Scientific Research and Essays Vol. 5(16), pp. 2148-2154, 18 August, 2010
Available online at http://www.academicjournals.org/sre
ISSN 1992-2248 ©2010 Academic Journals

Full Length Research Paper

Adaptive network based on fuzzy inference system estimates of geoid heights interpolation

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Accepted 18 March, 2010

Soft computing methods such as fuzzy logic and artificial neural networks (ANN) have gained popularity in solving engineering problems. Particularly, fuzzy logic and especially developments in uncertainty assessment, enable us to construct and validate precise geoid models. We investigate three different point densities, five variations in the numbers of subsets and five different membership functions when forming the fuzzy model to calculate the geoid heights in the Istanbul (Turkey) area. The results of the fuzzy model are compared with geoid heights obtained using GPS and leveling. The fuzzy model has been verified against the test points. The results indicate that constructing the fuzzy model with a point density of at least one point in 25 km^2 , carefully selected number of subsets in accordance with point density and a Gaussian membership function, gives superior performance.

Key words: ANFIS, fuzzy logic, type of membership function, number of fuzzy subsets, point density, geoid height.

INTRODUCTION

The geoid is the equipotential surface of the earth's gravity field which coincides with mean sea level. The geoid provides the reference surface for heights which is typically used in surveying and engineering. The importance of accurately knowing the geoid has increased with the advent of satellite positioning systems, such as the Global Positioning System (GPS). GPS provides ellipsoidal heights, or ellipsoidal height differences, relative to a geocentric ellipsoid. In order to convert ellipsoidal heights, reckoned along the ellipsoidal normal from the ellipsoid to orthometric heights and the reckoned along the curved plumb line from the geoid, the geoid heights (undulations) must be known. The following well-known relation exists.

$$N = h - H \qquad (1)$$

Where, N denotes the geoid height, h and H are the ellipsoidal and orthometric heights, respectively. Orthometric heights can be readily computed from (1) if the geoid and ellipsoidal height are known. Ellipsoidal heights, or ellipsoidal height differences, can be derived from GPS more economically than orthometric heights. Determination of the latter requires time-consuming leveling. More details can be found in Wellenhof and Moritz (2006); Featherstone (2001); Engelis et al. (1985); Torge (2001); Yilmaz and Arslan (2008).

Physical geodesy deals with the methods for determining the geoid and as such the geoid heights. All methods require either explicitly or implicitly the use of gravity. However, if we have a set of stations with known ellipsoidal and orthometric heights, then, we can use the difference h-H at these stations to compute N at other locations using appropriate interpolation techniques and the given ellipsoidal heights at these stations. In this study, we compute geoid heights using fuzzy logic by means of adaptive network based fuzzy inference systems (ANFIS). Specifically, we studied factors that affect the outcome of such computations. We used data from the region of Istanbul, Turkey.

MATERIALS AND METHODS

Materials

This study included 443 points with known latitude, longitude, ellipsoidal height and orthometric height in the area of Istanbul, Turkey (Ayan et al., 1999). We randomly selected 50, 200 and 393 points (models 1, 2, and 3) to examine the effect of point density in ANFIS. In case of model 1, we selected 43 reference (cardinal) points and 7 densification points as model points to form fuzzy model to compute the geoid heights. Similarly, the distribution of reference points and densification points were (44, 156) and (48, 345) for models 2 and 3, respectively. The points are homogenously distributed in the region, resulting in point densities of one point in about 100, 25 and 13 $km^{2,}$ respectively. The data covers the region 41° 29´11″ > φ > 40° 45´11″ in latitude and 29° 41´ 50″ > λ > 27° 57´ 36″ in longitude. The orthometric heights vary between 1.254 m and 484.981 m. The average standard deviation of the ellipsoidal heights after the GPS network adjustment is ± 3.0 cm. We have selected 50 points which had not been included in the fuzzy models (Ayan et al., 1999) to check the results of the computations. The distribution of the 50, 200 and 393 model points together with the 50 test points is seen in Figure 1 (a-c): Distribution of 50, 200, 393 model points (shown as +) and 50 test point (shown as +) in Istanbul

Methods

Adaptive network based fuzzy inference systems (ANFIS)

The subject of fuzzy logic and ANFIS is extensively treated in the literature. We only make a few remarks as related to this application, using freely some of the vanacular terminology. ANFIS are feed-forward adaptive networks which are functionally equivalent to fuzzy inference systems. The basic idea of ANFIS can be described as follows: a fuzzy inference system is typically designed by defining linguistic input and output variables and an inference rule base. However, the resulting system is just an initial guess for an adequate model. Hence, its premise and consequent parameters have to be tuned based on the given data in order to optimize the system performance. In ANFIS, this step is based on a supervised learning algorithm (Jyh- Shing, 1993).

Output membership functions are either linear or constant in ANFIS. The Sugeno fuzzy model is called a zero degree model when the output membership function is constant; it is called a first degree model when the output membership function is a first degree polynomial. A first degree Sugeno fuzzy model can be defined as follow:

A two-fuzzy ruled first-degree Sugeno fuzzy model can be defined as shown in Figure 2, where, the ANFIS structure was taken into account. It is assumed that the rule base contains two fuzzy if-then, rules of Takagi and Sugeno's type (Takagi and Sugeno, 1983).

Rule 1: If x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$
Rule 2: If x is A_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$

The symbols A and B denote the fuzzy sets defined for membership functions of x and y in the premise parts. The symbols p, q and r denote the consequent parameters. With this design, an output value can be obtained for every rule. The results in the Sugeno fuzzy model can be obtained as first degree polynomial $f = px + qy + r$ or as constant $f = r$. Studies have shown that the first degree polynomial gives better results. Therefore, the fuzzy model is formed using the first degree polynomial in this study.

In our application latitudes and longitudes are divided into 5 subsets, obtaining 5 × 5 = 25 rules. These rules are graphically shown in Figure 3. Some of these rules can be expressed verbally as:

If latitude is B1 and longitude is L1 then geoid height is N1
If latitude is B1 and longitude is L2 then geoid height is N2
If latitude is B2 and longitude is L3 then geoid height is N8
If latitude is B3 and longitude is L4 then geoid height is N14
If latitude is B4 and longitude is L5 then geoid height is N20

Figure 3, rules obtained when latitude and longitude are divided into 5 subsets of each. A first degree polynomial is written for each rule. For example, an equation can be written for rule number 3 stated above as $f_8 = p_8B + q_8L + r_8$ (B = latitude, L = longitude) and rule number 4 as $f_{14} = p_{14}B + q_{14}L + r_{14}$. The unknown of p, q and r will be solved for each of the 25 rules. To do this, model data is used and the values of membership functions are determined according to latitude and longitude. One or several rules are triggered by the data, including the determination of weights (w) and normalized weights $(\overline{w})$. At the end, the first degree polynomial coefficients related to this data are obtained.

We note that the product of normalized weights and first degree polynomial coefficients gives geoid heights. The results are obtained with 5 layers (Figure 2) in ANFIS. The membership functions of inputs are determined in the first layer. The firing strength (weight) of each rule is calculated in the second layer. In the third layer, the normalized firing strength of each rule is calculated. The output for corresponding rules, weighted by its normalized firing strength, is calculated in the fourth layer. In the last layer, the overall output using the weighted average defuzzification is computed. Each fuzzy model consists of these five layers. Details about the ANFIS structure and operations done in 5 layers can be found in Jyh- Shing (1991); Jyh- Shing (1993); Akyilmaz et al. (2003); Walid (2005); Yilmaz (2005).

The hybrid learning algorithm used in ANFIS modeling is the combination of least-squares estimation and gradient descent method. The advantage of this combined algorithm versus the pure gradient descent method is that the rapid convergence to the global minimum is guaranteed. The gradient descent is usually slow and likely to become trapped in local minima. More information about hybrid learning method in ANFIS can be found in Jyh- Shing (1993); Jyh- Shing (1995); Takagi and Sugeno (1985); Akyilmaz et al. (2003); Goodwin and Sin (1984); Ljung (1987); Strobach (1990).

Factors affecting results in ANFIS

In this study, we examine the effect of point density, the number of subsets and type of membership function. Therefore, three different groups of calculations are performed. In the first group, we vary the point density, but use the same number of subsets and the same Gaussian membership function. Three different point densities are considered in investigating the effect of point density. We use 50 points (one point per 100 km^2), 200 points (one point per 25 km^2) and 393 points (one point per 13 km^2). In the second group of calculations, we vary the number of subsets, but use the same point density (393 points) and the Gaussian membership function. Five different numbers of subsets are used; the numbers are 3, 5, 8, 10 and 13. Finally, in the third group of calculations the fuzzy models are formed in ANFIS with different membership functions, but using the same number of subsets and the same point densities.

A membership function (MF) is a curve that defines how each point in the input space and is mapped to a membership value (or degree of membership) between 0 and 1. We use five different types of membership functions: triangle (Trimf), trapezoidal (Trapmf), generalized bell (Gbellmf), Gaussian (Gaussmf), and the difference between two sigmoidal functions (Dsigmf). Brief definitions about these membership functions are given thus:

Triangular membership function (Trimf): The triangular curve is a function of a vector, x, and depends on three scalar parameters a, b, and c, given by:

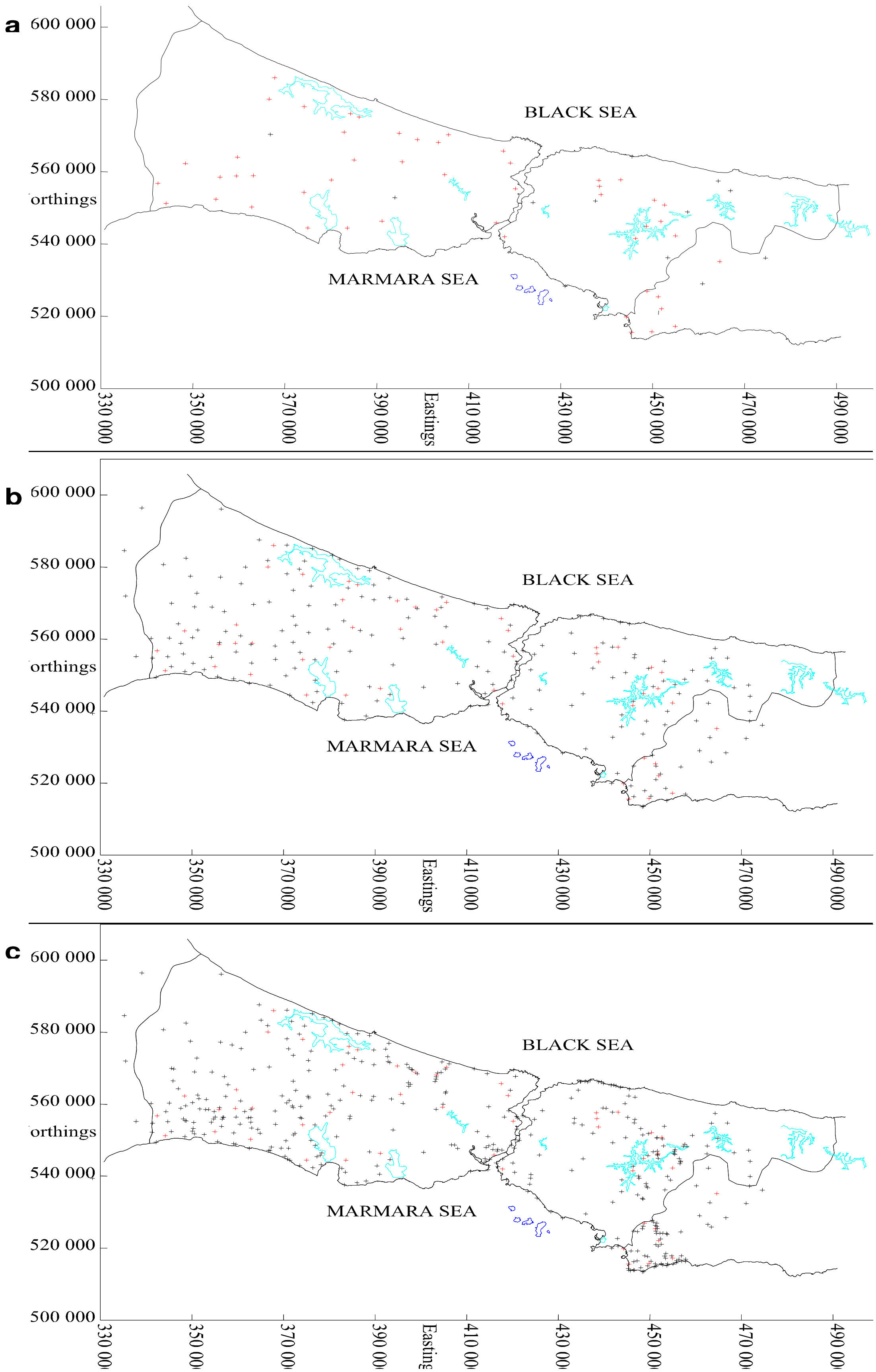
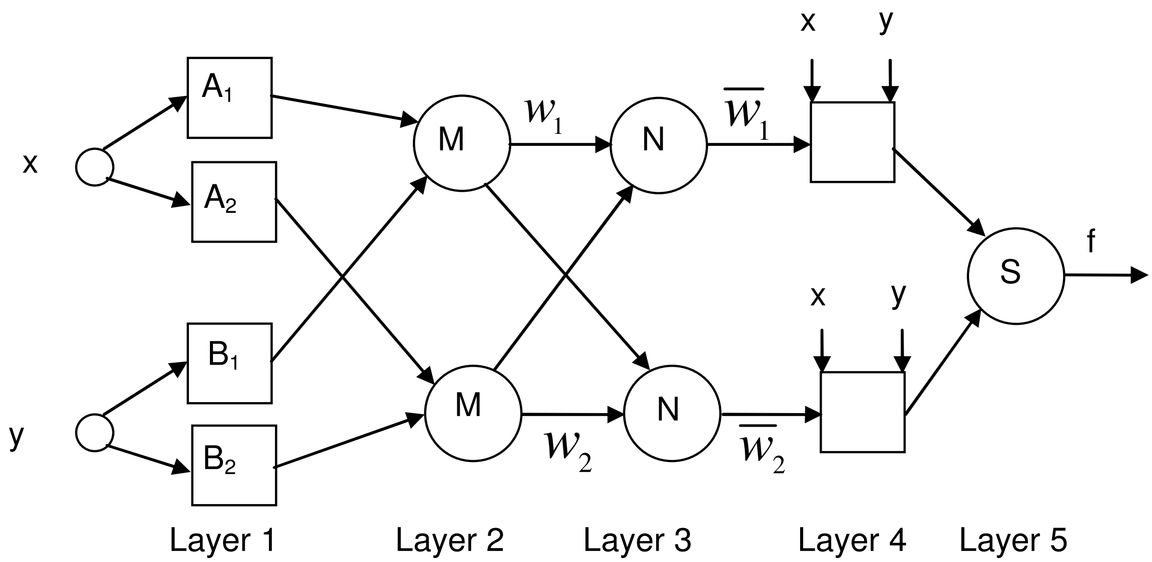


Figure 1. Distribution of 50, 200, 393 model points (+) and 50 test point (+) in Istanbul.



x, y : inputs (latitude and longitude)
w : firing strength
f : final output
S : summation
A, B : fuzzy sets
$\overline{w}$: normalized firing strength
M : multiplication of incoming signals
N : calculation of the ratio of ith rule's firing strength

Figure 2. A simple two-input, two-rule and single-output ANFIS structure (Yilmaz, 2005).

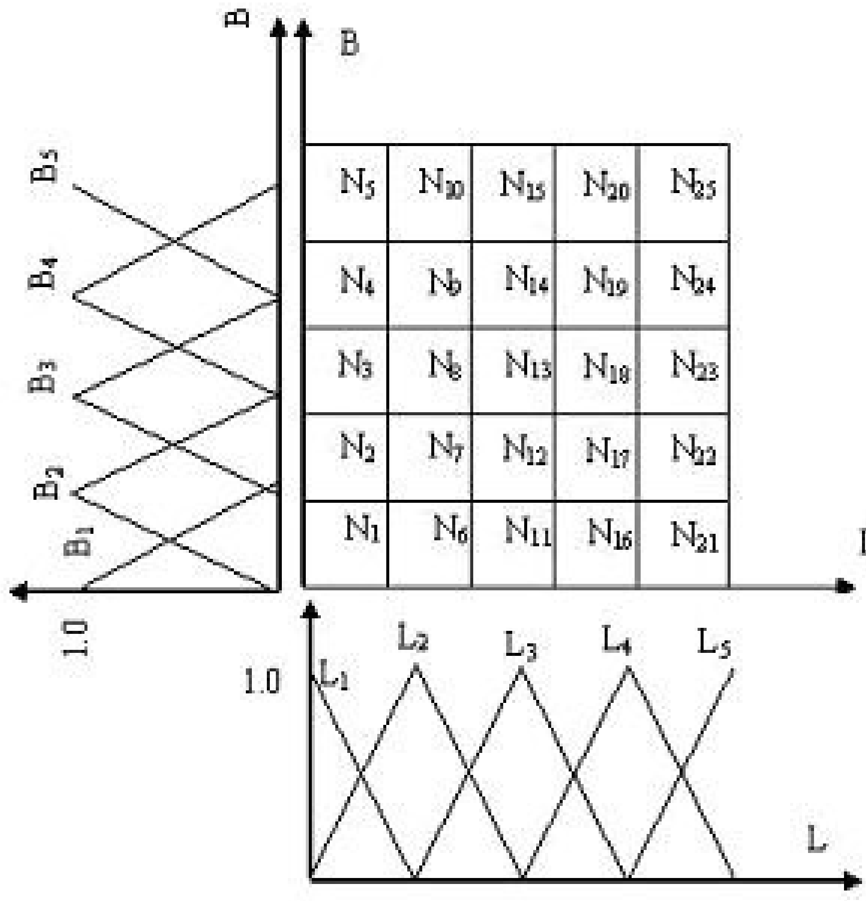


Figure 3. Rules obtained when latitude and longitude are divided into 5 subsets of each.

Table 1. Comparison of point densities (seven subset and Gaussian membership function).

Training data						
Point density (km^{2})	Minimum residual (m)	Maximum residual (m)	RMSE (m)	Minimum residual (m)	Maximum residual (m)	RMSE (m)
1/100	-0.000	0.000	0.000	-0.238	0.165	0.075
1/25	-0.088	0.067	0.025	-0.069	0.071	0.034
1/13	-0.099	0.090	0.026	-0.079	0.075	0.036

(iii)

$$f(x;a,b,c)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a\le x\le b\\ \dfrac{c-x}{c-b}, & b\le x\le c\\ 0, & c\le x\end{cases} \qquad (2)$$

The parameters a and c locate the "feet" of the triangle and the parameter c locates the peak.

Trapezoidal membership function (Trapmf): The trapezoidal curve is a function of a vector, x, and depends on four scalar parameters a, b, c, and d, given by:

$$f(x;a,b,c,d)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ \dfrac{d-x}{d-c}, & c\le x\le d\\ 0, & d\le x\end{cases} \qquad (3)$$

The parameters a and d locate the "feet" of the trapezoid and the parameters b and c locate the "shoulders."

Gaussian membership function (Gaussmf): The symmetric Gaussian function depends on two parameters σ and c as given by:

$$f(x;\sigma,c)=e^{\frac{-(x-c)^2}{2\sigma^2}} \qquad (4)$$

where, c represents the MF's center and σ determines the MF's width.

Generalized membership function (Gbellmf): The generalized bell function depends on three parameters a, b, and c as given by

$$f(x;a,b,c)=\frac{1}{1+\left|\dfrac{x-c}{a}\right|^{2b}} \qquad (5)$$

Where, the parameter b is usually positive. The parameter c locates the center of the curve.

(v) Difference between two sigmoidal membership functions (Dsigmf): The sigmoidal membership function used here depends on the two parameters a and c and is given by

$$f(x,a,c)=\frac{1}{1+e^{-a(x-c)}} \qquad (6)$$

The membership function Dsigmf depends on four parameters, a_1, c_1, a_2, and c_2, and is the difference between two sigmoidal functions $f_1(x; a_1, c_1) - f_2(x; a_2, c_2)$.

In this study we use ANFIS which is available under fuzzy toolbox of Matlab. While forming fuzzy model, we can select how many inputs and output we have, what kind of membership we will use, how we want to obtain the output (either f= px + qy + r, first degree Sugeno model or f = r, zero order Sugeno model), and we can train the model using model data.

RESULTS AND DISCUSSION

The accuracy of the ANFIS approximation method is investigated by comparing the computed values with results obtained from GPS and leveling, for at all model and test points. While forming the fuzzy model in ANFIS, the latitudes and longitudes are input parameters, and geoid heights are output parameters. In the first group, the calculations are performed using 50 (one point per 100 km^2), 200 (one point per 25 km^2) and 393 (one point per 13 km^2) model points, respectively. The differences between geoid heights obtained by GPS/leveling and the fuzzy model are summarized in Table 1. Root Mean Square Error (RMSE) is calculated with $\sqrt{\sum vv/n}$, where, v = error and n = number of points. Table 1 shows RMSE values of 0.000, 0.025 and 0.026 m for model points, and 0.075, 0.034, and 0.036 m for the test points. The respective results obtained using 200 and 393 points do not differ significantly. However, for the case of 50 points, the results obtained from models and test points differ; they also differ compared to the results obtained using 200 and 393 points. In case of 50 model points, the minimum, maximum and RMSE are 0.000 m for model points, they are + 0.165, -0.238 and 0.075 m for the test points. The large difference in the results between the model and test points clearly shows an inadequacy of the fuzzy model formed by 50 points. This difference confirms that 50 points is not enough to characterize the region of interest.

The second group of calculation uses different number

Table 2. Comparison of number of subset (point density is one point in 13 km^2 and Gaussian membership function).

			Training data			
Number of subset	Minimum residual (m)	Maximum residual (m)	RMSE (m)	Minimum residual (m)	Maximum residual (m)	RMSE (m)
3	-0.157	0.128	0.044	-0.114	0.079	0.050
5	-0.104	0.091	0.032	-0.094	0.083	0.038
8	-0.095	0.096	0.025	-0.077	0.071	0.035
10	-0.088	0.092	0.022	-0.095	0.084	0.037
13	-0.056	0.072	0.016	-0.127	0.096	0.040

Table 3. Comparison of different type of membership functions (point density is one point in 13 km^2 and 7 subsets).

			Training data			
Membership type	Minimum residual (m)	Maximum residual (m)	RMSE (m)	Minimum residual (m)	Maximum residual (m)	RMSE (m)
Trimf	-0.094	0.095	0.020	-0.129	0.078	0.041
Trapmf	-0.114	0.083	0.021	-0.118	0.119	0.042
Gbellmf	-0.095	0.077	0.020	-0.120	0.081	0.044
Gaussmf	-0.103	0.089	0.020	-0.096	0.091	0.037
Dsigmf	-0.111	0.083	0.020	-0.108	0.084	0.038

of subsets. Five different fuzzy models are formed and their results are given in Table 2. This table indicates that when the number of subsets is increased, the RMSE values are getting smaller at model points. However, the same can not be said about the test points. The lowest RMSE value on test points is obtained when the number of subsets is 8. Maximum and minimum residuals are obtained from subsets 3 (-0.157, 0.128 m) and subsets 13 (-0.127 m, 0.096 m) for the model and the test points. Increasing the number of subsets also increases the number of unknown parameters in the premise and consequent parts. Therefore, it is very important to select an optimal number of subsets in which the data will be divided. The best results are obtained with 8 subsets, since the difference between RMSE values of model and test points is the smallest.

Final, the calculation is performed using different membership functions. We form five different fuzzy models. The results are shown in Table 3. The table shows that the RMSE vary between ± 0.020 - ± 0.021 m, ± 0.037 - ± 0.044 m for model and test points. The maximum residual is found to be 0.095 m with Trimf membership; the minimum residual is found to be –0.114 m with Trapmf membership for model points. Similarly, maximum residual are 0 119 m for Trapmf membership; the minimum residual is - 0.129 m with Trimf membership for the test points.

The RMSE for the model points in Table 3 do not differ significantly. However, in case of the test points, the Gbellmf gives the highest value (0.044 m). This seems to indicate that the model formed by Gbellmf membership function is more inconsistent than the other fuzzy models. The Gaussmf membership function has given the smallest RMSE for the test points. The graphical representations of geoid height errors using different membership functions in ANFIS at 50 test points is shown in Figure 4 (a - e).

Conclusions

Some factors effecting results of ANFIS on geoid height interpolation were examined. The factors were point density, number of data subset, and type of membership function used to form the fuzzy model. When using ANFIS, one has to pay attention to several aspects, e.g. the number of parameters (premise and consequent) must be less than the number of training data pairs. This is to avoid the over fitting phenomenon, which does not allow generalization of the established fuzzy inference system. According to this study, the point density must be at least one point per 25 km^2 or higher in order to achieve good results.

Furthermore, the number of subsets used to construct the fuzzy model depends on the available data. We determined 8 to be the best number of subsets for the 393 points. Five different membership functions are used to form the fuzzy model. The results seem to suggest that using Gaussmf as membership function gives superior performance compared to other types of membership functions. The difference between the RMSE of both model and test points must be very small in order to obtain

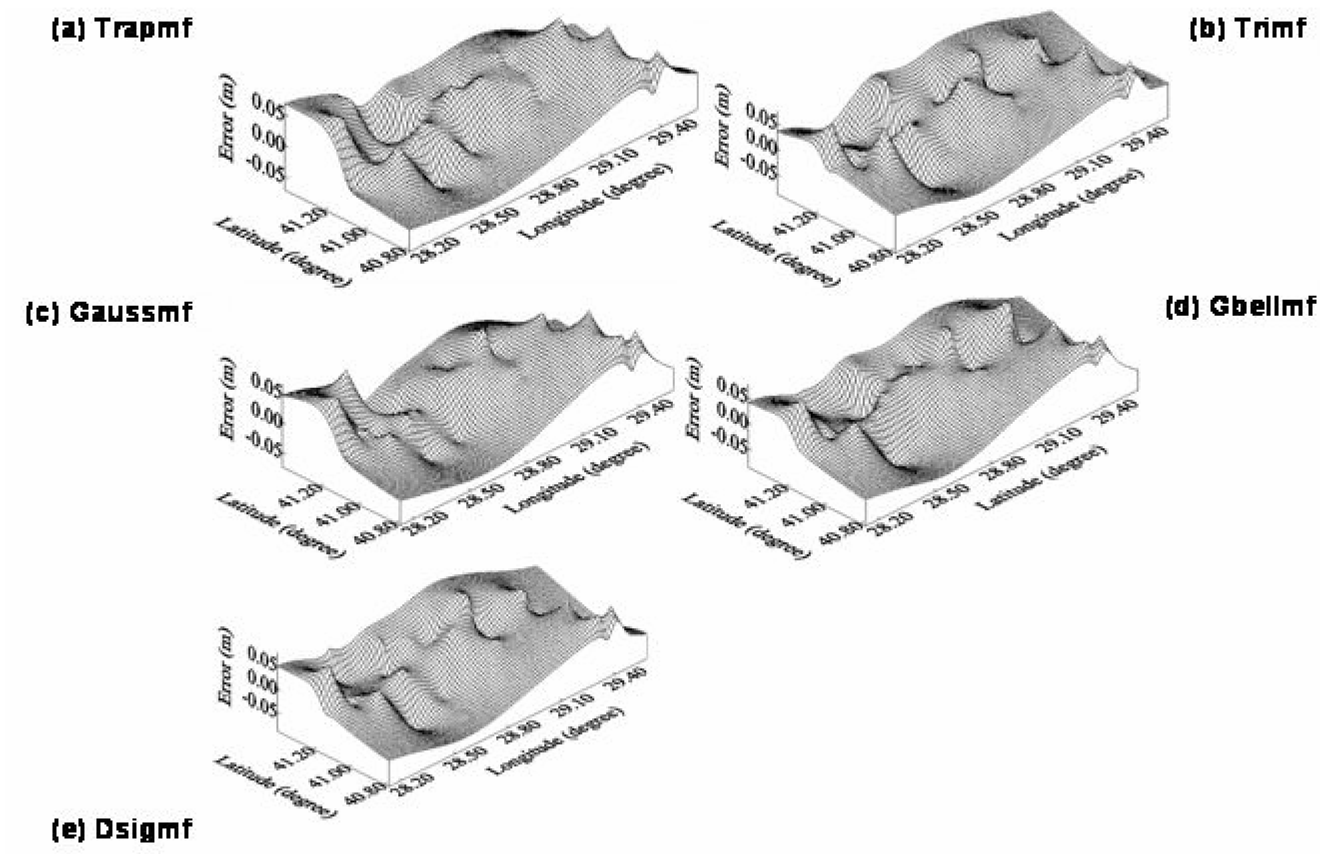


Figure 4. Graphical representations of geoid height errors by using different membership functions in ANFIS at 50 test points.

obtain meaningful results. Additional studies are needed to better understand the selection of the various factors that impact ANFIS when applied to geoid interpolation.

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