

*Full Length Research Paper*

## Two-phase closed-loop system identification method based on the auxiliary model

Jie Jia<sup>1</sup>, Jiao Cao<sup>1</sup>, Yong Yang<sup>2\*</sup>, Zhiteng Liu<sup>1</sup> and Shuying Huang<sup>3</sup>

<sup>1</sup> College of Information Engineering, Nanchang Hangkong University, Nanchang 330063, China.

<sup>2</sup>School of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330032, China.

<sup>3</sup>School of Software and Communication Engineering, Jiangxi University of Finance and Economics, Nanchang 330032, China.

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The two-stage recursive extended least squares parameter estimation algorithm based on the auxiliary model of a class of closed-loop identification system model is presented in this paper. The basic idea of the algorithm is a combination of the auxiliary model identification ideas and the decomposition technique in which the closed-loop system is converted to a two-step process and each step identified model is an open-loop system. The more mature open-loop identification method is dealt with, the closed-loop system parameter identification problem is known in this way. The proposed algorithm has been proven to have high computational efficiency and effectiveness by the simulation examples.

**Key words:** Auxiliary model, least squares method, closed-loop system.

### INTRODUCTION

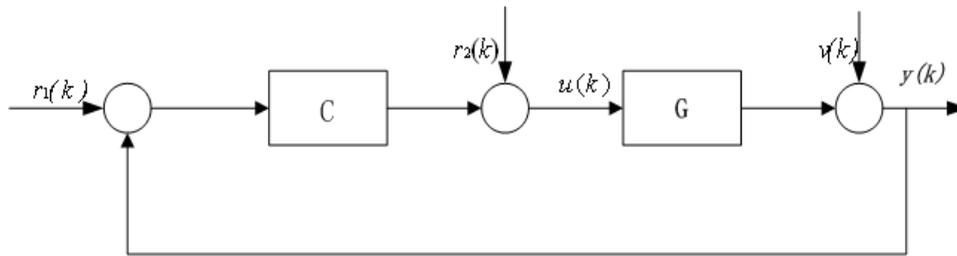
System identification in closed-loop conditions has always been of particular concern in the field of industrial applications. The main problems of using the closed-loop data identification system is that unpredictable noise of the system output signal is related to the system control input signal through the feedback link. It is because of the existence of this correlation, resulting in many classic identification algorithms (Landau, 2001; Ding and Chen, 2005; de Klerk and Craig, 2002; Besterfield-Sacre et al., 2000) (such as: the method of least squares, instrumental variables, correlation analysis, spectrum analysis, etc.) used in the identification of the closed-loop system, that we have a greater estimation error. In recent years, many closed-loop identification algorithms have been proposed

around the closed-loop data acquisition system parameters consistent unbiased estimate of the subject. Zhang and Feng (1995) and Ljung (1999) obtained the uniform convergence of the closed-loop parameters, by selecting the pre-data filter and compensating the noise covariance term.

Van Den Hof and Schrama (1993) and Landau and Karimi (1996) made full use of process control input, the relationship between the process output and the setting, a closed-loop identification algorithm-two-stage closed-loop identification algorithm based on open-loop conversion was proposed. The algorithm will convert the closed-loop identification problem for the two open loop identification problem. By constructing the no noise

\*Corresponding author. E-mail: yangyong5080@126.com

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**Figure 1.** Closed-loop control system structure.  $r_1(k)$  is the system settings,  $r_2(k)$  for the applied test signal,  $u(k)$  is the control input signal of the controlled object,  $v(k)$  is a random white noise,  $y(k)$  is the system output signal,

pollution intermediate signal instead of the actual control inputs, the correlation between the noise and the control input was removed. Then using the existing classic open-loop, identification algorithm can help to obtain system parameters consistent unbiased estimates. The algorithm idea is simple, convenient and does not require controller prior knowledge. But the original is not the final recognition result of the structure of the intermediate identification model parameters and model.

Accordingly, two stages of the closed-loop identification algorithm are given in this paper. Combined with the auxiliary model identification ideas and decomposition technique, the intermediate model is decomposed into two subsystems, each subsystem contains a parameter vector. With the auxiliary model-based recursive extended least squares theory, using the output of the auxiliary model instead of unknown intermediate variable vector, and using the estimated residuals instead of the information vector unpredictable noise term, which can use the idea of recursive identification to estimate the parameters of the intermediate model.

**CLOSED-LOOP CONTROL SYSTEM MODEL DESCRIPTION AND IDENTIFICATION MODEL**

First, define the symbol, "A: = X" or "X: = A" means that A is equal to X.

The symbol  $I(I_n)$  indicate the unit matrix of dimension  $n \times n$ ,  $I_n$  indicate n-dimensional unit column vector, superscript  $T$  represents the transpose of the matrix or vector.

Wherein  $y(k)$  is the system output signal,  $u(k)$  is the control input signal of the controlled object,  $v(k)$  is a random white noise,  $r_1(k)$  is the system settings,  $r_2(k)$  for the applied test signal,  $G(z)$  is a mathematical model of the controlled object,  $C(z)$  is the controller.

The closed-loop identification task is that using the

observational data sequence  $[r_1(k), r_2(k), u(k), y(k)]$  estimate the parameters of the controlled object  $G(z)$ , and hope to get them consistent unbiased estimator. Known from Figure 1, the control system output equation can be written as:

$$y(k) = G(z)u(k) + v(k) \tag{1}$$

Due to the presence of a unit feedback loops, control of the system equation can be written as:

$$u(k) = r_2(k) + C(z)[r_1(k) - y(k)] \tag{2}$$

(1) Substituting into (2), elimination system output  $y(k)$  can be obtained:

$$u(k) = \frac{C(z)r_1(k) + r_2(z)}{1 + C(z)G(z)} - \frac{C(z)}{1 + C(z)G(z)}v(k) \tag{3}$$

From Equation 3, it can be seen that random white noise  $v(k)$  directly related to the control input  $u(k)$  of feedback loops, which is the key that recognition results are biased when many identification algorithm about closed-loop system. Two-phase closed-loop identification algorithm by constructing intermediate signal no-noises pollution, overcome this correlation parameter identification which can be consistent unbiased estimate of the parameters.

**Two-phase closed-loop identification algorithm**

For convenience of description and without loss of generality, let  $r(k) = r_2(k) + C(z)r_1(k)$ , Equations 2 and 3 can be rewritten as:

$$u(k) = r(k) - C(z)y(k) \tag{4}$$

$$u(k) = \frac{1}{1+C(z)G(z)}r(k) - \frac{C(z)}{1+C(z)G(z)}v(k) \quad (5)$$

Equation (5) into (1), elimination of the control input  $u(k)$ , simplification was:

$$y(k) = \frac{G(z)}{1+C(z)G(z)}r(k) - \frac{C(z)}{1+C(z)G(z)}v(k) \quad (6)$$

Consider the system, Equations 5 and 6 make the following assumptions: (1)  $r(k)$  is persistently exciting, and can be measured; (2)  $r(k)$  and  $v(k)$  are independent and not related in any moment. The closed-loop identification is done in two steps, that  $1+C(z)G(z) = A(z)$ ,  $-C(z) = B(z)$  at the same time. Shift operator polynomial  $A(z)$ ,  $B(z)$  and  $G(z)$  are the units  $[z^{-1}z(k) = z(k-1)]$ , and

$$\begin{aligned} A(z) &:= 1 + a_1z^{-1} + a_2z^{-2} + \Lambda + a_{n_a}z^{-n_a} \\ B(z) &:= b_1z^{-1} + b_2z^{-2} + \Lambda + b_{n_b}z^{-n_b} \\ G(z) &:= 1 + g_1z^{-1} + g_2z^{-2} + \Lambda + g_{n_g}z^{-n_g} \end{aligned}$$

Suppose when  $k \leq 0$ , the  $u(k) = 0$ ,  $y(k) = 0$ , the  $V(k) = 0$ , and the order of  $n_a, n_b, n_g$  are known:

Step 1: In formula (5), by assuming that the conditions, independent of  $r(k)$  and  $v(k)$  irrelevant, and  $u(k)$  and  $r(k)$  are measured, it can be that Equation (5) as an open-loop system, using the least squares identification unanimously unbiased estimate. The least squares parameter identification expression:

$$u(k) = \frac{1}{A(z)}r(k) + \frac{B(z)}{A(z)}v(k) \quad (7)$$

Step 2: According to the recognition result of Formula (7), to construct a no noise pollution control input  $u(k)$ , that is:

$$u(k) = \frac{1}{A(z)}r(k) \quad (8)$$

Equation 8 into 6,  $u(k)$ ,  $v(k)$  is not relevant, so Formula 13 can be seen as open-loop system processing, least squares identification can be consistent and unbiased estimates. Parameter identification expression:

$$y(k) = \frac{G(z)}{A(z)}r(k) - \frac{1}{A(z)}v(k) \quad (9)$$

At this point, the entire closed-loop identification task is complete, the controlled object parameters consistent unbiased estimator.

### Simulation examples

Consider the following closed-loop system model,

$$\begin{aligned} A(z) &:= 1 + a_1z^{-1} + a_2z^{-2} = 1 + 1.06z^{-1} + 0.8z^{-2} \\ B(z) &:= 1 - b_1z^{-1} = 1 - 0.64z^{-1} \\ G(z) &:= 1 + g_1z^{-1} + g_2z^{-2} = 1 + 0.412z^{-1} - 0.309z^{-2} \end{aligned}$$

The model parameters are

$$\theta^T = [1.60, 0.8, -0.64, 0.412, -0.309]$$

Simulation for the first step,  $\{r(k)\}$  is selected white noise of zero mean and unit, the input  $\{u(k)\}$  is a zero mean and unit variance irrelevant the measurable random signal sequence, and  $\{v(k)\}$  is a zero mean and variance white noise sequence. When the noise variance parameters  $\sigma^2 = 0.4^2$ , recursive least squares (RLS) estimated error vs. time curve is shown in Figure 2. In particular, using the recursive extended least squares (RELS) (Duan et al., 2012; Yue et al., 2009; Yao and Ding, 2012) algorithm simulate the first step of identification model for comparing the effectiveness of the proposed algorithm in this paper. The RELS estimated error with the time curve is shown in Figure 3:

i. With the increase in the length of data, the parameter estimation errors of RLS algorithm based on the auxiliary model tended to be stable, but parameter estimation error of the RELS algorithm is very unstable. Therefore, the second step in terms of follow-up closed-loop identification is bound to generate a lot of bad influence. According to the recognition result of the first step, construction of a noise pollution control entry is  $u(k)$  in the second step.

Simulation  $\{r(k)\}$  is chose zero mean and unit white noise  $\{v(k)\}$  is a white noise sequence of zero mean and variance. In the type of noise variance parameters of  $\sigma^2$ , RLS estimate error vs. time curve is shown in Figure 4.

ii. With the increase in the length of the data, parameter estimation error is getting smaller and smaller, and gradually stabilized, reflecting the algorithm efficiency and stability.

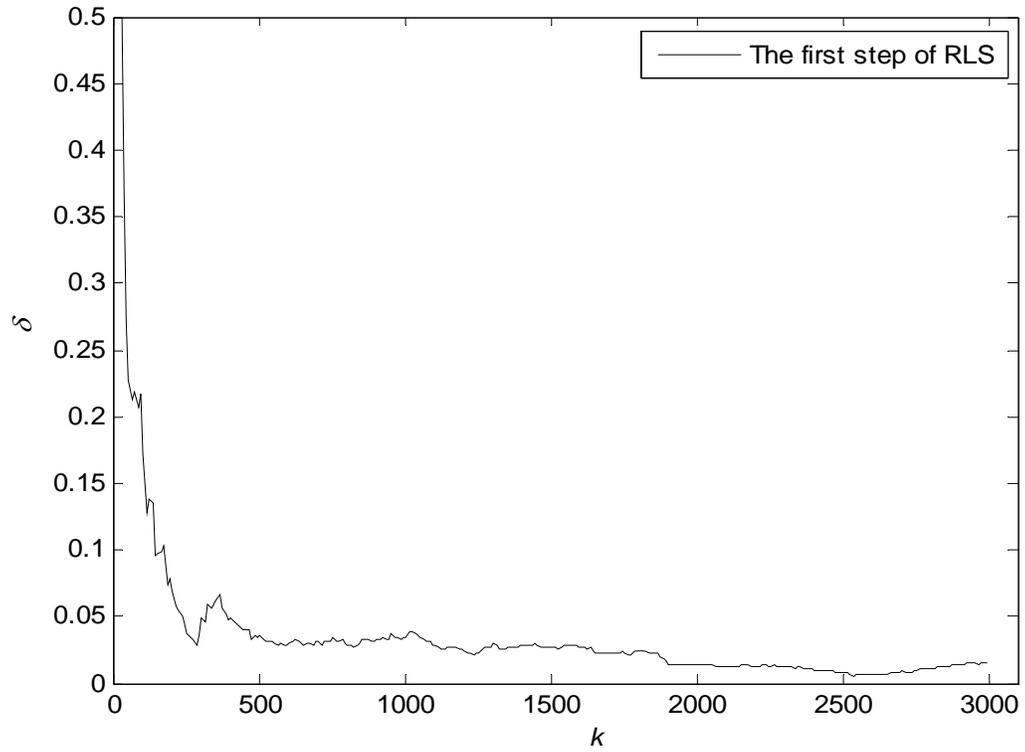


Figure 2. Recursive least squares (RLS) method of parameter estimation error change over time k.

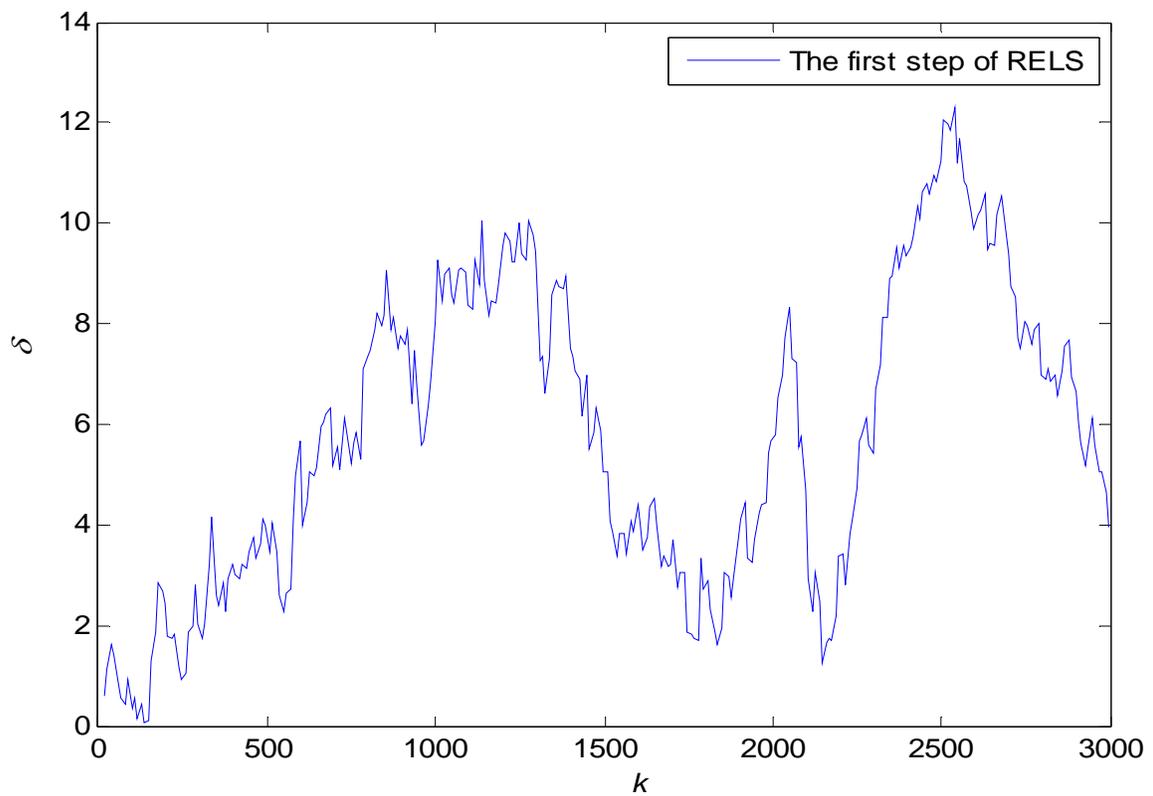
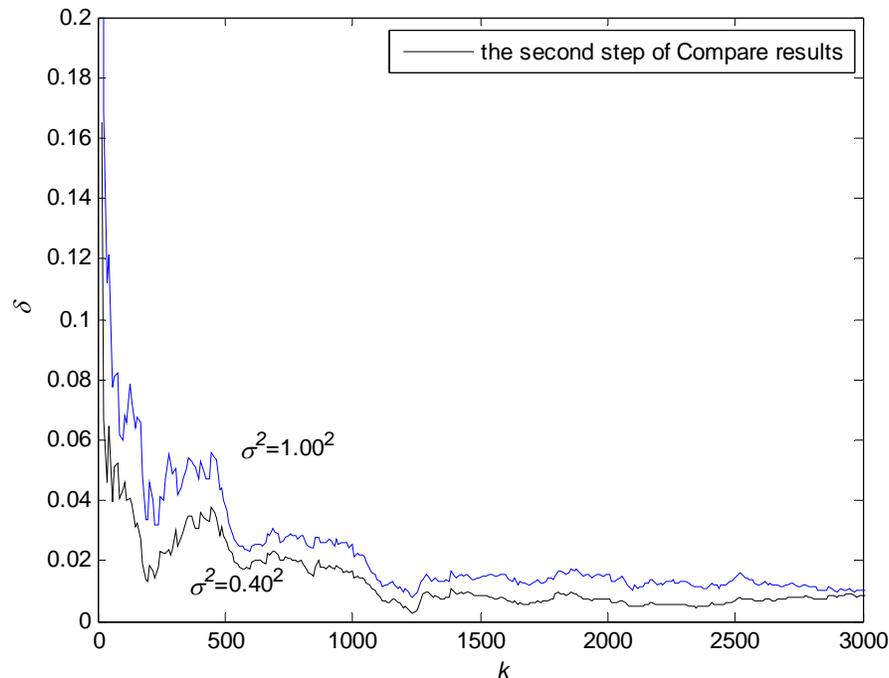


Figure 3. The recursive extended least squares (RELS) method parameter estimation error changes over time.



**Figure 4.** Parameter estimation error in the second step under different noise variance.

## Conclusion

In this paper, for the converted first step in open-loop subsystem with the help of the auxiliary model and recursive extended least squares theory, the unknown intermediate variable vector was used instead of the output of the auxiliary model identification model, vector unpredictable noise term was used instead of estimated residuals; so using the recursive identification thought, estimated all the parameters of the system. In the second step, using recognition result of the first step constructed, the no noise input model, and then using the recursive extended least squares theory derived system model parameters.

## Conflict of Interests

The author(s) have not declared any conflict of interests.

## ACKNOWLEDGMENTS

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