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Full Length Research Paper

# An optimization study on the delivery distance of colloidal silica

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The low initial viscosity of colloidal silica enables the use of low injection rates during soil treatment against liquefaction. Several large scale 3-D laboratory experiments show that colloidal silica treatment provides successful treatment coverage when the distance between injection and extraction wells is about 2.5 m. However the repeatability of such large scale experiments to optimize the injection procedure is cumbersome. Therefore such optimization process is achieved by 3-D flood simulator UTCHEM. This study discusses the required injection and extraction rates and well setups for optimum coverage considering different soil conditions.

Key words: Colloidal silica injection, MODFLOW, chemical grouting, liquefaction, numerical analysis.

### INTRODUCTION

Colloidal silica is an aqueous dispersion of fine-sized, amorphous, nonporous, and typically spherical silica particles in a liquid phase ranging from 5 to 100 nm in diameter (Du Pont, 1997). When diluted to 6%, the initial viscosity is measured between 1.05 and 1.6 cP, which is slightly greater than the initial viscosity of water (viscosity of tap water = 0.92 cP at 23°C). Colloidal silica can be made to gel by adjusting the pH or the ionic strength. The time to gelation can range from a few minutes to a few months (Gallagher and Mitchell, 2002). During the time between mixing and gelation, the viscosity of colloidal silica remains close to that of water until just prior to gelling, after which it increases very rapidly.

Colloidal silica has been shown to reduce liquefaction risk in laboratory, centrifuge and field applications (Maher et al., 1994; Gallagher and Mitchell, 2002; Gallagher et al., 2007; Conlee et al., (2010). In these studies, colloidal silica treated loose sands showed excellent performance under cyclic loading and had an excellent resistance to liquefaction. Loose sand samples treated with 20% CS remained intact after 1000 cycles and experienced less than 2% strain, whereas untreated sand samples failed after 13 cycles (Gallagher, 2000). The unconfined strengths of sand samples treated with 5 to 20% colloidal silica ranged between 40 to 230 kPa (Gallagher, 2000).

The low initial viscosity of colloidal silica makes it an attractive stabilizer for long distance grouting. The ease in colloidal silica groutability through the soil brings up the question of: How far can colloidal silica horizontally be delivered using one set of injection and extraction wells? Lin (2006) investigated the delivery distance in laboratory using 1-D column tests. It is reported that colloidal silica could successfully be delivered to 9 m distance. One of the shortcomings of this study is that the flow was 1dimensional and established along bottom up direction. However on site, the injected grout is free to advance in any direction. Hamderi (2010) showed that Ludox<sup>®</sup> SM colloidal silica could horizontally be delivered to about 2.5 m of distance using injection/extraction wells in a 3.6 m long x 2.4 m wide x 1.2 m sand box along with a numerical simulation in UTCHEM.

Ludox colloidal silica was also used for *in situ* hot spot stabilization and horizontal grouting by Noll et.al. (1993). MODFLOW in conjunction with MODPATH was used for calibration and simulation. Hot spot stabilization was accomplished by an injection well located at the center and 6 extraction wells in a 6 m diameter treatment area. The injection rate of the center well and the total extraction rate of 6 wells were set equal to 16.35 m<sup>3</sup>/day. A total of 13.6 m<sup>3</sup> of 5% colloidal silica was batched and the injection duration was planned to continue for 20 h. After completion of treatment, gel treatment area was scanned by Ground Penetrating Radar (GPR). According to the MODFLOW prediction, grout advanced towards the extraction wells covering the cylindrical volume between wells. In contrast, GPR results indicated that the grout was directed towards the lower levels and never reached the extraction wells.

One other study on colloidal silica flow modeling in porous media was performed by Bolisetti and Reitsma (2003). They developed a grout aging module to study injection processes in MODFLOW in conjunction with MT3D. They considered barrier formation in a 4.2 m x 2.40 m x 1 m domain in which colloidal silica was injected at 25 different points consecutively. The grout aging module was used to simulate the initiation of gelation as soon as 5% grout concentration had been reached. One of the conclusions of this study was that the target reduced permeability was not reached in a highly variable soil (Bolissetti and Retisma, 2003).

In this study, UTCHEM will be used to predict the advancement of colloidal silica plume which is horizontally induced by injection and extraction wells in a fully saturated sand aquifer. The main goal of this numerical study is to evaluate the efficiency of the treatment using different injection rates, permeabilities and injection-extraction well distances.

#### MATERIALS AND METHODS

A satisfactory permeation grouting scheme requires determination of two important parameters: 1-) Injection rate, 2-) Injection pressure. A higher injection rate is preferable for a shorter grouting duration. However, excessive injection rates can increase the water level around the wells and the grout may return to the surface. Similarly, extraction rates should be adjusted in a way that water level should not drop below the level of extraction wells. In this study MODFLOW code was used to find maximum possible injection and extraction rates.

MODFLOW is a 3-dimensional finite-difference ground water code which was published by U.S. Geological Survey in 1984. There is now a family of MODFLOW-related programs which can simulate various phenomena such as variable density and viscosity however MODFLOW itself can only calculate the water levels in steady state or transient conditions.

The mathematical model of MODFLOW includes the governing partial differential equation by McDonald and Harbaugh (1988):

$$\frac{\partial}{\partial x} \left( K_{xx} \cdot \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \cdot \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{zz} \cdot \frac{\partial h}{\partial z} \right) - w = 0$$
(1)

where  $K_{xx}$ ,  $K_{yy}$  and  $K_{zz}$  = the hydraulic conductivities along the x, y and z directions (m/day), h = water head (m), w = sources and sinks (m<sup>3</sup>/day).

After the optimization of injection and extraction rates in MODFLOW, grout penetration was modeled in UTCHEM. The 3-D flood simulator UTCHEM simulates major physical phenomena including 3-phase flow, diffusion, dispersion, dilution effects, adsorption for oil, surfactants and polymers, phase density, component density and composition phase viscosity. Further information about the governing equations in UTCHEM can be found in CPGE (2000).

### Numerical model setup

The numerical study was performed in a reservoir which was consisted of a single uniform sand layer with full water saturation. The reservoir had plan dimensions of 100 m by 25 m and a depth of 25 m (Figures 1 and 2). Such a depth made it possible to simulate the catastrophic liquefiable depth of 15 m reported by Seed et al. (2003). With the intention of minimizing the boundary effects, the target treatment volume was located in the middle of the simulation reservoir leaving enough space between the treatment volume and the side boundaries. The treatment volume had 6 m by 6 m fixed width and depth respectably where as the length of the treatment volume varied from 4 to 20 m (Figures 1 and 2). 4 injection and 4 extraction wells with 2 m intervals were located on the either side of the treatment volume. Constant pressure boundaries were located on the left and right side of the simulated reservoir. The remaining sides were designated as closed flow boundaries. The simulated reservoir was generated with 2 m x 1 m x 1 m blocks (on x, y and z directions respectively). The simulation details given above were identically applied both to MODFLOW and UTCHEM models.

## Determining the maximum injection and extraction rate using MODFLOW

It is preferable to inject the grout into the ground as quickly as possible. The injection rate is usually limited by injection depth, permeability and the strength of the soil formation. On site, the assessment of maximum possible injection and extraction rates can accurately be made by several injection trials. On the other hand, numerical simulations can also be useful in determining maximum possible rates. They can be estimated dependent on the desired steady state water levels around the injection and extraction wells. The steady state water level is related to the injection and extraction rates and the permeability of the reservoir. In this study, MODFLOW will be used to find the maximum possible injection and extraction rate that will maintain the water table elevation at a certain level above the injection and extraction wells.

### **MODFLOW** simulation

In a well-controlled grouting scheme, the groundwater line should be kept at a certain profile while achieving maximum possible injection and extraction rates. Therefore in this study, the injection and extraction rates were designated as the maximum rates that are theoretically possible for a certain hydraulic conductivity level. Groundwater profile was kept constant at different hydraulic conductivity levels by adjusting the injection and extraction rates. The hydraulic conductivity levels were 0.001 (let's say k=0.001), 0.05 (2k), 0.01 (10k), 0.1(100k) cm/s respectively. 3-dimensional version of Darcy's law in Equation 1 can be used to determine the rates which will produce an identical groundwater profile at different permeability levels.

Since the hydraulic conductivities in our numerical model are the same at all directions, we can use a common notation "K" instead of the hydraulic conductivity notations  $K_{xx}$ ,  $K_{yy}$  and  $K_{zz}$ . By plugging "K" in Equation 1 and we can rewrite Equation 1 as:



Figure 1. 3D reservoir setup in UTCHEM and MODFLOW.



Figure 2. Details of the reservoir setup in UTCHEM and MODFLOW.

$$\frac{\partial}{\partial x} \left( K \cdot \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \cdot \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \cdot \frac{\partial h}{\partial z} \right) - w = 0$$
(2)

K is constant therefore we can move it out from the derivative function parenthesis. By rearranging Equation 2, we finally write Equation 3:

$$\frac{\partial}{\partial x} \cdot \left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y} \cdot \left(\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z} \cdot \left(\frac{\partial h}{\partial z}\right) = \frac{w}{K}$$
(3)

In our model, term "w" is the net flow input (w = injection rate – extraction rate) induced by injection and the extraction wells. The left side of Equation 3 represents the water profile at 3-D media.

Max. Drawdown=1.5 m



Figure 3. Drawdown observed in MODFLOW under steady-state conditions a) Profile view b) Plan view.

According to Equation 3, to achieve an identical water profile in the system at different hydraulic conductivity levels,  $\frac{W}{K}$  ratio should be kept constant. Only way to achieve this is, to proportionally increase the net flow input "w" by the increasing hydraulic conductivity level "K".

As a first step in MODFLOW, a suitable steady-state injection profile was determined by assigning trial injection and extraction rates. The trial injection and extraction rates were lowered starting from high initial rates until a stable ground water profile was achieved. The first trials were performed by 0.001 (k) cm/s hydraulic conductivity. In the first trials, it was found out that, when the extraction rate was about two times of the injection rate, the best augmented flow towards the extraction wells was observed. According to the trial results. 6.6 (let's say R=6.6) m<sup>3</sup>/dav/well of extraction and 3.3 (0.5R) m<sup>3</sup>/day/well of injection developed about 1.5 m drawdown in steady state condition (Figure 3). 1.5 m drawdown was enough to maintain the water level about 0.5 m above the extraction wells. Such a steady state water profile was designated as the target water profile for other permeability levels. For soils with higher permeabilities, it is expected that 1.5 m of maximum drawdown can be achieved by higher extraction rates. Using such hypothesis, extraction and injection rates were increased 5 times (33 (5R) m3/day/well and 16.5 (2.5R) m3/day/well respectively) for the reservoir with 0.005 (5k) cm/s hydraulic conductivity. As expected, 1.5 m target drawdown was achieved by proportionally increased extraction and injection rates. For higher hydraulic conductivities such as 0.01 (10k) m<sup>3</sup>/day/well and 0.1 (100k) m<sup>3</sup>/day/well, the extraction rates were increased proportionally and 1.5 m target drawdown was also achieved. The direct proportionality between hydraulic conductivity and the extraction rate to obtain similar drawdown is summarized in Table 1.

# Parametric UTCHEM simulation with upper boundary injection rates

The maximum injection and extraction rates calculated in MODFLOW simulations were used as an input for UTCHEM simulations. UTCHEM simulations were run for 4 different injection-extraction well distances: 4, 6, 10 and 20 meters. Therefore, the treatment volume changed according to the distance between injection and extraction wells. The reservoir dimensions in UTCHEM were 100 m x 25 m x 25 m as they were in MODFLOW simulation. The left and the right boundaries were constant pressure boundaries. Extraction and injection rates are applied according to the MODFLOW rates tabulated in Table 2. In UTCHEM, intrinsic permeability is used, therefore hydraulic conductivity values given in Table 2, were converted to millidarcies (mD), assuming that the viscosity of the water is 1 cP. As an example, a 0.1 cm/sec hydraulic conductivity value corresponds to 100 000 mD.

16 different run configurations were established. The differences between runs were the distance between the injection and extraction wells (4 combinations) and the hydraulic conductivity (4 combinations). The treatment durations changed according to the size of the treatment volume whereas the injection/extraction rates were constant for each permeability level. The treatment durations were various between 0.9 h and 17 days (Table 2).

### RESULTS

The results of the parametric study were presented in terms of concentration distributions after 1 treatment pore volume of colloidal silica injection. 16 run configurations

Hydraulic	Extraction	Injection rate (a half of the	Observed drawdown
conductivity (cm/s)	rate (m <sup>3</sup> /day/well)	extraction) (m <sup>3</sup> /day/well)	above the extraction wells (m)
0.1	-6.6 x 10 <sup>2</sup>	3.3 x 10 <sup>2</sup>	~1.5
0.01	-6.6 x 10 <sup>1</sup>	3.3 x 10 <sup>1</sup>	~1.5
0.005	-3.3 x 10 <sup>1</sup>	1.65 x 10 <sup>1</sup>	~1.5
0.001	-6.6 x 10 <sup>0</sup>	3.3 x 10 <sup>0</sup>	~1.5

Table 1. Maximum allowable drawdown for various hydraulic conductivities.

Table 2. Maximum theoretical injection and extraction rates.

Distance between wells (m)	4	4	e	6	1	0	2	0	
1 treatment volume (m <sup>3</sup> )	14	44	21	6	36	60	72	20	
Porosity	0	.3	0.	.3	0	.3	0	.3	
I treatment pore volume (m <sup>3</sup> )		43.2		64.8		108		216	
Inj. Duration for 1 treat. vol. (day(s)) with 0.1 cm/s hyd. cond.		0.03 0.0		05	0.08		0.16		
Inj. Duration for 1 treat. vol. (day(s)) with 0.01 cm/s hyd. cond.	l. 0.33		0.49		0.82		1.64		
Inj. Duration for 1 treat. vol. (day(s)) with 0.005 cm/s hyd. cond.	0.65		0.98		1.64		3.27		
Inj. Duration for 1 treat. vol. (day(s)) with 0.001 cm/s hyd. cond.	3.27		4.91		8.18		16.36		
	Inj.	Ext.	Inj.	Ext.	Inj.	Ext	Inj.	Ext.	
Total <b>inj./ext.</b> rate for 4 wells (m <sup>3</sup> / day) at <b>0.1</b> cm/s hyd.cond.	1320	2640	1320	2640	1320	2640	1320	2640	
Total inj./ext. rate for 4 wells (m <sup>3</sup> / day) at 0.01 cm/s hyd.cond.	132	264	132	264	132	264	132	264	
Total inj./ext. rate for 4 wells (m <sup>3</sup> / day) at 0.005 cm/s hyd.cond.	66	132	66	132	66	132	66	132	
Total inj./ext. rate for 4 wells (m <sup>3</sup> / day) at 0.001 cm/s hyd.cond.	13.2	26.4	13.2	26.4	13.2	26.4	13.2	26.4	

are summarized in Table 3. The concentration plots with same injection-extraction distance produce identical plots regardless of the varying soil hydraulic conductivity value (e.g. Run1, Run 2, Run 3 and Run 4 produced the same plot). This was expected because the injection rates were proportionally adjusted according to the soil permeability. Therefore, only one typical plot for certain injectionextraction distance is illustrated. Figures 4, 5, 6 and 7 show the amount of concentration filling the treatment volume after 1 treatment pore volume of colloidal silica injected at 4, 6, 10 and 20 m injection and extraction well distances respectively. According to the figures, the volume filled with colloidal silica changes with the distance between injection and extraction wells. The increasing injection-extraction well distance decreases the amount filled with colloidal silica in the treatment volume.

The concentration plots are useful for visualization of injection but do not provide any quantitative comparison. Instead of plotting the entire data obtained from all cells, we can define a single value to represent the efficiency of the treatment. This will be called "Treatment Ratio" and it is given as below:

 $\label{eq:treatmentRatio} \mbox{TreatmentRatio} = \frac{\mbox{Sumof the concentrations in the treatment volume}}{\mbox{Number of cells in the treatment volume}} \ \ (4)$ 

Note that the source concentration in UTCHEM is "1" and they are located in the middle of the cells. In other words, if full concentration (=1) is achieved in a cell, the treatment ratio is 1 for that cell. Figure 8 shows the Treatment Ratios for all 16 Runs.

According to Figure 8, Treatment Ratios of the same injection-extraction spacing regardless of soil permeability are fairly equal. In contrast, Treatment Ratio decreases by increasing injection-extraction distance. In other words, with 4 m spacing, about 60% percent of the treatment pore could be filled by using 1 treatment pore volume of colloidal silica whereas this is only 30% for 20 meter spacing. Low treatment ratio is attributed to the decreasing flow attraction towards extraction wells considering the fact that they are farther away from injection wells.

It is evident from Figure 8 that 1 treatment pore volume of grout is not enough to treat the prismatic target zone. Next step should be calculating the required amount of grout to fully saturate the voids with colloidal silica. Therefore, UTCHEM was run until considerable amount of coverage was achieved within the treatment zone. For each run, time needed to cover the treatment zone was recorded. Figure 9 shows the amount of pore volumes needed to fully cover the treatment zone.

The amount needed is "two treatment pore volumes"

Distance		Hydraulic Conductivity (cm/s)						
Distance	0.1	0.01	0.005	0.001				
	4	Run 1	Run 2	Run 3	Run 4			
Distance between	6	Run 5	Run 6	Run 7	Run 8			
wells (m)	10	Run 9	Run 10	Run 11	Run 12			
	20	Run 13	Run 14	Run 15	Run 16			

Table 3. Run configurations.



**Figure 4.** Concentration distribution after 1 treatment pore volume injected; the distance between injection and extraction well groups is 4 m. The plot is similar for different hydraulic conductivities (0.1, 0.01, 0.005 and 0.001 cm/s).



### Source Concentration = 1

**Figure 5.** Concentration distribution after 1 treatment pore volume injected; the distance between injection and extraction well groups is 6 m. The plot is similar for different hydraulic conductivities (0.1, 0.01, 0.005 and 0.001 cm/s).



Source Concentration = 1

**Figure 6.** Concentration distribution after 1 treatment pore volume injected; the distance between injection and extraction well groups is 10 m. The plot is similar for different hydraulic conductivities (0.1, 0.01, 0.005 and 0.001 cm/s).



**Figure 7.** Concentration distribution after 1 treatment pore volume injected; the distance between injection and extraction well groups is 20 m. The plot is similar for different hydraulic conductivities (0.1, 0.01, 0.005 and 0.001 cm/s).



Figure 8. Treatment ratios for different injection and extraction distances and hydraulic conductivities using maximum possible injection and extraction rates.



**Figure 9.** Relationship between number of treatment pore volume of grout needed to fully saturate the treatment volume and the distance between injection and extraction wells.

when injection - extraction distance is 4 m. This requires only 1 extra pore volume to saturate the prismatic treatment space with colloidal silica. The required amount of grout increases to 10 pore volumes when the injectionextraction well distance is 20 m. In other words, 9 pore volumes are wasted to saturate 1 pore volume of prismatic space in the configuration with 20 m injectionextraction well distance. The trend line in Figure 9 shows that when the distance between injection and extraction wells is 2 m, only 1 pore volume of grout is needed.

As an example, Figure 10 shows the concentration distribution after 5 treatment pore volumes of colloidal silica are injected at the configuration with 10 m injection - extraction distance. It can be seen that the injected grout fully covered the treatment volume, at the same time, spread in a great amount on the other directions.

### Lower boundary injection - extraction rates

In the concentration distributions illustrated between Figures 4 to 7, typically, the injected grout formed a bulb bulging towards the extraction well dependent on the extraction distance. The rates used were the fastest theoretical rates, however these high rates may not be achieved on site at all times. To investigate the effect of lower injection rates, the configuration with 10 m injection-extraction distance at 0.01 cm/s hydraulic conductivity was run at one tenth (3.3 m<sup>3</sup>/day/well instead of 33 m<sup>3</sup>/day/well) of the rate given in Table 1. The

comparison of two runs with different injection and extraction rates is illustrated in Figure 11. The investigation indicated that the injected grout sank towards the bottom of the aquifer when a lower injection rate was used.

### Conclusion

A parametric study was conducted using MODFLOW and UTCHEM programs. Some of conclusions from this study can be given as follows:

(i) Extraction rate is important for horizontal colloidal silica delivery. The maximum extraction rate that can be established in an aquifer is dependent on the hydraulic conductivity of the aquifer.

(ii) Using the maximum available injection-extraction rates provided, considerable amount of success is achieved when the injection - extraction well distance is between 2 and 4 m. In that case, 1-2 pore volumes of grout are enough to cover the treatment volume. Configuration with 6 m injection-extraction distance required 3 pore volumes of injection. For 10 and 20 m injection-extraction distance, the required injection volume was excessive. Overall trend shows that exceeding 4 m of injection-extraction distance is most likely to be uneconomical.

(iii) In cases where low injection or extraction rates have to be applied, sinking problem can occur. Proper colloidal



**Figure 10.** Concentration distribution after 5 treatment pore volumes of injection at the configuration with 10 m injection - extraction distance.



**Figure 11.** Concentration distribution with different injection rates; upper one established with theoretical maximum extraction rate, bottom one established with 1/10 of that rate at 10 times of injection duration.

silica grout delivery requires high injection-extraction rates. The minimum required injection rate should be estimated before site injection.

(iv) Although this study covered soils with 0.1 cm/s hydraulic conductivity, Hamderi (2010) reports that high injection and extraction rates such as 330/-660 m<sup>3</sup>/day/well rates at 0.1 cm/s hydraulic conductivity may not be practically achieved on site conditions in loose saturated deposits due to instability. Therefore, it is questionable to use colloidal silica in such conductive soils.

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