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Development of a linear model for computational simulation of an induction motor driven by frequency inverter

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Frequency inverters are largely used as induction motor drives, so that a linear model for the torque of an induction motor can be useful for making computational simulations about that control technique. This paper propose to develop a model for a torque of an induction motor based on the torque versus speed characteristic, using the linearization method applied in a specific operating range, which normally work the frequency inverters. This model becomes interesting for evaluating the performance and calculating parameters of the motor, when it's driven by the frequency inverter. From a specific expression for the torque versus speed, the application of the linearization method leads to a linear expression to the torque relating the induction motor parameters. The validation of this linear model is obtained comparing with some original torque curves of the induction motor given by manufacturers. Results showed accordance with curves values in the range of interest.

Key words: Induction motors, linearization methods, mathematical models.

INTRODUCTION

Three-phase induction motor is the most common engine used in drives with speed control, and it is a more attractive option because of its lower maintenance costs. Frequency inverter is its typical control option; also its ease of installation and parameterization constitutes a gain for operation and maintenance of the drive system.

There are several possibilities of three-phase induction motors with frequency inverters application associated to the growing demand for renewable energy systems, that is, to actuate an electric vehicle.

There are several ways to control and to vary speed in electric motors, that is, the control of field current in direct current (DC) motors and the control of voltage and frequency in synchronous and induction motors.

The dynamics and steady-state characteristics of induction motor drive systems and frequency converters were analysed by Shinohara and Nonaka (1987), Liu et al. (1996), and Hu et al. (2000).

The PWM (Pulse Width Modulation) inverter control technique to drive induction motors was studied throughout experimental results and simulations for controlling the speed of three phase induction motor by maintaining v/f ratio at constant value by Trovao et al. (2002), Salerno et al. (2003), and Patil and Kurkute

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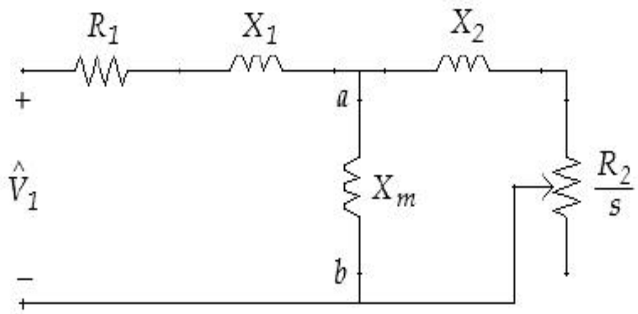

Figure 1. Typical equivalent circuit per phase for a three-phase induction motor.

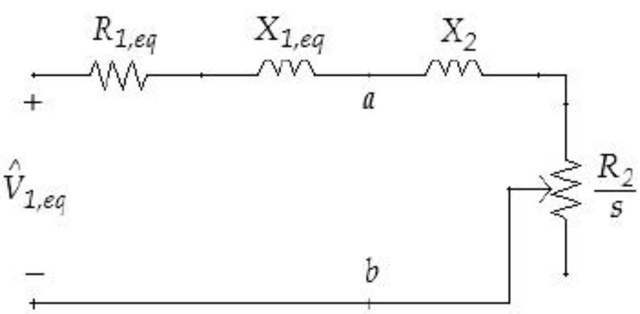

Figure 2. Thevenin's equivalent circuit per phase for a three-phase induction motor.

(2006). Other control techniques were presented, such as rotor field-oriented control (RFOC) scheme by Bojoi et al. (2006).

Modelling and control approaches for problems associated to three-phase induction motor systems, including dynamic models of frequency controlled drive, were proposed and simulation results were provided and compared to current industrial ones by Papafotiou et al. (2004), Mastorakis et al. (2009), Razik et al. (2009), Rinkeviciene and Petrovas (2009), and Bernal et al. (2010).

The results of simulations in SIMULINK and LabVIEW for open and closed loop system were presented in works such as Arumugam and Ramareddy (2010), and Hosek and Diblik (2011).

Those approaches are based in direct and quadrature axis theory, also called dq0 transformation. In fact, this theory is embracing, but otherwise it's also complex for simplified applications, depending on parameters not always available.

In order to present a simple and functional alternative in modelling, this work is consists of an establish mathematical linear model for the torque of a three-phase induction motor to be used in simulations involving frequency inverters. Starting of the equivalent circuit of the induction motor, the objective is obtaining a linear expression of torque in function of the speed (angular speed) relating the usual motor parameters.

However the linear model proposal is applicable only on a range often used by the frequency inverters, a linear region of a torque versus speed curve of the motor, in other words the range beginning from the 150% of nominal torque until the zero torque on synchronous speed. It can be also applied in a range from zero torque until -150% of nominal torque, considering the operation of the motor in generator mode.

MATERIALS AND METHODS

This study is based on linearization method to determinate a representative expression for torque versus speed characteristic of a three-phase induction motor in a particular region of operation with frequency inverter. The validation of this linear model is obtained in comparison to original curves of the induction motor chosen for simulation. Due to the model used, this method is not applicable for transient analysis, and then is only applied to steady-state analysis.

Linear model for three-phase induction motor torque

The mechanical torque in a three-phase induction motor can be represented by equivalent circuit with Thevenin's theorem analysis. Figure 1 shows the equivalent circuit per phase for a three-phase induction motor. Losses were neglected in the stator core, but the associated error is relatively small (Fitzgerald et al., 2006).

The fractional slip, or simply slip, can be presented as Equation (1).

$$s = \frac{\omega_s - \omega_r}{\omega_s} \tag{1}$$

From Equation (1), where,
s – Fractional slip (%);
ω_s – Stator angular synchronous speed (rad/s);
ω_r – Rotor angular speed (rad/s).
The representative scheme of equivalent circuit using Thevenin's theorem is shown in Figure 2.

The equivalent phase voltage in phasor form is presented by Equation (2).

$$\hat{V}_{1,eq} = \hat{V}_1 \left(\frac{jX_m}{R_1 + j\left(X_1 + X_m\right)} \right) \tag{2}$$

From Equation (2), where,
R_1 – Effective resistance of the stator (Ω);
$\hat{V}_1$ – Phase voltage at the stator (V);
$\hat{V}_{1,eq}$ –Thevenin's equivalent phase voltage in the stator (V);
X_1 – Leakage reactance of the stator (Ω);
X_m – Magnetizing reactance (Ω).

The equivalent resistance and reactance of the stator can be expressed according to the equivalent impedance of the stator ($Z_{1,eq}$), which is presented in its phasor form by Equation (3).

$$\hat{Z}_{1,eq} = R_{1,eq} + jX_{1,eq} = \frac{jX_m\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_m\right)} \quad (3)$$

From Equation (3), where,

$X_{1,eq}$ – Thevenin's equivalent inductive reactance in the stator (Ω);
$\hat{Z}_{1,eq}$ – Stator impedance (Ω).

According to the Thevenin's equivalent circuit analysis, a characteristic equation of mechanical torque (T_{mec}) of three-phase induction motor (Fitzgerald et al., 2006) can be obtained through Equation (4), where the magnitude (n_{phases}) represents the number of phases of three-phase induction motor.

$$T_{mec} = \frac{1}{\omega_s}\left\{\frac{n_{phases}\hat{V}_{1,eq}^2\left(\frac{R_2}{s}\right)}{\left[R_{1,eq} + \left(\frac{R_2}{s}\right)\right]^2 + \left(X_{1,eq} + X_2\right)^2}\right\} \quad (4)$$

From Equation (4), where,
T_{mec} – Mechanical torque (Nm);
n_{phases} – Number of phases (-);
$R_{1,eq}$ – Thevenin's equivalent resistance in the stator (Ω)
R_2 – Resistance of the related rotor (Ω);
X_2 – Leakage reactance of the related rotor (Ω).

Assuming that the effective stator resistance is negligible ($R_1 \approx 0$), the Equations (2) and (3) take forms of Equations (5) and (6).

$$\hat{V}_{1,eq} = \hat{V}_1\left(\frac{X_m}{\left(X_1 + X_m\right)}\right) \quad (5)$$

$$Z_{1,eq} = X_{1,eq} = \frac{X_1X_m}{X_1 + X_m} \quad (6)$$

Being the reactance expressed by X = ωL, the Equations (5) and (6) can be presented as Equations (7) and (8).

$$\hat{V}_{1,eq} = \hat{V}_1\left(\frac{\omega_e L_m}{\left(\omega_e L_1 + \omega_e L_m\right)}\right) = \hat{V}_1\frac{L_m}{L_1 + L_m} \quad (7)$$

$$Z_{1,eq} = X_{1,eq} = \frac{\omega_e L_1 \omega_e L_m}{\omega_e L_1 + \omega_e L_m} = \omega_e\frac{L_1L_m}{L_1 + L_m} \quad (8)$$

From Equations (7) and (8), where,
L_1 – Stator leakage inductance (H);
L_m – Magnetizing inductance (H);
ω_e – Angular speed of the supply network of the stator (rd/s).

Equations (7) and (8) can be simplified as Equations (9) and (10), introducing the constants k_a and k_b, as they follow.

$$k_a = \frac{L_m}{L_1 + L_m}$$

$$V_{1,eq} = k_a V_1 \quad (9)$$

$$k_b = \frac{L_1L_m}{L_1 + L_m}$$

$$X_{1,eq} = k_b\omega_e \quad (10)$$

Separately, constants k_a and k_b do not represent a physical meaning, but it can be observed that the ratio k_b/k_a leads to L_1, the leakage stator induction. Also, the sum $L_1 + L_m$ is associated with the apparent stator reactance X_{vz}, measured at the stator terminals in motor under blank operating conditions, in other words $X_{vz} = \omega_e(L_1+L_m)$ (Fitzgerald et al., 2006).

Applying Equations (9) and (10) into Equation (4), with $R_1 \approx 0$, the mechanical torque expression can be rewritten as Equation (11).

$$T_{mec} = \frac{1}{\omega_s}\left[\frac{n_{phases}k_a^2V_1^2\left(\frac{R_2}{s}\right)}{\left(\frac{R_2}{s}\right)^2 + \left(k_b\omega_e + X_2\right)^2}\right]. \quad (11)$$

Putting s² in evidence at Equation (11) denominator, considering $X_2 = \omega_e L_2$ and simplifying terms, becomes Equation (12).

$$T_{mec} = \frac{1}{\omega_s}\left[\frac{n_{phases}k_a^2V_1^2R_2s}{R_2^2 + \omega_e^2\left(k_b + L_2\right)^2s^2}\right] \quad (12)$$

Knowing that the source angular frequency is $\omega_e = 2\pi f_e$, with the stator angular synchronous speed defined as $\omega_s = 2\omega_e/p$, where p is the motor poles number, therefore $\omega_s = 4\pi f_e/p$. Considering this and according the Equation (1), after some algebraic manipulations in the Equation (12), the mechanical torque expression turns to Equation (13).

$$T_{mec} = \frac{1}{\omega_s^2}\left[\frac{n_{phases}k_a^2V_1^2R_2\left(\omega_s - \omega_r\right)}{R_2^2 + \omega_e^2\left(k_b + L_2\right)^2\left(\omega_s - \omega_r\right)^2/\omega_s^2}\right]$$

$$T_{mec} = \frac{n_{phases}p^2k_a^2V_1^2R_2\left(\omega_s - \omega_r\right)}{16\pi^2f_e^2\left(R_2^2 + p^2\left(k_b + L_2\right)^2\left(\omega_s - \omega_r\right)^2/4\right)} \quad (13)$$

Dividing numerator and denominator of Equation (13) by p²ka²R₂, and rearranging the terms.

$$T_{mec} = n_{phases}\left(\frac{V_1}{f_e}\right)^2\frac{\left(\omega_s - \omega_r\right)}{\frac{16\pi^2R_2}{p^2k_a^2} + \frac{4\pi^2\left(k_b + L_2\right)^2}{R_2k_a^2}\left(\omega_s - \omega_r\right)^2} \quad (14)$$

Making $k_1 = 16\pi^2R_2/k_a^2$ and $k_2 = 4\pi^2\left(k_b + L_2\right)^2/\left(R_2k_a^2\right)$, the Equation (14) is simplified as Equation (15).

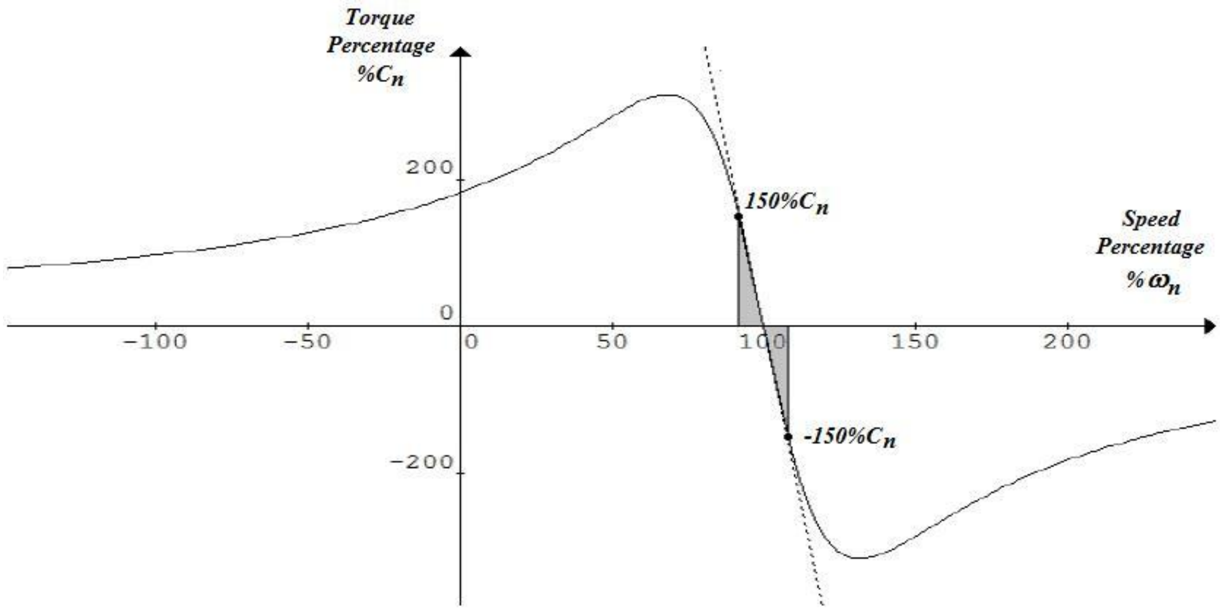

Figure 3. Typical conjugate (%C) versus speed (%ω) curve for a three-phase induction motor. Operation range with frequency inverter is detached.

$$T_{mec} = n_{phases}\left(\frac{V_1}{f_e}\right)^2 \frac{(\omega_s - \omega_r)}{\frac{k_1}{p^2} + k_2(\omega_s - \omega_r)^2} \quad (15)$$

From Equation (15), where,
k_1 – Resistance of induction motor (Ω);
k_2 – Reactance of induction motor (H^2/Ω).

For k_1 and k_2 with appropriate substitutions, are presented Equations (16) and (17).

$$k_1 = 16\pi^2 R_2\left(1 + \frac{L_1}{L_m}\right)^2 \quad (16)$$

$$k_2 = \frac{4\pi^2}{R_2}\left[L_1 + L_2\left(1 + \frac{L_1}{L_m}\right)\right]^2 \quad (17)$$

The both constants k_1 and k_2 can be determined from the values of resistance and reactance characteristics of three-phase induction motor, as shown in Equations (16) and (17). Normally, are done no-load and locked rotor tests to determine the parameters.

A constraint for use of Equation (15) is that it represents approximately the characteristic of mechanical torque of a motor class A, and it should be used with caution for classes B, C, and D. That is, the operation region of three-phase induction motor for do not take divergences should be analysed.

It can be seen from Equation (15) that, if the ratio V_1/f_e and the difference $\omega_s - \omega_r$ are constant, the mechanical torque applied to the load will be also constant. This characteristic is normally used in three-phase induction motor operation driven by a frequency inverter. The shaded region in Figure 3 indicates the operating range of a three-phase induction motor driven by a frequency inverter, in other words, the region of ± 150 %C_N (WEG, 2011). It is noticed that in this region, the three-phase induction motor has a nearly linear behaviour.

Owing to the frequency of use of drives with inverter-motor configuration, the linearization method is used to simplify the mechanical torque curve of an induction motor, in order to generate a simple model for simulation with frequency inverter.

Once the three-phase induction motor curve must cross through the point (ω_s; 0), for configuration with frequency inverter can be determined by a linear equation represented by a tangent line at this point. The principle of linearization (Nise, 2009), based on Taylor's series expansion, for a region near a specific point, can be presented as Equation (18).

$$f(x) = f(x_0) + \left[\frac{df}{dx}\right]_{x=x_0}(x - x_0) \quad (18)$$

The linearised torque model of three-phase induction motor is determined through the derivative of the Equation (15), presented as Equation (19).

$$\frac{dT_{mec}}{d\omega_r} = n_{phases}\left(\frac{V_1}{f_e}\right)^2 \frac{-\frac{k_1}{p^2} + k_2(\omega_s - \omega_r)^2}{\left(\frac{k_1}{p^2} + k_2(\omega_s - \omega_r)^2\right)^2} \quad (19)$$

From Equation (19), f_e – Frequency of the supply network (Hz).

Considering the linearisation represented by Equation (18) and the point where $\omega_r = \omega_s$, the mechanical torque of three-phase induction motor is evaluated through Equation (20).

Table 1. Results for linear model developed.

Motor #	1	2	3	4
Model	1LA7075-2EA90	1LA7131-2EA90	1LA7083-4EB90	1LA7131-4EA90
Power (hp)	1	10	1	10
Number of poles	2	2	4	4
Nominal torque (Nm)	2.1	20	4.18	40.9
Nominal rotation (rpm)	3,410	3,530	1,715	1,755
Synchronous rotation (rpm)	3,600	3,600	1,800	1,800
Determination of k_1 (Ω)	1,527.8	59.132	1,374.2	74.354
Torque curve (C_{mec})	$-0.0111(n_r - 3,600)$	$-0.2857(n_r - 3,600)$	$-0.0492(n_r - 3,600)$	$-0.9089(n_r - 3,600)$

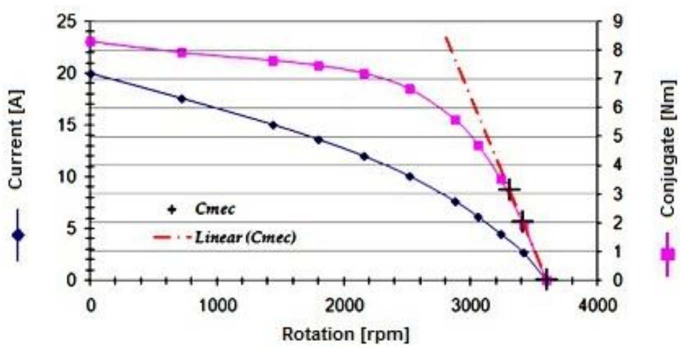

Figure 4. Comparison of motor torque #1 and the linearised model, at the operation region with frequency inverter.

$$T_{mec} = T(\omega_s) + \left[\frac{dT}{d\omega_r}\right]_{\omega_r=\omega_s} (\omega_r - \omega_s)$$

$$T_{mec} = -n_{phases}\frac{p^2}{k_1}\left(\frac{V_1}{f_e}\right)^2 (\omega_r - \omega_s) \qquad (20)$$

Therefore, Equation (20) can be used as an alternative approximation of the linear part of steady-state torque versus speed curve for constant V/f ratio in the region ±150%C_N. The magnitude of k_1 can be determined through nominal parameters of the motor and obtained from manufacturers' data sheets. If it is need to work with rotations, knowing that $\omega = 2\pi n/60$, with *n* being the rotation in rpm, Equation (20) can be used as shown in Equation (21).

$$T_{mec} = -\frac{\pi}{30} n_{phases}\frac{p^2}{k_1}\left(\frac{V_1}{f_e}\right)^2 (n_r - n_s) \qquad (21)$$

RESULTS AND DISCUSSION

Application and validation

The linear model was compared to its equivalent motor torque curves (Siemens, 2011b) in the region of operation with frequency inverter for validation of Equation (21). The three-phase induction motors with nominal voltage of 220 V_{AC} and nominal frequency of 60 Hz was considered. Equation (22) was used to determinate the synchronous rotation.

$$n_s = \frac{120 f_e}{p} \qquad (22)$$

The curves of three-phase induction motors of 1 and 10 hp, with 2 and 4 poles, according to the manufacturer's data sheets (Siemens, 2011a), were analysed. Table 1 shows nominal and evaluated values.

The comparison of the results for motor #1 is showed in Figure 4.

The points indicated by (+) in Figure 4 represent, from bottom to top, the null torque (synchronous speed), the nominal torque and the limit of 150%C_N, and it can be observed that there is a correspondence between the curves, presenting a very low diversion.

The points shown in Figure 4 begin to deviate more than others and it becomes inconvenient to represent them by linearisation. There are other results that prove the applicability of the linear model.

The comparative of the results for motor #2, #3, and #4 are shown in Figures 5, 6, and 7. It is worth observing that, normally, the value of L_m is large enough compared with L_1, or $L_1/L_m \approx 0$, which results in a simplification of Equations (16) and (17). Thus, the constants k_1 and k_2 are only a function of R_2 and $L_1 + L_2$, respectively. Therefore, calculating k_1 by the Equation (20), it is obtained an approximation to R_2 from the Equation (16). Similarly, calculating k_2 from the Equation (15) it is obtained an approximation for $L_1 + L_2$ by Equation (17), so that the parameters of the equivalent circuit of three-phase induction motor can be determinate. This theoretical method can serve as a reasonable alternative when it is not possible to execute essays at the equipment.

Conclusions

The linear model described by Equations (20) and (21) is

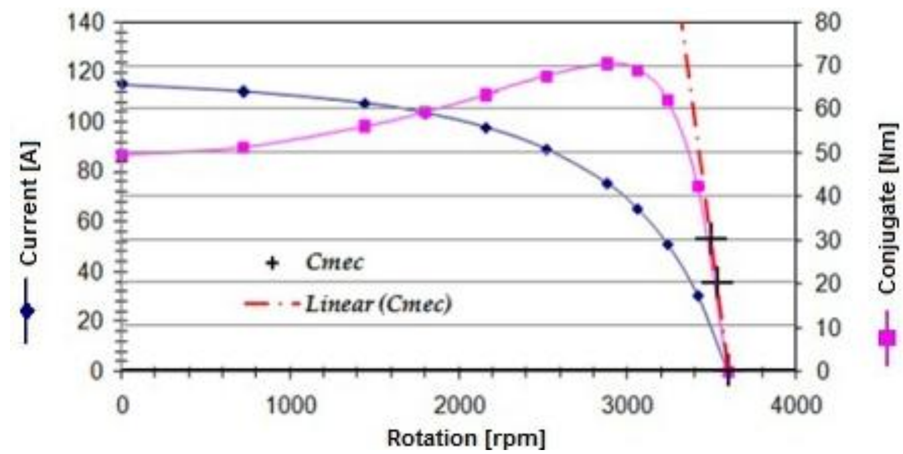

Figure 5. Comparison of motor torque #2 and the linearised model, at the operation region with frequency inverter.

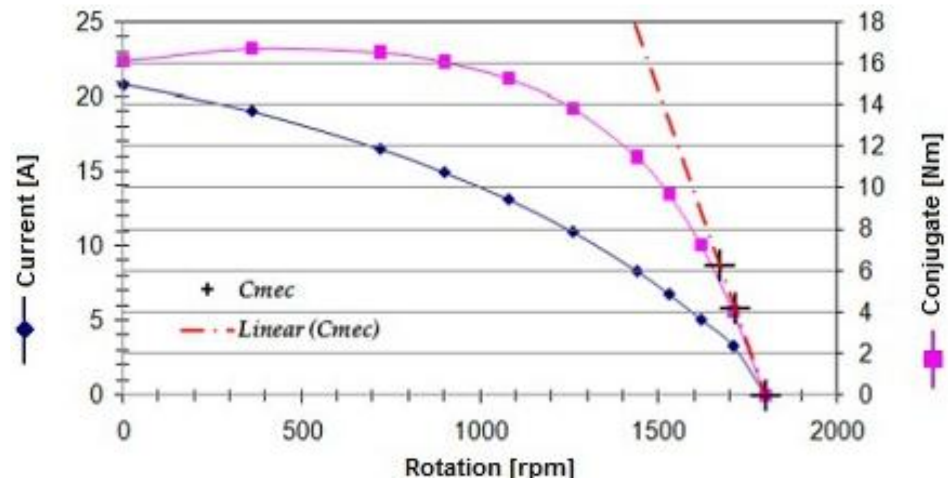

Figure 6. Comparison of motor torque #3 and the linearised model, at the operation region with frequency inverter.

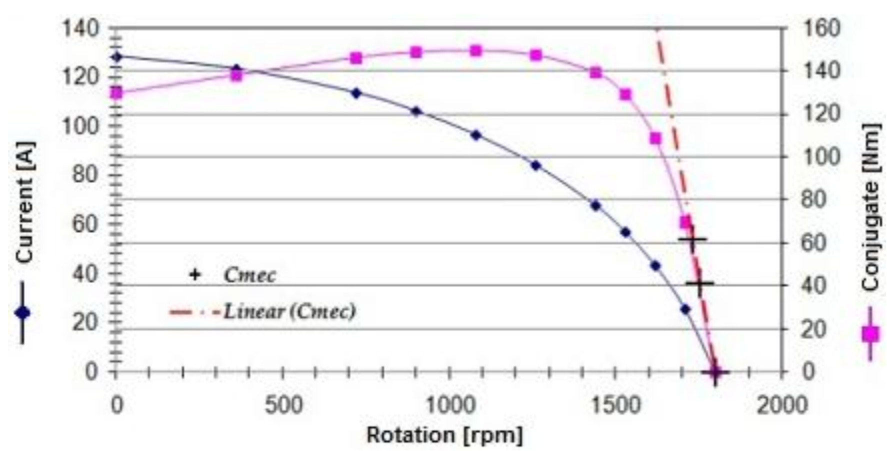

Figure 7. Comparison of motor torque #4 and the linearised model, at the operation region with frequency inverter

convenient for the representation of the characteristic curve of the torque versus speed of a three-phase induction motor, in the region of 150%C_N, which was proposed for analysis. The simplification provided results in a practice to represent the behaviour of a three-phase induction motor, in applications in simulation software such as MATLAB© or SCILAB©, both in analysis and time domain to frequency domain.

Based on the principle that the torque curve of the three-phase induction motor shows some symmetry in relation to the point of synchronous speed, the linear representation is also applicable for the region up to 150%C_N, whose three-phase induction motor's behaviour takes the form of a generator. The linear configuration makes clear the ratio V/f and the difference $\omega_s - \omega_r$, which can serve as entry for the application of a frequency inverter in a constant torque configuration.

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