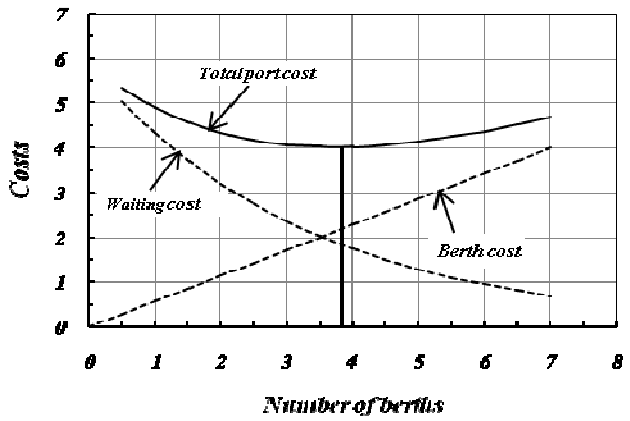

Figure 4. Total usage cost, hypothetical port.

Thus, r_s will be adopted hereafter as a measure to determine the optimum number of berths.

From the preceding information the procedure can be standardized as follows when given the data Q, R, c_b, c_s, λ, μ, k:

Step 1. Calculate the minimum number of berths from the equation, $S_{\min} = \frac{Q}{R.T}$.

Step 2. Determine the value of traffic intensity σ as; ($\sigma = \frac{\lambda}{\mu}$).

Step 3. Compute the value of berth-ship cost ratio r_{bs} from the given data c_b and c_s.

Step 4. For each number of berths, with S greater than the minimum value, estimate the number of ships present in port ns, and predict the ratio r_s.

Step 5. The number of berths which satisfies the optimization condition ($r_s < r_{s+1}$, and $r_s < r_{s-1}$) is optimum.

Step 6. Compute the average berth utilization, β: ($\beta = \frac{\sigma}{S}$).

Step 7. Summarize the queuing results (average number of ships present in the port, average number of ships at berths, average number of waiting ships, average waiting time).

APPLICATION OF THE PROPOSED METHODOLOGY TO ALEXANDRIA SEAPORT

The foregoing methodology is applied to investigate the movements of ships in Alexandria Port and to predict the future capacity. The application is restricted to general cargo ships, excluding full-container, bulk, and RO/RO ships which have particular berths at the port.

Alexandria Port is the major port in Egypt. About 40.80 million tons passed through the port in the year 2007/2008, that is, 36% of the total volume of the foreign trade. The amount of general cargo handled in the port in that year was 4.326 million tons (Egyptian Maritime Data Bank, 2008).

Alexandria port is constituted of an old and complicated layout with short quays and too narrow or too long piers. A large number of quays has limited drought less than 8.0 meters, and only a lower number of berths is capable to receive ships with more than 130.00 meter length. The number of berths available for general cargo in the port is 32 berths.

Data base

The daily "log books" of the traffic department of the Alexandria Port Authority include (among others) the arrival time of each ship at the pilot vessel. In addition, detailed information concerning the movement of each ship in the port is also available in the so-called "ship log sheets". Every sheet is a ship report, and it contains the following data:

a) Ship name, nationality, type of cargo, and total tonnage.
b) Berth occupancy, including berth changes during the period in port.
c) Date and time of arrival, berthing, and quitting the port.

Ships arrivals

If the distribution of ships arrivals can be predicted reliably, port planner can proceed with great confidence in making development plans that may avoid over-building or under-building the port facilities.

The actual pattern of ship arrivals at the port of Alexandria is compared with the theoretical function prognosticated mathematically by Poisson distribution of random occurrences. The application includes a specific analysis of the number of ships present, day by day, over a period of one year (from July 1, 2007 to June 30, 2008).

The number of ships present in the port, each day, was transcribed from the port "log books" and then summarized to obtain the number of days, that various number of ships were present during the period studied. The theoretical distribution, Poisson is computed.

Table 1 compares the predicted distribution with the actual one. The average arrival rate was 5.68 ships per day. Table 1 shows a good agreement between actual and predicted distributions. The number of days that various numbers of ships are predicted to be present in the port is in agreement with the actual distribution on 336 days of 360 days, that is, on 92% of days.

To judge whether the observed frequencies of ship arrival distribution is compatible with the predicted theoretical frequencies, Chi-square is computed, and the result, x^2 = 20.0 with 10% probability, indicates a good fit. From the statistical standpoint, probability values between 5 and 95% designate good fit from which it is concluded that this theoretical distribution is plausible (Tadashi, 2003). Figure 5 demonstrates the goodness of fit between actual and predicted distributions.

Berth service times

Information giving the date and time of arrival at a berth and the date and time of departure from the berth were obtained from the "ship log sheets". A total of 315 observations, including those general cargo ships which were tied up at the berths between July 1, 2007 and June 30, 2008 were randomly selected to be analyzed. A class interval of 15 hours was selected for such analysis.

Search for a suitable model for the distribution of the durations at berths led to an Erlang distribution giving K = 3. The mean time

Table 1. Comparison of actual versus predicted ship arrival distribution.

Arrival rate (Ships/day)	Actual number of days (A)	Predicted number of days (B)	Minimum (A) or (B)
0	1	1	1
1	6	7	6
2	18	20	18
3	27	37	27
4	49	54	49
5	73	62	62
6	60	59	59
7	61	47	47
8	37	34	34
9	15	21	15
10	9	12	9
11	5	6	5
12	2	3	2
13	2	2	2
Total	365	365	336

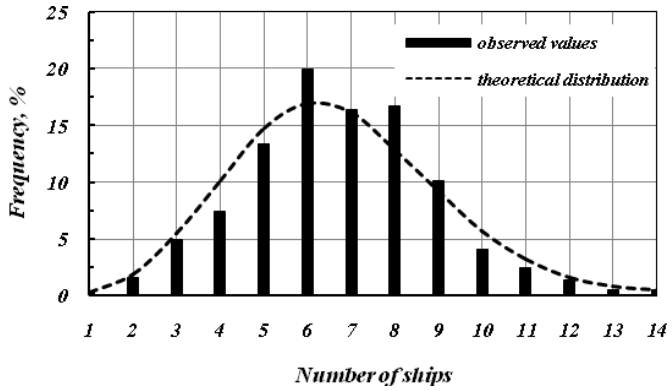

Figure 5. Frequency Distribution of Ships Arrivals, Alexandria Port 2007/2008

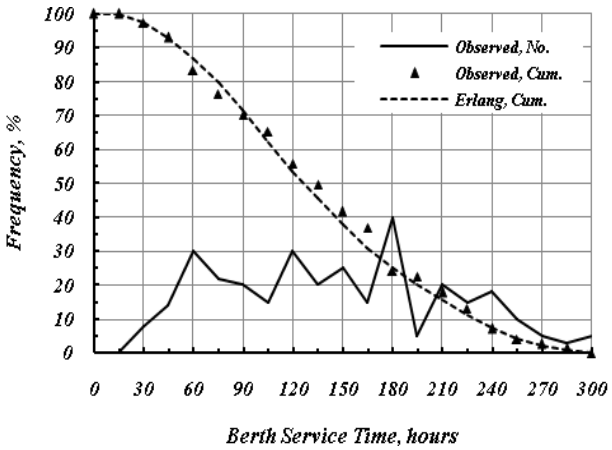

Figure 6. Frequency distribution of berth service time, Alexandria port 2007/2008.

spent at a berth was found 5.58 days for the 315 observations. The standard deviation of the distribution was computed and found to be ± 1.43 days. Figure 6 presents the frequency and the cumulative distributions of the observed data and compares the values of the cumulative distribution with those of the Erlang function having K = 3.

A Chi-square test was also performed to test the goodness of fit between the observed frequency distribution and the postulated Erlang function, and a value X^2 = 14.87 for 42% probability was found. Comparison with other Erlang functions (K = 1, K = 2, and K = 4) indicates that K = 3 is the best choice for this distribution function. Figure 6 also shows the observed data points and a plot of the selected function.

Optimum number of berths

To establish the optimum number of berths needed for general cargo handling at Alexandria port in the year 2017, applying the proposed procedure, the following input data are used:

i) Due to the further development of the Egyptian ports, particularly the Dekheila port, the annual general cargo tonnage to be handled at the berths of Alexandria port will be only about 4.00 million tons at the target year (tonnage in year 2007/2008 = 4.326 million tons) (Egyptian Maritime Data Bank, 2008).
ii) The average arrival rate of general cargo ships will be 5.68 ships /day, assuming that the average ships load equals 2084 tons (the present value).
iii) The average rate of cargo handling at a general cargo berth R = 373.5 tons per day (the existing rate).
iv) The average cost of a berth c_b = 2000 per day (approximately $600 per day), based on the development program of the Alexandria port (Egyptian Maritime Data Bank, 2008).
v) The average delay cost of a general cargo ship c_s = $ 6000 per day.

The calculations are carried out as follows:

S_{min} = 4000 000/(373.5 × 365) = 29.34 = 30 berths
λ = 5.68 ships/day

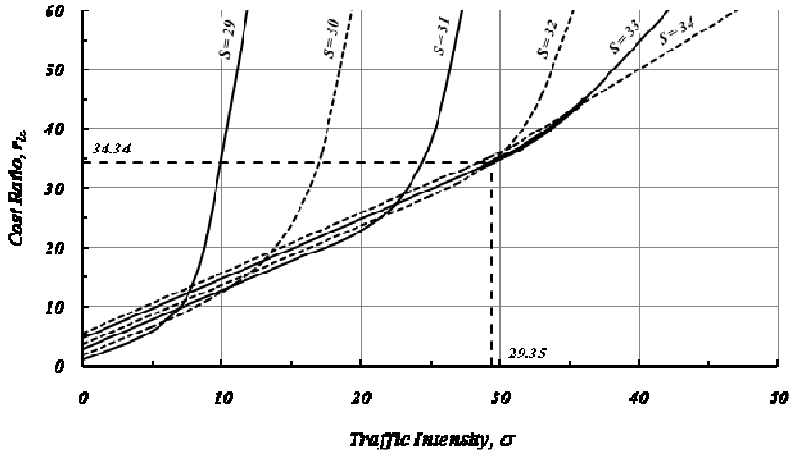

Figure 7. Determination of optimum number of berths, σ, Alexandria port, case study.

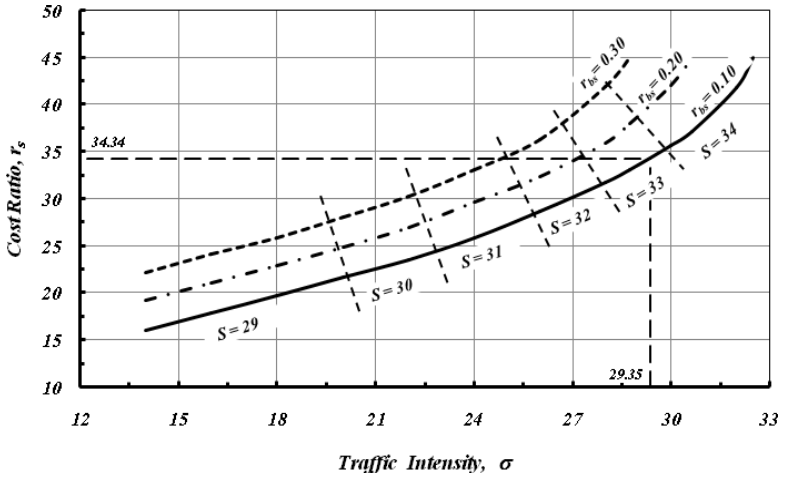

Figure 8. Determination of Optimum Number of Berths, σ due to different cost ratio, r_{bs}, Alexandria Port, Case Study.

μ = 1/5.58 = 0.18 ships/day
σ = 5.68/0.18 = 29.35
r_{bs} = 600/6000 = 0.10

Figure 7 shows the relationships between traffic intensity and the cost ratio r_s for a proper number of berths (from S = 29 to S = 34) The optimum port capacity is 33 berths. In this instance, r_s = 34.34, and the total port costs C = $75.200 million (the development of 33 berths plus the annual maintenance costs).

The average berth utilization = 29.35/33 = 0.89

Queuing results:

Average number of ships present in port n_s = 31.04
Average number of ships served at berths n_b = 2935
Average number of waiting ships n_w = 1.69
Average waiting time per ship w_k = 0.32 days

The relationships in Figure 7 are prepared as design curves derived to determine the optimum number of berths for Alexandria port by changing the traffic intensity and/or the cost ratio r_{bs}.

The optimum number of berths corresponding to a suitable cost ratio r_{bs} values (from 0.10 to 0.30) is noted in Figure 8. It can also be seen that a 33-berths set is the optimum port capacity in case of traffic intensity values varying between 27.58 and 29.60.

Table 2 shows the calculation of the costs of idle berths and idle ships for 33 berths in view of the expected frequency (number of days per year). It also presents the combined costs (vacant berths and ships) in case of port size 31, 32, 33 and 34 berths. The cost comparison indicates that the total port cost is least when there are 33 berths. This conclusion confirms the result previously obtained by applying the proposed methodology.

RESULTS CONCLUSIONS

This paper presents a methodology proposed to predict the optimum number of berths required in a sea port to meet the future traffic volumes. The methodology is based on the hypothesis that the number of berths can

Table 2. Cost calculation in case of 33 berths, and the comparison of the resulting value with those for 31, 32 and 34 berths.

Arrival rate (ships/day)	Predicted frequency (in days)	Berth utilization	Required number of berths	Over-building		Under-building	
				Number of berths	Berths-days	Number of ships	Ships-days
μ	F	$\beta = \lambda \div \mu$		$33 - \sigma$	$F . (33 - \sigma)$	$\lambda - X^*$	$F . (\lambda - X^*)$
0	1	0.00	0	33	33		
1	7	0.17	6	27	189		
2	20	0.34	12	21	420		
3	37	0.51	17	16	592		
4	54	0.68	23	10	540		
5	62	0.85	28	5	310		
6	59	1.00	33	0	0	0	0
7	47					1	47
8	34					2	68
9	21					3	63
10	12					4	48
11	6					5	30
12	3					6	18
13	0					7	0
Total	365				2086		274
Cost in Million \$ (using c_b = 600, c_s = 6000)					1.25		1.64
Total costs for 33 berths in Million dollars						2.89	
Total costs for 31 berths in Million dollars						3.15	
Total costs for 32 berths in Million dollars						3.08	
Total costs for 34 berths in Million dollars						2.93	

X^* = Number of available berths × maximum berth utilization / average service time = 33 × 1/5.58 = 6.00

be increased as long as the marginal cost of berths (construction and maintenance) is less than the delay costs of waiting ships.

The "Queuing theory" has been employed to derive the number of waiting ships and the average ship delays. The usage of queuing theory is subjected to the following two assumptions:

i) Ships arrivals at a sea port can be described as a negative exponential distribution, and,
ii) Berth service time yields to a multi-exponential function.

The employment of the queuing theory to study the movements of general cargo ships at Alexandria port was profitable. The observed pattern of ships arrivals appears to agree with Poisson's law of random distribution. In addition, the berth service time for 315 ships was found to conform most closely to an Erlang distribution with K = 3. The usage of an approximate model of queuing theory led to acceptable results. The criterion for acceptance of this model was the reasonable agreement achieved between the computed and observed values of average waiting time and average number of waiting ships in queues at berths.

Thus, there is no doubt that ships arrive at Alexandria port in accordance with a random pattern and that the degree of accuracy compares favorably with the accuracy that may be realized in estimating future traffic.

The application to Alexandria port verifies the anticipated benefit of using the suggested methodology to evaluate the port size in the best interests of both ship operators and the port authority. The evaluation is settled on the premise that maximum port efficiency results when the total port cost is minimum, that is, the cost of vacant berths over a substantial period plus the time cost of ships waiting for a berth during the same period.

REFERENCES

Gokkup U (1995). Application of queuing theory on the design of fishing Harbors, Transactions on the Built Environment. 11: 711-719.

Branislav D, Nam KP (2006). Modeling of ship-Berth-Yard Link Performance and Throuput Optimizatin, IAME 2006 Conference, Melbourne, Australia,

Tadashi Y (2003). Optimizing the handling Capacity in a Container Terminal for Investigating efficient Handling Systems, J. Eastern Asia Society for Transportation Studies, 5: 597–608.

Son JH, Kim MH (2004). An analysis of the optimal number of servers in distributed client/servers, Environ. Decision Support Syst., 36: 297-312.

Zoran R, Branislav D (2005). Optimal Number and Capacity of Servers in Queuing Systems, Inf. Manage. Sci., 16(3): 1-16.

Wen-Chih H, Tu-Cheng K, Sheng-Chieh W (2007). A Comparison of Analytical Methods and Simulation for Container Terminal Planning, 200 J. Chinese Institute of Indus. Engineers, 24(3): 200-209.

Jan OJ, Robert E (2002). Unification of Accounts and Marginal Costs for Transport Efficiency Swedish Seaport Case Study: Price-relevant marginal cost of Swedish seaport services, ITS, University of Leeds,

Funded by the European Commission, 5th Framework – Transport RTD.

Egyptian Maritime Data Bank (2008). Annual Statistical Report, National Planning Institute, Cairo.