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A fractal approach for estimating soil water retention curve

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Direct measurement of soil water retention curve (SWRC) is time consuming and expensive, therefore many empirical models have been developed to describe it. Also fractal models have been proposed in the last decades for estimating SWRC. In this study, the fractal dimension of SWRC (D_{SWEC}) and the mass fractal dimension of soil texture for 30 soil samples (with a spread range of soil texture) were determined. Fractal dimension of soil texture and a relationship among the fractal dimension of SWRC, Soil texture data and bulk density in a fractal base model was used to estimate SWRC, as it was estimated by Campbell empirical model. The estimated results were compared with the validation data set for validation and model comparison. The results indicated that the fractal base model was capable of predicting SWRC with reasonable accuracy, also the difference between estimation results of fractal base models and Campbell empirical model decrease when the fractal dimension of soil texture $D_{texture}$ was used, instead of D_{SWEC}. The estimation of D_{SWRC} by using of clay and silt content and soil bulk density as a regression relationship had better results than the fractal dimension of soil texture as replacement of D_{SWRC} was used in the fractal model. Therefore, it could be used for estimating SWRC, using easily measured data.

Key words: Fractal, soil water retention curve, estimation.

INTRODUCTION

Soil water retention curve (SWRC) is one of the main hydraulic properties of the soil which represents the relationship between two parameters: water content and matric potential. Although SWRC is one of the most important hydraulic functions for modeling flow transport in porous media, it describes the availability of soil water to plants and to model movement of water through unsaturated soils, its direct measurement is time consuming and expensive. It is necessary to use the function utilizing readily available data, for measuring SWRC. Many empirical models for SWRC have been developed (Brooks and Corey, 1964; van Genuchten, 1980; Russo, 1988; Leij et al., 1997). In these models parameters were usually estimated by fitting the functions with measured data, and for describing the relationship between the parameters and readily available soil data the pedotransfer functions (PTFs) were often used (Wösten and van Genuchten, 1988; Vereecken et al., 1989; Schaap et al., 1998; Minasny et al., 1999; Wösten et al., 2001).

For mathematical explanation of SWRC and recognizing the heterogeneity in many of the related properties and processes that we study in soil, new developments in other disciplines that deal with heterogeneity such as fractal geometry can be used (Perfect and Kay, 1995). Many efforts including fractal methods have been made in last decades for estimating SWRC (Tyler and Wheatcraft, 1990; Rieu and Sposito, 1991a, b; Perrier et al., 1996; Kravchenko and Zhang, 1998; Perfect, 1999; Liu and Xu, 2003). Several models have been derived either using the fractal nature of the solid or the void phases or both (de Gennes, 1985; Tyler and Wheatcraft, 1990; Rieu and Sposito, 1991a; Perrier et al., 1996; Perfect et al., 1996, 1998; Huang and Zhan, 2002). Tyler and Wheatcraft (1990) applied the Sierpinski Carpet pattern to describe the soil pore size distributions and developed a power-law form for SWRC, similar to the

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Table 1. General characteristics of studied soils.

Parameter	Max.	Min.
% sand	74.5	43.1
% silt	35.9	10.8
% clay	25.5	9.1
% $CaCo_3$	13.5	1.7
%OC	0.5	1.3
EC (dS/m)	1.8	0.18
pH	7.9	7
Bulk density (g.cm^3)	1.73	1.34

functions of Brooks and Corey (1964) and Campbell (1974). Perrier et al. (1996) developed a general expression for SWRC with the fractal scaling nature of the soil pore space in the three-dimensional Euclidian domain. Based on the fractal nature of full self-similarity of aggregates and the pore space of structured soils, Rieu and Sposito (1991a) and Perfect et al. (1996, 1998) developed SWRC models, which were in a power-law form but differed from the Brooks and Corey (1964) and Campbell (1974) models.

Fractal dimensions of the solid matrix (that is, soil particle size distribution and soil texture) and the void phase (that is, soil pore size distribution and soil pore surface) can characterize the fractal nature of soils. Nevertheless, further study is required to quantify the relationship among the fractal dimensions of the soil solid and void phases and the fractal dimension used in the SWRC (Huang et al., 2006). A sensitivity analysis that was carried out on Tyler and Wheatcraft (1990) model showed that the most sensitive parameter in it, is the SWRC fractal dimension (D_{SWRC}); whereas this model is less sensitive to the saturated water content and air entry value (Ghanbarian-Alavijeh et al., 2008). Although the exponent of soil water retention curve, D_{SWRC}, is physically meaningful, its direct measurement is difficult as laboratorial measurement and also field soil water retention experiments are labor and time consuming. It could be estimated by readily available parameters. The objectives of this study were: (1) determining the D_{SWRC} form SWRC experimental data and fractal dimension of soil texture, (2) establishing a relationship among D_{SWRC}, and readily available characteristics (that is, clay, silt, sand content and bulk density), (3) validating the developed relationship in estimation of SWRC and comparing the results with the results of Campbell model.

METHODS

Samples and measurements

A total of 40 soil samples, collected from different locations (Mashhad, Isfahan, Tehran) of Iran, were used to measure soil texture, and the water retention curve. The soils covered all range of texture classes from sandy to clayey. Table 1 shows the general characteristics of soils. Five undisturbed samples were taken directly from each soil sample site with samplers (50 mm in inner diameter and 10 mm in height) for measuring SWRC and bulk density. Also, 500 g of disturbed soil samples were collected from each sample site used for the measurement of soil texture according to the US Department of Agriculture (USDA) texture classification standards (Hillel, 1998). Soil water retention data were measured using the pressure plate apparatus. For this purpose first, the undisturbed soil samples were saturated from the bottom with distilled water. Then, the samples were put in the pressure plate and were equilibrated at the desired matric potential (10, 30, 300, 500, 1000 and 1500 kPa). Finally, the samples were weighed with the remaining moisture, dried in an oven at 105°C overnight and weighed again and SWRC for each soil was determined.

Determination of D_{SWRC} and soil texture fractal dimension

soil texture data and the log-transformed form of the fractal scaling of soil particle-size distribution (Tyler and Wheatcraft, 1992) is expressed as Equation (1), were used to determine the fractal dimension of soil texture, by employing a linear regression between the cumulative log mass fractions and log characteristic particle radius for all soils.

$$\frac{M(r<R)}{M_T} = \left(\frac{R}{R_{\max}}\right)^{3-D_M} \quad (1)$$

Where r is the grain size, R represents the characteristic particle radius, M (r < R) is referred to as the mass of grain radius r less than R, M_T is the total mass, R_{max} is the maximum characteristic particle radius, and D_M, is the mass fractal dimension. Soil water retention data with a log-transformed of 3D form of Tyler and Wheatcraftt (1990) that is expressed as Equation (2) were used to determine D_{SWRC}.

$$\theta = \theta_S \left(\frac{\psi}{\psi_a}\right)^{D_M - 3} \quad (2)$$

Where ψ, is the capillary tension head (cm), θ is the soil water content (cm^3 cm^{-3}), θs, is the saturated soil water content (cm^3 cm^{-3}) and ψ a is the air entry pressure. A regression analysis was used to determine the relation among D_{SWRC}, soil texture (that is, clay and sand content) and soil bulk density. In this study, 30 soil samples were used to analyze the regression relationship among D_{SWRC}, soil texture (that is, clay and sand content) and soil bulk density.

Estimation of soil water retention curve

Two fractal base methods and an empirical model were used for estimating of SWRC, The first method is base on the using of the fractal dimension of soil texture as a replacement for D_{SWRC} (method 1) and the second method is on the basis of estimating D_{SWRC} by using of a regression relationship among D_{SWRC}, soil texture (that is, clay content and sand content) and soil bulk density (method 2), and also Campbell empirical model (1974) that is expressed as Equation (3) was used (Method 3). Saturated water content, θs, assumed to be equal to porosity and the air entry value, ψa, were taken from literatures based on soil texture (Rawls et al., 1982).

$$\frac{\psi}{\psi_a} = \left(\frac{\theta}{\theta_s}\right)^{-b_c} \quad (3)$$

Where b_c, is an empirical coefficient.

Methods comparison and validation

Estimated SWRCs by employing three different methods for the data set (10 test soils), were compared with measured data. The measured data of ten soil samples differ in textures, were used to validate the results of estimated SWRCs. the difference between estimated and measured data was then quantified by using the Mean absolute error (MAE) and Mean Square error (MSE) as follows:

$$MAE = \frac{\sum_{i=1}^{n} \left| Z^*(x_i) - Z(x_i) \right|}{n} \quad (4)$$

$$MSE = \frac{\sum_{i=1}^{n} \left(Z^*(x_i) - Z(x_i) \right)^2}{n} \quad (5)$$

Where Z*(x) is the estimated value, Z(x) is the measured value and n is the number of observation points in the independent test data set (here n = 10). Linear regression was also performed between measured and estimated water content for all tested soils and coefficients of determination (r^2), was determined.

RESULTS AND DISCUSSION

Fractal dimension of SWRCs and soil textures

Figure 1 shows a linear regression between the cumulative log mass fractions and log characteristic particle radius for two typical soils: soil 1 (clay) and soil 40 (sandy). The slope of fitted regression line for the coarse-textured sandy soil, with $D_{texture}$ = 2.66, was steeper than the fine textured, clay soil, with $D_{texture}$ = 2.92. Also a linear regression between ($\frac{\theta}{\theta_s}$) and ($\frac{h}{h_0}$) for soil 5 (clay) and soil 25 (sandy) has been showed in Figure 2. The largest soil texture fractal dimension was 2.95 for soil 6 (a clay soil) and a sandy soil (soil 39) had the smallest soil texture fractal dimension as 2.55. The D_{SWRC} ranges from 2.55 to 2.87 for a clay soil (6) and a sandy soil (39), respectively.

The estimated fractal dimension values depended on soil texture as soils with coarse texture had lower fractal dimension values than soils with fine texture. Tyler and Wheatcraft (1992), Rieu and Sposito (1991b) as well as Filgueira et al. (1999) and Kravchenko and Zhang (1998) using respectively, the soil mass distribution, the aggregate size distribution and the particle-size distribution in the three-dimensional Euclidian domain, have found that the fractal dimension of soils were in the range of 2 to 3. The values of determined D_{SWRC} were usually smaller than mass fractal dimensions of soil texture. Filgueira et al. (1999) investigated the fractal behavior of the aggregate mass-size distribution and observed no statistical difference between the fractal dimension of mass and the fractal dimension of pore space. Most estimated D was within the range 2.5≤ D ≤3. The fractal dimensions estimated by Filgueira et al. (1999) from the mass fractal model of Rieu and Sposito (1991b) were smaller than those determined by Millán et al. (2006) from a prefractal pore–solid interface model of soil-water retention which also accounts for lower and upper scale cutoffs of the prefractal domain. These previous results agree to some extent with Dathe and Thullner (2005) findings from binary images.

The coefficient of correlation between mass fractal dimension of soil texture and D_{SWRC}, 0.83, showed a high correlation between these parameters. Also the relationship among the fractal dimension, soil texture and bulk density were established as follows:

$$D_{SWRC} = 3.32 + 0.0012\,C - 0.0012\,S - 0.394\,\rho b \qquad r^2 = 0.80 \quad (6)$$

in which, D_{SWRC} is the estimated fractal dimension of soil water retention curve, C and S are clay and silt content and ρb is soil bulk density (cm^3/cm^3).

Results showed that the D_{SWRC} could be approximated by the mass fractal dimension of soil texture, (Kravchenko and Zhang, 1998) or it could be estimated by using clay and silt contents and soil bulk density as Equation (6). Table 2 shows the MAE, MSE and R^2 obtained from comparing all data of the measured soil water content versus the estimated by using three different methods for 10 test soils. The results showed a reasonably good estimation of soil water retention curves for the most of soils. Method 1 showed a better estimation for light-textured.

Comparison of the statistical parameter values, MSE and MAE, showed that Method 2 was able to estimate the soil water retention curve more accurate than Methods 1 and 3 were. It revealed that 80% of the MAE for soil water content was less than 3% for Method 2 while it was 40 and 20% for Methods 1 and 3, respectively.

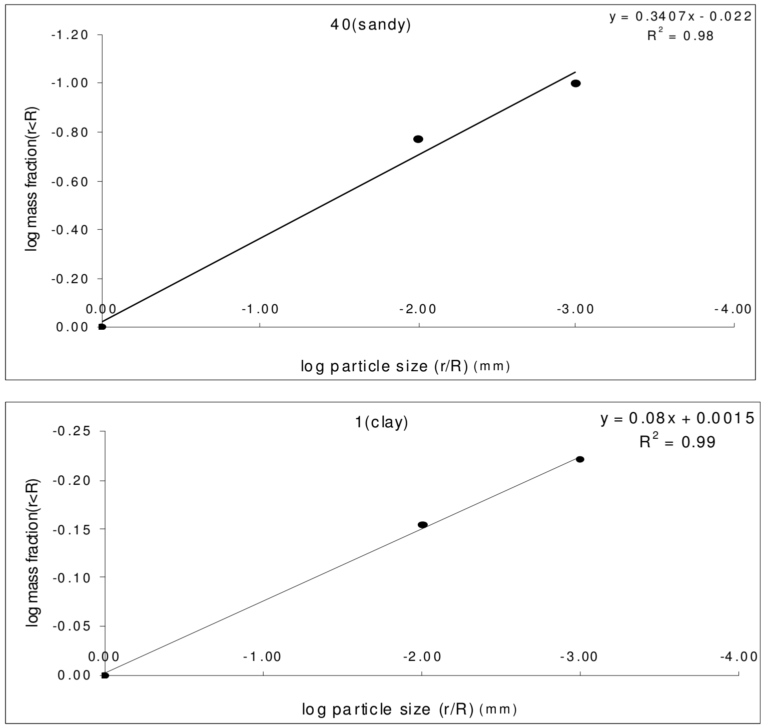

Figure 1. Linear regression between the cumulative log mass fractions and log characteristic particle radius for soils 1 (clay) and soil 40 (sandy).

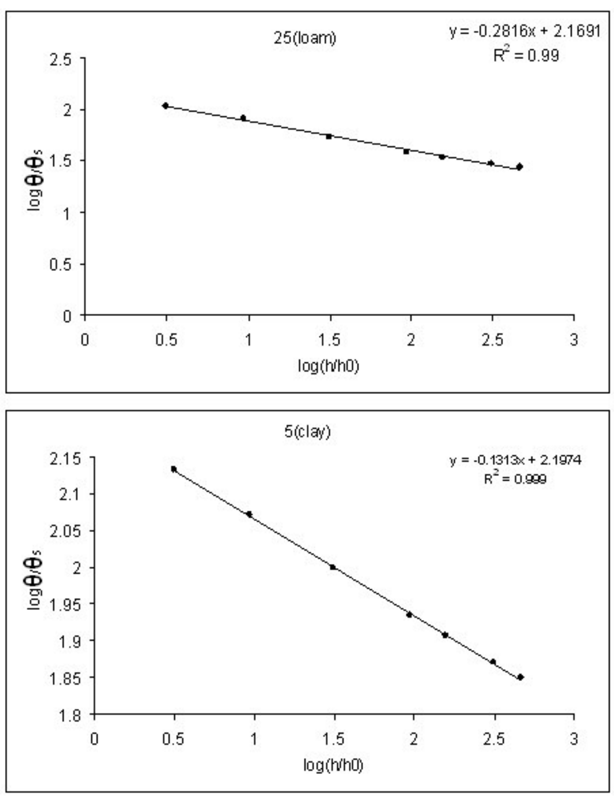

Figure 2. Linear relation between (θ/θ_s) and (h/h_0) for soil 5 (clay) and soil 25 (loamy).

Table 2. The MAE, MSE and R^2 obtained form comparing the measured soil water content data versus the estimated one by using three different methods.

	MAE			MSE			R^2		
Soil No.	**Method 1**	**Method 2**	**Method 3**	**Method 1**	**Method 2**	**Method 3**	**Method 1**	**Method 2**	**Method 3**
4	0.111	0.033	0.059	0.013	0.001	0.004	0.994	0.998	0.994
7	0.034	0.006	0.030	0.001	0.000	0.001	0.996	0.999	0.992
10	0.114	0.007	0.085	0.014	0.000	0.009	0.989	0.999	0.987
15	0.030	0.013	0.045	0.001	0.000	0.002	0.993	0.995	0.978
20	0.050	0.015	0.062	0.003	0.001	0.004	0.986	0.994	0.977
23	0.084	0.011	0.074	0.008	0.000	0.006	0.985	0.995	0.980
26	0.024	0.011	0.037	0.001	0.000	0.002	0.991	0.996	0.981
30	0.027	0.005	0.023	0.001	0.000	0.002	0.999	0.998	0.991
34	0.012	0.023	0.028	0.000	0.001	0.001	0.997	0.998	0.986
38	0.016	0.032	0.061	0.001	0.001	0.004	0.994	0.995	0.977
Mean	0.050	0.016	0.050	0.004	0.000	0.004	0.992	0.997	0.984

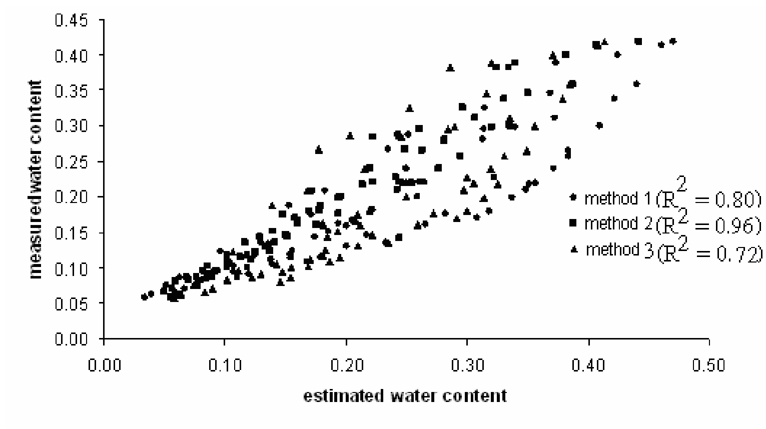

Figure 3. Estimated water content vs. measured one for all 10 test samples using three different methods.

The coefficients of determination (r^2) mean for estimated data using Methods 1, 2 and 3, were 0.992, 0.997 and 0.984 respectively. Also estimated water content versus measured one for all 10 samples using different methods and their R^2 values as 0.80, 0.96 and 0.72 for Methods 1, 2 and 3, respectively, was presented in Figure 3. Figure 4 shows measured and estimated SWRC, for two typical soils: soil 7 (sandy clay), soil 30 (sandy loam).

The results showed for most of the soils, using regression relationship, Equation 6, gave a better estimation of SWRC. Additionally, linear regression of the measured and estimated water contents using Method 2 for validation data set showed that the intercept values for all tested soils were close to zero, most of the slope values were close to unity, and the coefficients of determination (r^2) between the estimated results and measured data for all soils ranged from 0.994 to 0.999. Therefore, Method 2 (clay and silt content and bulk density), could be recommended for estimating SWRC.

Conclusion

Soil water retention data and soil texture data were used to estimate the mass fractal dimension of soil. The fractal dimensions for both SWRC and soil texture ranged between 2.5 to 2.95 and increased with clay content and decreased with sand content. The fractal dimension of SWRC was linearly related to clay and silt contents and bulk density of soil. The fractal dimension of soil texture, $D_{texture}$ and the estimated D_{SWRC} by using a linear regression equation, were used in a fractal base model, for estimating SWRC. Also SWRC was estimated by using the Campbell empirical model.

A reasonable agreement between the estimated and measured results was achieved for the tested soils. The results indicated that the model with the fractal dimension calculated from two fractal methods was capable of predicting SWRC with reasonable accuracy. The difference between estimation results of fractal base methods and

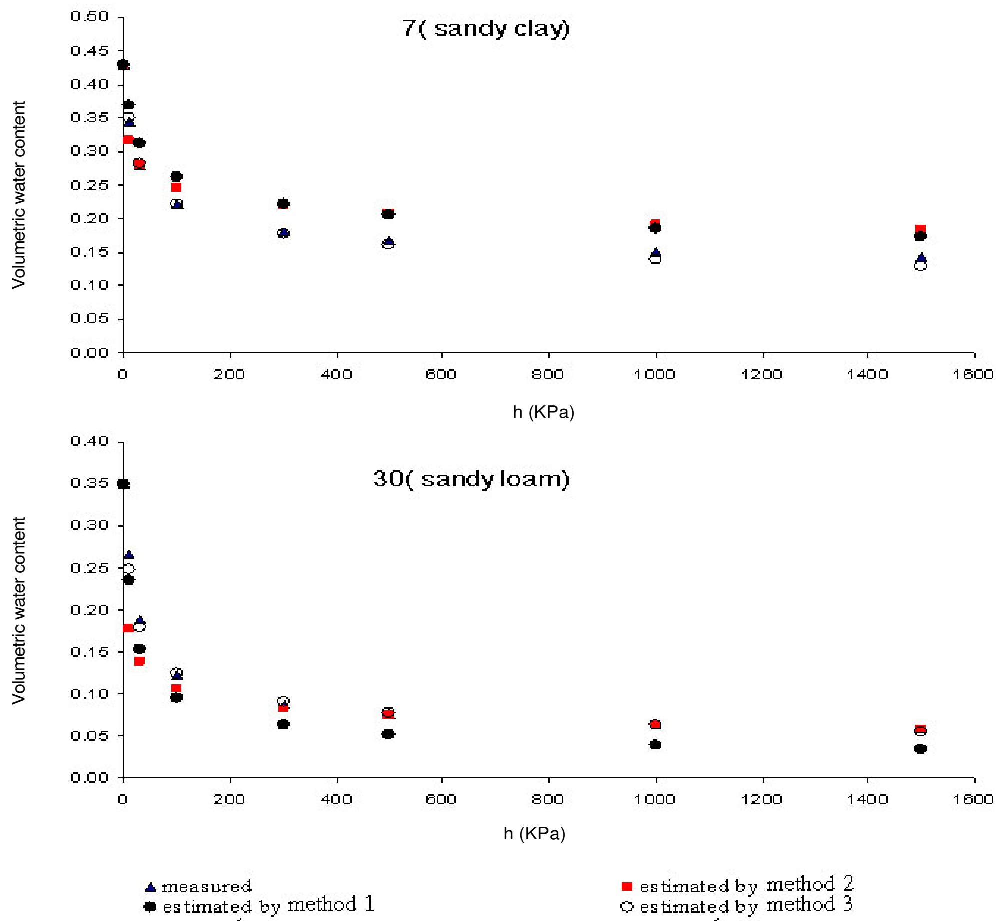

Figure 4. Estimated SWRC by using of fractal model (methods 1 and 2) and Campbell empirical model (Method 3) and experimental measured SWRC for soil 7 (sandy clay), soil 30 (sandy loam).

Campbell empirical model decrease when the fractal dimension of soil texture $D_{texture}$ was used, instead of D_{SWEC}. The estimation of D_{SWRC} by using clay and silt content and soil bulk density as a regression relationship (method 2), had better results than the fractal dimension of soil texture as replacement of D_{SWRC} was used in fractal model. Therefore, method 2 is capable for estimating SWRC, using easily measured data.

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