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Full Length Research Paper

Bivariate lognormal distribution model of cutoff grade impurities: A case study of magnesite ore deposit

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One of the most important aspects of magnesite ore deposits is for assessment of the cutoff grade impurities. The impurities in the magnesite ore deposit with grade higher than cutoff grade is waste, which is sent to the waste dump; the impurities lower than the cutoff grade are sent to the processing plant. The bivariate lognormal distribution will serve as an important tool for analysis impurities. In this paper, the Beylikova magnesite ore deposit impurities in Eskisehir (Turkey) were assessed by bivariate lognormal distribution model. This article presents a procedure for using the bivariate lognormal distribution to describe the joint distribution of correlated SiO_2% and Fe_2O_3% rates of ore deposit. Through the results from the model, it was determined that there are magnesite tonnage rate, mean of SiO_2% and mean of Fe_2O_3% involves the identification of cutoff SiO_2% and cutoff Fe_2O_3%. These analyses are believed to assist the management of magnesite ore deposits and determine priorities to improve mining issues.

Key words: Magnesite, cutoff grade impurity, bivariate lognormal distribution, beylikova magnesite ore deposit, Turkey.

INTRODUCTION

Cutoff grade is defined as the grade, which discriminates between ore and waste (Dagdelen, 1992). However, it is critical that the ore deposit classified as waste today could become economical to be process in future (Asad, 2005). Mine planning of ore deposits that contain more than one mineral are generally done on the basis of parametric cutoff grade (Cetin and Dowd, 2002).

In the literature, there are many studies based on cutoff grade theories developed by Lane (1964) and Taylor (1972), which are applicable to multiple ore deposits. Some studies conducted in multiple ore deposits by using cutoff grade theories are as follows; Cetin and Dowd (2002) describe the general problem of cutoff grade optimization for multi-mineral deposits and outline the use genetic algorithms for optimal cutoff grade schedules for deposits with up to three constituent minerals. Ataei and Osanloo (2003) presented an optimum cutoff grade of multiple metal deposits by using the golden section search method. Osanloo and Ataei (2003) selected cutoff grades with the purpose of maximizing net presented value subject to the constraints of mining, concentrating, and refining capacities of multiple metal deposits will be discussed. Asad (2005) presented the ease of operation for the second case becomes a reason of choice for the development of the cutoff grade optimization algorithm with a stockpiling option for deposits of two economic mineral. But all of above studies weren't considering the grade distribution of the ore deposits.

In magnesite ore deposit, cutoff impurity rates such as SiO_2% and Fe_2O_3% rates have more importance than the rate of MgO%. In this study, the case in which two important impurities SiO_2% and Fe_2O_3% rates are joint grade distributed is considered. To achieve this goal, a bivariate lognormal distribution model was developed and the application of the model was made by drillholes data of a Beylikova magnesite ore deposit in Eskisehir, Turkey.

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METHODOLOGY

Bivariate lognormal distribution

A positive random variable x is said to be lognormally distributed with two parameters mean (μ) and standard deviation (σ) if $y = \log(x)$ is normally distributed with μ and σ. The probability density function of the random variable x is given equation 1.

$$f(x)=\frac{1}{x\sigma_y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(x)-\mu_y}{\sigma_y}\right)^2\right] \quad x>0 \tag{1}$$

where μ_y and σ_y are the mean and standard deviation of y, respectively. The cumulative distribution function of x can be computed through the normal distribution as follows

$$F(x)=\Phi\left[\frac{\log(x)-\mu_y}{\sigma_y}\right] \quad x>0 \tag{2}$$

In which Φ is the cumulative distribution function of the standard normal distribution. As there is no analytical form of the cumulative distribution function, it can be calculated by directly integrating the corresponding probability density function (Yue, 2002).

If two correlated continuous random variables x_1 and x_2 are lognormally distributed with different parameters (mean and standard deviation) as follows

$$f(x_1)=\frac{1}{x_1\sigma_{y_1}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(x_1)-\mu_{y_1}}{\sigma_{y_1}}\right)^2\right] \quad x_1>0 \tag{3}$$

$$f(x_2)=\frac{1}{x_2\sigma_{y_2}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(x_2)-\mu_{y_2}}{\sigma_{y_2}}\right)^2\right] \quad x_1>0 \tag{4}$$

Then the joint distribution of these two variables can be represented by the bivariate lognormal distribution. The probability density function of the bivariate lognormal distribution can be derived using the Jacobian of the transformation and is given by

$$f(x_1,x_2)=\frac{1}{2\pi x_1 x_2\sigma_{y_1}\sigma_{y_2}\sqrt{1-\rho^2}}\left(\exp-\frac{1}{2(1-\rho^2)}\times\left[\left(\frac{\log(x_1)-\mu_{y_1}}{\sigma_{y_1}}\right)^2-2\rho\left(\frac{\log(x_1)-\mu_{y_1}}{\sigma_{y_1}}\right)\left(\frac{\log(x_2)-\mu_{y_2}}{\sigma_{y_2}}\right)+\left(\frac{\log(x_2)-\mu_{y_2}}{\sigma_{y_2}}\right)^2\right]\right) \tag{5}$$

Where μ_{y_i} and σ_{y_i} are mean and standard deviation of y_i $(i=1,2)$ and they can be derived using the following formulae (Stedinger et al., 1993).

$$\mu_{y_i}=\log(\mu_{x_i})-\left(\frac{\sigma_{y_i}^2}{2}\right) \tag{6}$$

$$\sigma_{y_i}=\left[\log\left(1+\frac{\sigma_{x_i}^2}{\mu_{x_i}^2}\right)\right]^{\frac{1}{2}} \tag{7}$$

Here μ_{x_i} and σ_{x_i} are the mean and standard deviation of x_i and ρ is the correlation coefficient of y_1 and y_2, and ρ is estimated by equation 8.

$$\rho=\frac{E\left[\left(y_1-\mu_{y_1}\right)\left(y_2-\mu_{y_2}\right)\right]}{\sigma_{y_1}\sigma_{y_2}} \tag{8}$$

For conditional probability density function of x_2 given x_1 can be derived as follows

$$f(x_1\backslash x_2)=\frac{1}{x_1\sigma_{y_1\backslash y_2}\sqrt{2\pi}}\left(\exp\left[-\frac{1}{2}\left(\frac{\log(x_1)-\mu_{y_1\backslash y_2}}{\sigma_{y_1\backslash y_2}}\right)^2\right]\right) \tag{9}$$

$$\mu_{y_1\backslash y_2}=\mu_{y_1}-\left(\rho\frac{\sigma_{y_1}}{\sigma_{y_2}}\left[\log(x_2)-\mu_{y_2}\right]\right) \tag{10}$$

$$\sigma_{y_1\backslash y_2}=\sigma_{y_1}\sqrt{\left(1-\rho^2\right)} \tag{11}$$

Where $\mu_{y_1\backslash y_2}$ and $\sigma_{y_1\backslash y_2}$ are the mean and standard deviation of x_2 given x_1 and they can calculated using the equations 10 - 11. For the standard values (z) corresponding to cutoff x_2 given cutoff x_1 can be presented as equation 12 (Yerel, 2008).

$$z=\frac{\log(x_{c1})-\mu_{y_1\backslash y_2}}{\sigma_{y_1\backslash y_2}} \tag{12}$$

Which is the product of dependent probabilities, gives the total joint probability of cutoff impurity rates correspond to the tonnage rate (T_c) of ore deposit given by equation 13.

$$T_c=T_{x_1\backslash x_2}.T_{x_2} \tag{13}$$

Mean x_1 and x_2 of ore deposit under cutoff x_1 and x_2 values, can be calculated by the equations 14 - 15 (Clark, 2001).

$$\mu_{c_{x_1}}=\frac{B}{T_c}.\mu_{x_1} \tag{14}$$

$$\mu_{c_{x_2}}=\frac{B}{T_c}.\mu_{x_2} \tag{15}$$

From the above equations corresponding of the parameters B and $\sigma_{y_1y_2}$ are calculated equations 16 - 17.

$$B=\Phi(z-\sigma_{y_1y_2}) \tag{16}$$

Table 1. Descriptive statistics of SiO_2% and Fe_2O_3%.

Parameters	n	Min.	Max.	Mean	Std. deviation	Variance
SiO_2%	135	0.01	1.21	0.224	0.278	0.077
Fe_2O_3%	135	0.01	0.27	0.048	0.053	0.003

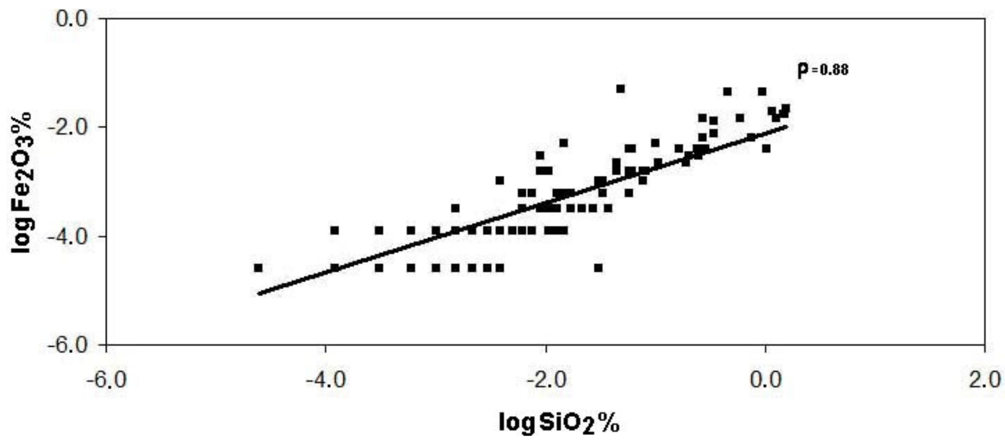

Figure 1. The correlation coefficient between the logSiO_2% and logFe_2O_3%.

$$\sigma_{y_1y_2} = \rho.\sigma_{y_1}.\sigma_{y_2} \tag{17}$$

Description of the study area

Eskisehir is an industrialized city located in the western part of Central Anatolia Region which has a population exceeding 600 thousand habitants and covers an area of approximately 13,700 km^2 (Orhan et al., 2007). The city is located at equal distance from the primary metropolitan city Istanbul and the capital Ankara (Uygucgil, et al., 2007). In this study area is Beylikova magnesite ore deposit is located in the southeast part of Eskisehir city, Turkey. The geological units are not complex in the study area. Metamorphic, volcanic and sedimentary rocks from Triassic to Quaternary age are the main geological units in the area (Gozler et al., 1997).

Dataset

The bivariate lognormal distribution model was applied to a Beylikova magnesite deposit in Eskisehir. In magnesite deposit, SiO_2% and Fe_2O_3% of the ore body have more importance than the MgO% (Yerel, 2008). In this study, SiO_2% and Fe_2O_3% data were obtained from 40 vertical drillholes. Of the 60 irregular drillholes perpendicular to the magnesite deposit, 40 penetrated drillholes. The drillholes contains information about the rock type and magnesite. Descriptive statistics of the SiO_2% and Fe_2O_3% were presented in Table 1.

RESULTS AND DISCUSSION

The evaluation of cutoff grade impurities for multiple ore deposits is significantly more complex than for a single mineral ore deposits. The most significant impurities of a magnesite ore deposits are the SiO_2% and Fe_2O_3%. The calculation of magnesite tonnage rate, mean of SiO_2% and mean of Fe_2O_3% involves the identification of cutoff SiO_2% and cutoff Fe_2O_3%.

In this study, correlation coefficient was calculated by using equation 8. A value of correlation coefficient indicated that 88% of the logFe_2O_3% variability is explained by the linear regression analysis. On the other hand, there is close correlation between logSiO_2% and logFe_2O_3% (Figure 1). Thus, we assume that these parameters are mutually dependent.

The magnesite tonnage rate, mean of SiO_2% and mean of Fe_2O_3% estimates can be used in mine planning. In cutoff grade policy, mean of dependent variables may be estimated by bivariate model. Investigation in the T_c, mean of SiO_2%, and mean of Fe_2O_3% with the Beylikova magnesite ore deposit may be determined by using bivariate lognormal distribution model. This model for the Beylikova magnesite ore deposit can be evaluation of dependent variables.

Considering that cutoff SiO_2% and cutoff Fe_2O_3% was joint bivariate lognormal distribution model, T_c, mean SiO_2% and mean Fe_2O_3% of the ore deposit were calculated by using the equations 13 - 15. These calculations were graphed and presented in Figures 2 - 4. The Figure 2 shows that, cutoff SiO_2% and Fe_2O_3% increases as the T_c increases, but over 0.4 cutoff SiO_2% not considerable variations is seen in T_c. Thus, the cutoff Fe_2O_3% exceeds

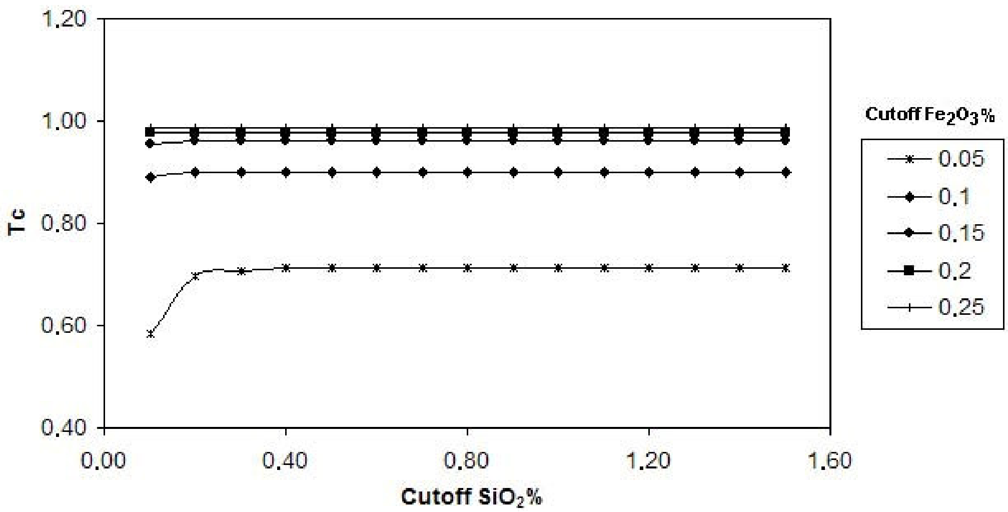

Figure 2. Cutoff SiO_2% versus T_c.

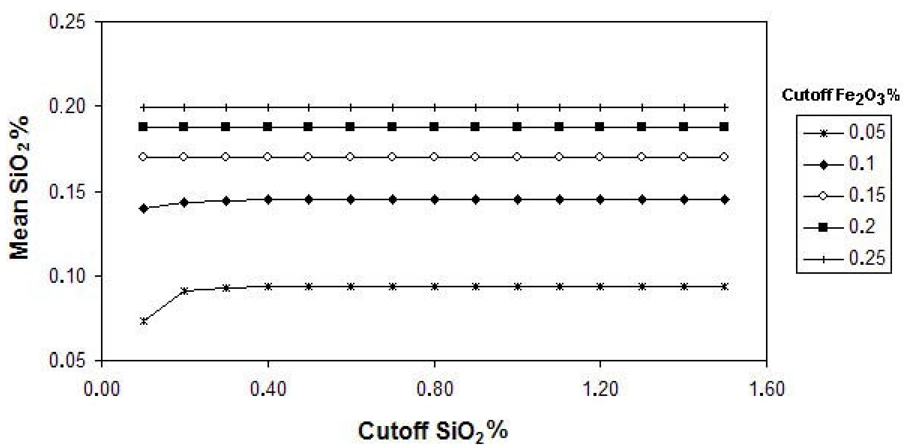

Figure 3. Cutoff SiO_2% versus mean SiO_2%.

0.15%, not variations at T_c is seen due to variations in cutoff SiO_2%.

Figure 3 indicates that cutoff SiO_2% and cutoff Fe_2O_3% increases as the mean SiO_2% increases, but over 0.2 SiO_2% not important variations are seen in mean SiO_2%. In addition these, the cutoff Fe2O3% increases, mean SiO_2% is seen due to increases in cutoff SiO_2%. Similarly, cutoff SiO_2% and cutoff Fe_2O_3% increases as the mean Fe_2O_3% increases, but over 0.2 SiO_2% not considerable variations are seen in mean Fe_2O_3% (Figure 4). Thus, with mean SiO_2% and mean Fe_2O_3% increases, the quality of the magnesite ore deposits are decreases.

Conclusion

The one of the most significant impurities of a magnesite ore deposits are the SiO_2% and Fe_2O_3%. The determination of T_c, mean of SiO_2% and mean of Fe_2O_3% involves the identification of cutoff SiO_2% and cutoff Fe_2O_3% are very important. In the ore deposit, as the cutoff impurity rates increase, T_c of the deposit also increase. But over 0.4 cutoff SiO_2% and exceeds 0.15% cutoff Fe_2O_3% aren't considerable variations in T_c. In addition, cutoff SiO_2% and cutoff Fe_2O_3% are increased as the mean Fe_2O_3% and mean SiO_2% are increased. However, with mean Fe_2O_3% and mean SiO_2% increase, the quality of the magnesite ore deposit is decreased.

This study shows that bivariate lognormal distribution model provide useful information for the cutoff impurities in helping them plan their ore deposits. These methods are believed to assist decision makers assessing cutoff impurities in order to improve the efficiently of mining

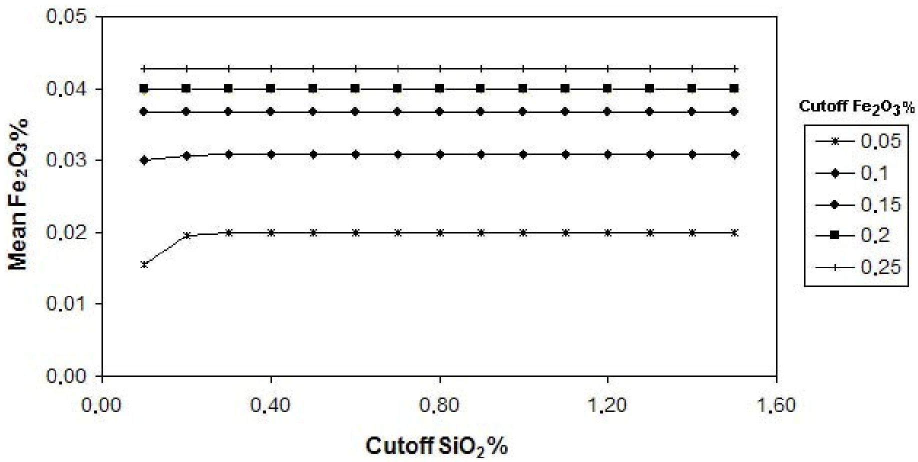

Figure 4. Cutoff SiO_2% versus mean Fe_2O_3%.

planning.

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