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The time delayed feedback control to suppress the vibration of the autoparametric dynamical system

Yasser A. Amer[1]* and Samira M. Soleman[2]

[1]Mathematics Department, Faculty of Science, Zagazig University, Egypt.
[2]Mathematics Department, Faculty of Science, EL-Zawya University, Libya.

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The response of a dynamical system of two-degree-of-freedom with parametrically excited pendulum is solved and studied. The delayed feedback control is applied to suppress or stabilize the vibration of the system. The case of 1:2 sub-harmonic resonances between pendulum and primary system is studied; the method of multiple scales is applied to obtain second-order approximations of the response of the system. It is shown that the delayed feedback control can be used to suppress the vibration or stabilize the system when the saturation control is invalid, the vibration of the system can be suppressed by the delayed feedback control. The effect of delay on the suppression is discussed; the vibration of the system can be suppressed at some values of the delay.

Key words: Frequency response, delayed feedback control, multiple times scale, vibration suppression.

INTRODUCTION

In the last years, many papers have been devoted to the control of resonantly forced systems in various engineering fields. In passive vibration absorbers a physical device is connected with the primary structure, while in the case of active absorbers the device is replaced by a control system of sensors, actuators and filters. Active control of mechanical and structural vibrations is superior to passive control, because the former is more flexible in many aspects.

Periodically forced nonlinear systems under delay control have been analyzed by Plaut and Hsieh (1987) in the case of nonlinear structural vibrations with a time delay in damping. The studying of an approach for implementing an active nonlinear vibration absorber (El-Bassiouny, 2005). The strategy exploits the saturation phenomenon that is exhibited by multi-degree-of-freedom systems with cubic nonlinearities possessing one-to-one internal resonance. The proposed technique consists of introducing a second- order controller and coupling it to the plant through a sensor and an actuator, where both the feedback and control signals are cubic. The vibration and chaos control of nonlinear torsion al vibrating systems is studied (El-Bassiouny, 2006).

The resonance, stability and chaotic vibration of a quarter-car vehicle model with time-delay feedback is investigated in (Naik and Singru, 2011). The primary, super harmonic and sub harmonic resonances of a harmonically excited nonlinear quarter-car model with

*Corresponding author. E-mails: yaser31270@yahoo.com

time delayed active control are investigated. They focused on the influence of time delay in the system. This naturally gives rise to a delay differential equation model of the system. It was found that proper selection of time-delay showed optimum dynamical behavior. The modeling and optimal active control with time delay dynamics of a strongly nonlinear beam. They investigated the control by a sandwich beam and one using piezoelectric absorber (Nana Nbendjo et al., 2003, 2009).

The effect of the time delay on the non-linear control of a beam when subjected to multi-external excitation forces is determined, multiple scale perturbation method is applied to obtain the solution up to the second order approximations (El-Gohary and El-Ganaini, 2011). Using the delayed feedback control and saturation control to suppress the vibration of the dynamical system under external force is studied (Zhao and Xu, 2012). The nonlinear dynamical analysis on four semi-active dynamic vibration absorbers with time delay is solved (Yongjun and Mehdi, 2013). Vibration reduction, stability and resonance of a dynamical system excited by external and parametric excitations via time-delay absorber is studied (Sherif and Sayed, 2014). The dynamical system of a twin-tail aircraft, which is described by two coupled second order nonlinear differential equations having both quadratic and cubic nonlinearities, solved and controlled (Amer et al., 2009). The effect of different controllers on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion is obtained (Sayed and Kamel, 2011, 2012). studied the primary resonance, stability and design methodology of a piecewise bilinear system under cubic velocity feedback control with a designed time-delay are investigated through combining multi-scale perturbation method with Fourier expansion, the effects of time delay on dynamics behaviors are explored (Gao and Chen, 2013).

The nonlinear analysis of time-delay position feedback control of container cranes studied (Nayfeh and Baumann, 2008). Stabilizability of the turning process subjected to a digital proportional- derivative controller is analyzed, the governing equation involves a term with continuous-time point delay due to the regenerative effect and terms with piecewise-constant arguments due to the zero-order hold of the digital control (Lehotzky et al., 2014). The dynamical system with time varying stiffness subjected to multi external forces studied. The system is written as two degree of freedom consists of the main system and absorber. The multiple time scale perturbation method is applied to get the approximate solution up to the third approximation. The stability of the system at the simultaneous primary resonance is investigated using both frequency response equations and phase-plane methods (Amer and Ahmed, 2014). The application of the renormalization group (RG) methods to the delayed differential equation is determined by analyzing the Mathieu equation with time delay feedback, get the amplitude and phase equations, and then obtain the approximate solutions by solving the corresponding R G equations (Wu and Xu, 2013).

In this paper, the delayed feedback control is applied to suppress the vibration of the system. The equation of motion and the perturbation analysis and the stability of the equilibrium solutions are given. All possible resonance cases are extracted and investigated at this approximation order. The stability of the system is studied using the phase plane and frequency response curves. The analytical solutions and numerical simulations of the delayed feedback control are presented and compared.

EQUATIONS OF MOTION

A two degree-of-freedom dynamic vibration absorber system with a parametrically excited pendulum is described in (Song et al., 2003). In the present paper, a position feedback with delay is introduced into the model to control the vibration of the system and the model is shown in Figure 1. The system consists of the mass M, the linear spring with stiffness k, and the viscous damper presented by coefficient c. The second system is a simple pendulum of mass m_p hinged at the system. The distance between the supporting point and the center of gravity of the pendulum is l. J and c_θ represent the inertia moment with respect to the supporting point and the damping coefficient of the pendulum respectively. The system is excited directly by a harmonic force $f(t) = Fx\cos(\omega t)$. a time delayed position feedback $g_1[x(t-\tau) - x(t)]$ is introduced into the auto parametric dynamic vibration absorber system. The equations of motion of the delayed feedback control system are

$$(M+m_p)\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx + m_p l\left(\frac{d^2\theta}{dt^2}\sin\theta + \left(\frac{d\theta}{dt}\right)^2\cos\theta\right) + g_1(x_\tau - x) = f(t) \quad (1)$$

$$J\frac{d^2\theta}{dt^2} + c_\theta\frac{d\theta}{dt} + m_p l\sin\theta\left(g + \frac{d^2x}{dt^2}\right) = 0\,, \quad (2)$$

where x the displacement of is M, θ is the angle of rotation of the pendulum, and t, $x_\tau = x(t-\tau)$. It should be noted that the delayed feedback disappears in (1) and (2) when $\tau = 0$, and (1) and (2) are identical to Song et al. (2003). Thus it is easy to observe effects of the delayed feedback on vibration suppression performance when $\tau \neq 0$.

Firstly, a set of dimensionless (or normalized) variables are defined as:

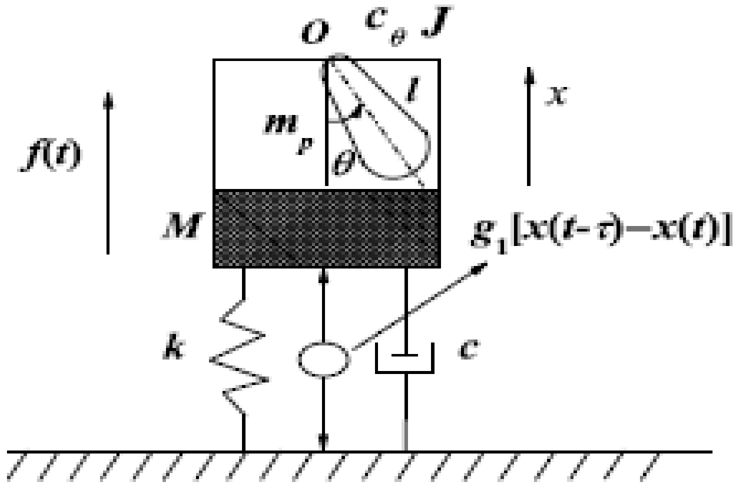

Figure 1. A model describing the autoparametric dynamical vibration absorber with delayed feedback.

$$t^* = \frac{\omega}{\Omega}\mathrm{t},\ \tau^* = \frac{\omega}{\Omega}\tau,\ \eta = \frac{x}{l},\ F = \frac{p_0}{kl},\ R = \frac{m_p}{M},\ \alpha = \frac{\omega_1}{\omega_3},\ \beta = \frac{\omega_2}{\omega_1},\ \gamma = \frac{\omega_4}{\omega_3},\ \mu = \frac{m_p l^2}{J},$$

$$\zeta_1 = \frac{c}{2M\,\omega_3(1+R)},\ \zeta_2 = \frac{c_\theta}{2J\,\omega_3},\ \omega_1 = \sqrt{\frac{k}{M+m_p}},\ \omega_2 = \sqrt{\frac{g}{l}},\ \omega_3 = \sqrt{\frac{k}{M}},$$

$$\omega_4 = \sqrt{\frac{g_1}{M+m_p}},\ \Omega = \frac{\omega}{\omega_3}.$$

Equations (1) and (2) can be written as the following non-dimensional forms by dropping the asterisk for convenience.

$$\eta'' + G_1\eta' + \Omega_1^2\eta + G_2(\theta''\sin\theta + \theta'^2\cos\theta) + H_3(\eta_\tau - \eta) = G_3\eta\cos(\Omega t), \tag{3}$$

$$\theta'' + H_1\theta' + (\Omega_2^2 + H_2\eta'')\sin\theta = 0, \tag{4}$$

where $\Omega_1^2 = \alpha^2,\ \Omega_2^2 = \mu\alpha^2\beta^2,\ G_1 = 2\zeta_1,$ $H_1 = 2\zeta_2,\ G_2 = \dfrac{R}{1+R},\ H_2 = \mu,\ G_3 = F\alpha^2,$ $H_3 = \gamma^2,\ \Omega$ is dimensionless frequency, $\eta_\tau = \eta(t-\tau),\ ()' = \dfrac{d()}{dt^*}.$

Equations (1) and (2) are the nonlinear delayed differential equation of the forced vibration system. The non-autonomous differential equations can be transformed into autonomous differential equations by the method of average or the multiple scales. The method of multiple scales is used to obtain the approximate analytical solutions. The bifurcation problem is investigated based on the new autonomous differential equations given thus.

PERTURBATION ANALYSIS

Since θ are very small, here it is set $\sin\theta \approx \theta - \dfrac{\theta^3}{6}$ and $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$. In the following part, the method of multiple scales is employed in the perturbation analysis. A small dimensionless perturbation parameter $\varepsilon\ (0 < \varepsilon < 1)$ is introduced into the equations used for bookkeeping only. A fast scale is characterized in $T_0 = t$ with the motion at Ω and a slow scale in $T_1 = \varepsilon t$. It can be set that $\eta = \varepsilon\hat{\eta},\ \theta = \varepsilon\hat{\theta},$ $F = \varepsilon\hat{F},\ \zeta_1 = \varepsilon\hat{\zeta}_1,\ \zeta_2 = \varepsilon\hat{\zeta}_2,\ \gamma^2 = \varepsilon\hat{\gamma}^2,$ and $\eta_\tau = \varepsilon\hat{\eta}_\tau$ in order to have the damp, nonlinear terms, and Sub-harmonic resonance force appear in the same perturbation magnitude. Dropping the ' ^ ' for convenience, (3) and (4) are written as:

$$\eta'' + \Omega_1^2 \eta = \varepsilon[-G_1\eta' - \mathrm{G}_2(\theta''\theta + \theta'^2) + \mathrm{G}_3\,\eta\cos(\Omega t) - H_3(\eta_\tau - \eta)] + \varepsilon^3\,\mathrm{G}_2(\theta''\frac{\theta^3}{6} + \theta'\frac{\theta^2}{6}) \tag{5}$$

$$\theta'' + \Omega_2^2\theta = \varepsilon[-H_1\theta' - H_2\theta\eta''] + \varepsilon^2\Omega_2^2\frac{\theta^3}{6} + \varepsilon^3 H_2\eta''\frac{\theta^3}{6} \tag{6}$$

The method of multiple scales is employed to seek second order approximate solutions of Equations (5) and (6) in the following form:

$$\eta(t,\varepsilon) = \eta_0(T_0,T_1) + \varepsilon\eta_1(T_0,T_1) + \varepsilon^2\eta_2(T_0,T_1) + O(\varepsilon^3) \tag{7}$$

$$\eta_\tau(t,\varepsilon) = \eta_{0\tau}(T_0,T_1) + \varepsilon\eta_{1\tau}(T_0,T_1) + \varepsilon^2\eta_{2\tau}(T_0,T_1) + O(\varepsilon^3) \tag{8}$$

$$\theta(t,\varepsilon) = \theta_0(T_0,T_1) + \varepsilon\theta_1(T_0,T_1) + \varepsilon^2\theta_2(T_0,T_1) + O(\varepsilon^3) \tag{9}$$

The derivatives with respect to time are expressed in terms of the new scales as:

$$\frac{d}{dt} = D_0 + \varepsilon D_1\,, \qquad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2, \tag{10}$$

Where $D_k = \dfrac{\partial}{\partial T_k},\ k = 0,1.$

Substituting Equations (7) to(10) into Equations (5) and (6) and equating coefficients of like powers of ε yield that:

Order ε^0:

$$\left(D_0^2 + \Omega_1^2\right)\eta_0 = 0 \tag{11}$$

$$\left(D_0^2 + \Omega_2^2\right)\theta_0 = 0 \tag{12}$$

Order ε^1:

$$\left(D_0^2 + \Omega_1^2\right)\eta_1 = -2D_0D_1\eta_0 - G_1D_0\eta_0 - G_2\theta_0D_0^2\theta_0 - G_2(D_0\theta_0)^2 + G_3\eta_0\cos(\Omega T_0) - H_3\eta_{0\tau} + H_3\eta_0 \tag{13}$$

$$\left(D_0^2 + \Omega_2^2\right)\theta_1 = -2D_0D_1\theta_0 - H_1D_0\theta_0 - H_2\theta_0D_0^2\eta_0 \tag{14}$$

Order ε^2:

$$(D_0^2 + \Omega_1^2)\eta_2 = -D_1^2\eta_0 - 2D_0D_1\eta_1 - G_1D_1\eta_0 - G_1D_0\eta_1 - G_2\theta_1D_0^2\theta_0 - 2G_2\theta_0D_0D_1\theta_0 - G_2\theta_0D_0^2\theta_1 - 2G_2D_0\theta_0D_1\theta_0 - 2G_2D_0\theta_0D_0\theta_1 + G_3\eta_1\cos(\Omega T_0) - \mathrm{H}_3(\eta_{1\tau} - \eta_1) \tag{15}$$

$$(D_0^2 + \Omega_2^2)\theta_2 = -D_1^2\theta_0 - 2D_0D_1\theta_1 - H_1D_1\theta_0 - H_1D_0\theta_1 - 2H_2\theta_0D_0D_1\eta_0 - H_2\theta_0D_0^2\eta_1 - H_2\theta_1D_0^2\eta_0 + \frac{1}{6}\Omega_2^2\theta_0^3 \tag{16}$$

The general solution of Equations (11) and (12) can be expressed as:

$$\eta_0(\mathrm{T}_0,\mathrm{T}_1) = A(\mathrm{T}_1)\exp(i\,\Omega_1 T_0) + cc \tag{17}$$

$$\theta_0(\mathrm{T}_0,\mathrm{T}_1) = B(\mathrm{T}_1)\exp(i\,\Omega_2 T_0) + cc \tag{18}$$

Where A and B are arbitrary functions at this level of approximation, cc denotes complex conjugate. The external excitation and the delayed feedback are expressed in complex forms.

$$G_3\cos(\Omega T_0) = \frac{1}{2}G_3\exp(i\,\Omega T_0) + cc \tag{19}$$

$$\eta_{0\tau}(\mathrm{T}_0,\mathrm{T}_1) = A_\tau(\mathrm{T}_1)\exp(i\,\Omega_1(T_0 - \tau)) + cc \tag{20}$$

A_τ can be expanded in a Taylor series (2002) under the assumption that the product of time delay and the small parameter ε is small compared to unity.

$$A_\tau(\mathrm{T}_1) = A(\mathrm{T}_1 - \varepsilon\tau) = \mathrm{A}(\mathrm{T}_1) - \varepsilon\tau\frac{dA(\mathrm{T}_1)}{dt} + \frac{1}{2}\varepsilon^2\tau^2\frac{d^2A(\mathrm{T}_1)}{dt^2} + \ldots \tag{21}$$

Substituting Equations (17) to (21) into Equations (13) and (14) yields

$$\left(D_0^2 + \Omega_1^2\right)\eta_1 = (-2i\,\Omega_1D_1A - G_1i\,\Omega_1A + H_3A)\exp(i\,\Omega_1T_0) + \frac{1}{2}G_3A\exp(i\,(\Omega_1 + \Omega)T_0) + 2G_2B^2\Omega_2^2\exp(2i\,\Omega_2T_0) + \frac{1}{2}G_3\bar{A}\exp(i\,(\Omega - \Omega_1)T_0) - H_3A\exp(i\,\Omega_1(T_0 - \tau)) + \mathrm{cc} \tag{22}$$

$$\left(D_0^2 + \Omega_1^2\right)\eta_1 = (-2i\,\Omega_1D_1A - G_1i\,\Omega_1A + H_3A)\exp(i\,\Omega_1T_0) + H_2\Omega_1^2\,\mathrm{AB}\exp(i\,(\Omega_1 + \Omega_2)T_0) + H_2\Omega_1^2\,\mathrm{A}\bar{\mathrm{B}}\exp(i\,(\Omega_1 - \Omega_2)T_0) + cc \tag{23}$$

The particular solutions of the above equations are:

$$\eta_1(T_0,T_1) = A_1\exp(i\,\Omega_1T_0) + E_1\exp(2i\,\Omega_2T_0) + \mathrm{E}_2\exp(i\,(\Omega_1 + \Omega)T_0) + E_3\exp(i\,(\Omega - \Omega_1)T_0) + \mathrm{cc} \tag{24}$$

$$\theta_1(T_0,T_1)=B_1\exp(i\,\Omega_2 T_0)+E_4\exp(i\,(\Omega_1+\Omega_2)T_0)$$
$$+E_5\exp(i\,(\Omega_1-\Omega_2)T_0)+\text{cc} \tag{25}$$

Substituting Equations (17), (18), (24) and (25) into Equations (15) and (16), hence solving the resulting equations, we get:

$$\eta_2(T_0,T_1)=E_6\exp(2i\,\Omega_2 T_0)+\mathrm{E}_7\exp(i\,(\Omega_1+\Omega)T_0)$$
$$+E_8\exp(i\,(\Omega-\Omega_1)T_0)+\mathrm{E}_9\exp(i\,(\Omega_1+2\Omega_2)T_0)$$
$$+\mathrm{E}_{10}\exp(i\,(\Omega_1-2\Omega_2)T_0)+\mathrm{E}_{11}\exp(i\,(2\Omega_2+\Omega)T_0)$$
$$+\mathrm{E}_{12}\exp(i\,(2\Omega_2-\Omega)T_0)+\mathrm{E}_{13}\exp(i\,(\Omega_1+2\Omega)T_0)$$
$$+\mathrm{E}_{14}\exp(i\,(2\Omega-\Omega_1)T_0)+A_2\exp(i\,\Omega_1 T_0)+\text{cc} \tag{26}$$

$$\theta_2(T_0,T_1)=E_{15}\exp(i\,(\Omega_1+\Omega_2)T_0)+E_{16}\exp(i\,(\Omega_1-\Omega_2)T_0)$$
$$+E_{17}\exp(3i\,\Omega_2 T_0)+\mathrm{E}_{18}\exp(i\,(\Omega+\Omega_1+\Omega_2)T_0)$$
$$+\mathrm{E}_{19}\exp(i\,(\Omega+\Omega_1-\Omega_2)T_0)+\mathrm{E}_{20}\exp(i\,(\Omega-\Omega_1+\Omega_2)T_0)$$
$$+\mathrm{E}_{21}\exp(i\,(\Omega-\Omega_1-\Omega_2)T_0)+\mathrm{E}_{22}\exp(i\,(2\Omega_1+\Omega_2)T_0)$$
$$+\mathrm{E}_{23}\exp(i\,(2\Omega_1+\Omega_2)T_0)+\mathrm{E}_{24}\exp(i\,(2\Omega_1-\Omega_2)T_0)$$
$$+B_2\exp(i\,\Omega_2 T_0)+\text{cc} \tag{27}$$

Where A_i, B_i, $(\mathrm{i}=1,2)$ and E_n, $(n=1,...,24)$ are complex functions in T_1 and cc denotes complex conjugate. From the above derived solutions, the reported resonance cases are:

1) Primary resonance: $\Omega\cong\Omega_1$.

2) Sub-harmonic resonance: $\Omega\cong 2\Omega_1$.

3) Internal resonance: $\Omega_1\cong\Omega_2,\ \Omega_1\cong 2\Omega_2$.

4) Combined resonance: $\Omega\cong\pm(2\Omega_2-\Omega_1),\ \Omega\cong(2\Omega_2+\Omega_1)$.

5) Simultaneous resonance: any combination of above resonance cases is considered as simultaneous resonance.

STABILITY ANALYSIS

From numerically studying the different resonance cases, we find that the worst is resonance case is the simultaneous resonance case $\Omega\cong 2\Omega_1,\ \Omega_1\cong 2\Omega_2$. So that we introduce the detuning parameters σ_1 and σ_2 according to the following:

$$\Omega=2\Omega_1+\varepsilon\sigma_1,\ \Omega_1=2\Omega_2-\varepsilon\sigma_2 \tag{28}$$

Substituting Equation (28) into Equations (22) and (23) and setting the coefficients of the secular terms to zero yield the solvability conditions given by:

$$-2i\,\Omega_1 D_1 A-G_1 i\,\Omega_1 A+\frac{1}{2}G_3\bar{A}\exp(i\,\sigma_1 T_1)$$
$$+2G_2 B^2\Omega_2^2\exp(i\,\sigma_2 T_1)-H_3 A\exp(-i\,\Omega_1\tau)+H_3 A=0 \tag{29}$$

$$-2i\,\Omega_2 D_1 B-H_1 i\,\Omega_2 B+H_2\Omega_1^2 A\bar{B}\exp(-i\,\sigma_2 T_1)=0 \tag{30}$$

We express the complex function A, B in the polar form as

$$A=\frac{1}{2}a\exp(i\,\theta_1),\ B=\frac{1}{2}b\exp(i\,\theta_2) \tag{31}$$

Where a, b, θ_1 and θ_2 are real-valued functions. Substituting Equation (31) into Equations (29) and (30) and separating real and imaginary parts yields:

$$a'=-\frac{1}{2}G_1 a-\frac{1}{2\Omega_1}G_2\Omega_2^2 b^2\sin\varphi_2+\frac{1}{4\Omega_1}G_3\,a\sin\varphi_1+\frac{1}{2\Omega_1}aH_3\sin\Omega_1\tau \tag{32}$$

$$a\varphi_1'=\sigma_1 a+\frac{1}{\omega_1}G_2\Omega_2^2 b^2\cos\varphi_2+\frac{1}{2\Omega_1}G_3\,a\cos\varphi_1-\frac{1}{\Omega_1}aH_3\cos\Omega_1\tau+\frac{1}{\Omega_1}aH_3 \tag{33}$$

$$b'=-\frac{1}{2}H_1 b+\frac{1}{4\Omega_2}H_2\Omega_1^2 ab\sin\varphi_2 \tag{34}$$

$$b(\frac{\varphi_1'}{4}+\frac{\varphi_2'}{2})=(\frac{\sigma_1}{4}-\frac{\sigma_2}{2})\mathrm{b}+\frac{1}{4\Omega_2}H_2\Omega_1^2 ab\cos\varphi_2 \tag{35}$$

Where $\varphi_1=\sigma_1 T_1-2\theta_1,\ \varphi_2=\theta_1-2\theta_2-\sigma_2 T_1$.

For the steady state solution $a'=b'=0$, $\varphi_m'=0;\ m=1,2$. Then it follows from Equations (32) to (35) that the steady state solutions are given by:

$$-\frac{1}{2}G_1 a+\frac{1}{4\Omega_1}G_3\,a\sin\varphi_1+\frac{1}{2\Omega_1}aH_3\sin\Omega_1\tau-\frac{1}{2\Omega_1}G_2\Omega_2^2 b^2\sin\varphi_2=0 \tag{36}$$

$$-\sigma_1 a-\frac{1}{2\Omega_1}G_3\,a\cos\varphi_1+\frac{1}{\Omega_1}aH_3\cos\Omega_1\tau-\frac{1}{\Omega_1}aH_3-\frac{1}{\Omega_1}G_2\Omega_2^2 b^2\cos\varphi_2=0 \tag{37}$$

$$\frac{1}{2}H_1 b-\frac{1}{4\Omega_2}H_2\Omega_1^2 ab\sin\varphi_2=0 \tag{38}$$

$$(\frac{\sigma_1}{4}-\frac{\sigma_2}{2})\mathrm{b}+\frac{1}{4\Omega_2}H_2\Omega_1^2 ab\cos\varphi_2=0 \tag{39}$$

From Equations (36) to (39), we have the following cases:

Case 1. $a \neq 0$ and $b \neq 0$: in this case, the frequency response equation are given by the following equations:

$$
\begin{aligned}
&(G_1^2+\sigma_1^2+\frac{1}{4\Omega_1^2}G_3^2+\frac{2}{\Omega_1^2}H_3^2-\frac{1}{\Omega_1}G_1G_3\sin\varphi_1 \\
&-\frac{2}{\Omega_1}G_1H_3\sin\Omega_1\tau-\frac{1}{\Omega_1^2}G_3H_3\cos(\varphi_1+\Omega_1\tau) \\
&-\frac{2}{\Omega_1}\sigma_1H_3\cos\Omega_1\tau+\frac{2}{\Omega_1}\sigma_1H_3+\frac{1}{\Omega_1^2}G_3H_3\cos\varphi_1 \\
&-\frac{2}{\Omega_1^2}H_3^2\cos\Omega_1\tau+\frac{1}{\Omega_1}\sigma_1G_3\cos\varphi_1)a^2-\frac{1}{\Omega_1^2}G_2^2\Omega_2^4b^4=0
\end{aligned}
\tag{40}
$$

$$
H_1^2b^2+(\frac{\sigma_1}{2}-\sigma_2)^2b^2-(\frac{1}{4\Omega_2^2}H_2^2\Omega_1^4)a^2b^2=0 \tag{41}
$$

Case 2: $a \neq 0$ and $b = 0$: in this case, the frequency response equation is given by:

$$
\begin{aligned}
&(G_1^2+\sigma_1^2+\frac{1}{4\Omega_1^2}G_3^2+\frac{2}{\Omega_1^2}H_3^2-\frac{1}{\Omega_1}G_1G_3\sin\varphi_1 \\
&-\frac{2}{\Omega_1}G_1H_3\sin\Omega_1\tau-\frac{1}{\Omega_1^2}G_3H_3\cos(\varphi_1+\Omega_1\tau) \\
&-\frac{2}{\Omega_1}\sigma_1H_3\cos\Omega_1\tau+\frac{2}{\Omega_1}\sigma_1H_3+\frac{1}{\Omega_1^2}G_3H_3\cos\varphi_1 \\
&-\frac{2}{\Omega_1^2}H_3^2\cos\Omega_1\tau+\frac{1}{\Omega_1}\sigma_1G_3\cos\varphi_1)a^2=0
\end{aligned}
\tag{42}
$$

Linear solution

Now to the stability of the linear solution of the obtained fixed let us consider A and B in the forms

$$
A(T_1)=\frac{1}{2}(p_1-iq_1)\exp(i\,\delta T_1)\,,
$$

$$
B(T_1)=\frac{1}{2}(p_2-iq_2)\exp(i\,\delta_1 T_1)
$$

Where p_1, p_2, q_1 and q_2 are real values and considering $\delta=\frac{\sigma_1}{2}$, $\delta_1=\sigma_2$.

Substituting Equation (42) into the linear parts of Equations (29) and (30) and separating real and imaginary parts, the following system of equations are obtained:

For the solution ($a \neq 0$ and $b \neq 0$), we get:

$$
\begin{aligned}
p_1'=&(-\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)p_1+(\frac{1}{4\Omega_1}G_3-\frac{\sigma_1}{2}+ \\
&\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau-\frac{1}{2\Omega_1}H_3)q_1
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
q_1'=&(\frac{1}{2}\sigma_1+\frac{1}{4\Omega_1}G_3-\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau+\frac{1}{2\Omega_1}H_3)p_1+(-\frac{1}{2}G_1 \\
&+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)q_1
\end{aligned}
\tag{44}
$$

$$
p_2'=(-\frac{1}{2}H_1)p_2-\sigma_2q_2 \tag{45}
$$

$$
q_2'=\sigma_2p_2-(\frac{1}{2}H_1)q_2 \tag{46}
$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$
\begin{vmatrix}
(-\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau-\lambda) & (\frac{1}{4\Omega_1}G_3-\frac{\sigma_1}{2}+\frac{1}{2\Omega_1}H_3(\cos\Omega_1\tau-1)) & 0 & 0 \\
(\frac{1}{2}\sigma_1+\frac{1}{4\Omega_1}G_3+\frac{1}{2\Omega_1}H_3(1-\cos\Omega_1\tau)) & (-\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau-\lambda) & 0 & 0 \\
0 & 0 & (-\frac{1}{2}H_1-\lambda) & -\sigma_2 \\
0 & 0 & \sigma_2 & (-\frac{1}{2}H_1-\lambda)
\end{vmatrix}=0
\tag{47}
$$

After extract we obtain that

$$
\lambda^4+r_1\lambda^3+r_2\lambda^2+r_3\lambda+r_4=0, \tag{48}
$$

Where $r_1=H_1+G_1-\frac{1}{\Omega_1}H_3\sin\Omega_1\tau$,

$$
\begin{aligned}
r_2=&\frac{1}{4}H_1^2+\frac{1}{4}G_1^2+\frac{1}{\Omega_1}(\frac{1}{4\Omega_1}H_3\sin\Omega_1\tau-H_1-\frac{1}{2}G_1)H_3\sin\Omega_1\tau \\
&+\frac{\sigma_1}{2}(\frac{\sigma_1}{2}+\frac{1}{\Omega_1}H_3)+\sigma_2^2-\frac{1}{16\Omega_1^2}G_3^2+\frac{1}{4\Omega_1^2}H_3^2 \\
&-\frac{1}{2\Omega_1}(\sigma_1-\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau+\frac{1}{\Omega_1}H_3)H_3\cos\Omega_1\tau+G_1H_1
\end{aligned}
$$

$$
\begin{aligned}
r_3=&\frac{1}{4}H_1(\frac{1}{\Omega_1}H_3+\sigma_1)^2-\frac{1}{2\Omega_1}H_1H_3\cos\Omega_1\tau(\frac{1}{\Omega_1}H_3+\sigma_1) \\
&+\frac{1}{4\Omega_1^2}H_1H_3^2+\frac{1}{4}H_1G_1^2+(\sigma_2^2+\frac{1}{4}H_1^2)(G_1-\frac{1}{\Omega_1}H_3\sin\Omega_1\tau) \\
&-\frac{1}{2\Omega_1}G_1H_1H_3\sin\Omega_1\tau-\frac{1}{16\Omega_1^2}H_1G_3^2
\end{aligned}
$$

$$r_4 = (\frac{1}{4}(\frac{1}{\Omega_1}\mathrm{H}_3+\sigma_1)^2 - \frac{1}{2\Omega_1}H_3\cos\Omega_1\tau(\frac{1}{\Omega_1}\mathrm{H}_3+\sigma_1)+\frac{1}{4}G_1^2 - \frac{1}{16\Omega_1^2}\mathrm{G}_3^2+\frac{1}{4\Omega_1^2}\mathrm{H}_3^2-\frac{1}{2\Omega_1}\mathrm{G}_1\mathrm{H}_3\sin\Omega_1\tau)(\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2)$$

According to the Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied:

$$r_1>0,\ r_1r_2-r_3>0,\ r_3(r_1r_2-r_3)-r_1^2r_4>0,\ r_4>0. \quad (49)$$

When Conditions (49) are not satisfied, the initial equilibrium solution is unstable, and bifurcations may occur. But if Conditions (49) are not satisfied.

Conditions (49) imply that all the eigenvalues of the Equation (47) have negative real parts. When Conditions (49) are not satisfied, this is not the case. First, we prove that zero cannot be a simple eigenvalue of the Equation (47). If zero is an eigenvalue of the Equation (47), then it is not simple. In fact, suppose zero is an eigenvalue of the Equation (47), then it follows $r_4=0$.

Denote $A=\frac{1}{\Omega_1}\mathrm{H}_3+\sigma_1,$ we have

$$r_4 = (\frac{1}{4}A^2 - (\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau)A+\frac{1}{4}G_1^2-\frac{1}{16\Omega_1^2}\mathrm{G}_3^2 + \frac{1}{4\Omega_1^2}\mathrm{H}_3^2-\frac{1}{2\Omega_1}\mathrm{G}_1\mathrm{H}_3\sin\Omega_1\tau)(\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2)=0 \quad (50)$$

Regard A as a variable, if Equation (50) has a real solution, it follows, the discriminate of quadratic Equation (50), $\Delta\geq 0.$ On the other hand,

$$\begin{aligned}\Delta &= \frac{1}{4\Omega_1^2}H_3^2(\cos\Omega_1\tau)^2(\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2)^2-(\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2)^2(\frac{1}{4}G_1^2 - \frac{1}{16\Omega_1^2}\mathrm{G}_3^2+\frac{1}{4\Omega_1^2}\mathrm{H}_3^2-\frac{1}{2\Omega_1}\mathrm{G}_1\mathrm{H}_3\sin\Omega_1\tau)\\ &= -[\frac{1}{4}(\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2)^2((\frac{1}{\Omega_1}\mathrm{H}_3\sin\Omega_1\tau-G_1)^2-\frac{1}{4\Omega_1^2}\mathrm{G}_3^2)]\leq 0\end{aligned}$$

Implies $(\frac{1}{\Omega_1}\mathrm{H}_3\sin\Omega_1\tau-G_1)\geq\frac{1}{2\Omega_1}\mathrm{G}_3$.

So $r_4=0$ implies $\frac{1}{4}\mathrm{H}_1^2+\sigma_2^2=0$ or $\frac{1}{\Omega_1}\mathrm{H}_3\sin\Omega_1\tau-G_1=\pm\frac{1}{2\Omega_1}G_3$, consequently,

$$A=\frac{1}{\Omega_1}H_3\cos\Omega_1\tau$$

Substituting into r_3, we have $r_3=0$, so zero is not simple.

(i) Double zero and two negative eigenvalues

Taking $r_1=0.4,\ r_2=0.04,\ r_3=r_4=0,$ then Equation (48) has a double zero and two negative eigenvalues, $\lambda_{1,2}=0,\ \lambda_{3,4}=-0.2$. One choice of the parameters that satisfy these conditions is: $\Omega_1=2.6,\ H_1=0.3,\ \mathrm{H}_3=0.2, G_1=0.1,$ $G_3=1.67,\ \tau=1.3,\ \sigma_1=0.1,$ $\sigma_2=0.1$.

(ii) Double zero and a pair of purely imaginary eigenvalues

Taking the following parameter values: $\Omega_1=2.6,\ H_1=0.037,\ \mathrm{H}_3=0.8, G_1=0.01,$ $G_3=0.8,\ \tau=1.3,\ \sigma_1=2,\ \sigma_2=0.1$, it yields that $r_1=r_3=0$, $r_2=1$ in Equation (49) has a double zero and two imaginary eigenvalues $\lambda_{1,2}=0,\ \lambda_{3,4}=\pm i$.

For the solution ($a\neq 0$ and $b=0$), we get:

$$p_1' = (-\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)p_1+(\frac{1}{4\Omega_1}G_3-\frac{\sigma_1}{2}+\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau-\frac{1}{2\Omega_1}H_3)q_1 \quad (51)$$

$$q_1' = (\frac{1}{2}\sigma_1+\frac{1}{4\Omega_1}G_3-\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau+\frac{1}{2\Omega_1}H_3)p_1+(-\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)q_1 \quad (52)$$

The stability of the linear solution is obtained from the zero characteristic equation

$$\begin{vmatrix} -\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau-\lambda & \frac{1}{4\Omega_1}G_3-\frac{\sigma_1}{2}+\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau-\frac{1}{2\Omega_1}H_3 \\ \frac{1}{2}\sigma_1+\frac{1}{4\Omega_1}G_3-\frac{1}{2\Omega_1}H_3\cos\Omega_1\tau+\frac{1}{2\Omega_1}H_3 & -\frac{1}{2}G_1+\frac{1}{2\Omega_1}H_3\sin\Omega_1\tau-\lambda\end{vmatrix}=0 \quad (53)$$

hence $\lambda^2+r_5\lambda+r_6=0,$

where $r_5=G_1-\frac{1}{\Omega_1}H_3\sin\Omega_1\tau$,

$$r_6=\frac{1}{4}G_1^2-\frac{1}{2\Omega_1}G_1H_3\sin\Omega_1\tau+\frac{1}{2\Omega_1^2}H_3^2-\frac{1}{16\Omega_1^2}G_3^2+\frac{1}{4}\sigma_1^2 + \frac{1}{2\Omega_1}\sigma_1H_3(1-\cos\Omega_1\tau)-\frac{1}{2\Omega_1^2}H_3^2\cos\Omega_1\tau.$$

Then we get $\lambda_{1,2}=\frac{1}{2}(-r_5\pm\sqrt{r_5^2-4r_6})$

The linear solution is stable in this case if and only if $r_5>0$, and otherwise it is unstable.

Non-linear solution

To determine the stability of the fixed points, one lets

$$a = a_{10} + a_{11},\ b = b_{10} + b_{11},\ \varphi_m = \varphi_{m0} + \varphi_{m1}, (m = 1, 2), \quad (54)$$

Where a_{10}, b_{10} and φ_{m0} are the solutions of Equations (36) to (39) and $a_{11}, b_{11}, \varphi_{m1}$ are perturbations which are assumed to be small compared to a_{10}, b_{10} and φ_{m0}. Substituting Equation (54) into Equations (32) to (35), using Equations (36) to (39) and keeping only the linear terms in $a_{11}, b_{11}, \varphi_{m1}$ we obtain:

For the solution ($a \neq 0$ and $b \neq 0$), we get:

$$a_{11}' = (-\frac{1}{2}G_1 + \frac{1}{4\Omega_1}G_3\sin\varphi_{10} + \frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)a_{11} - (\frac{1}{\Omega_1}G_2\Omega_2^2 b_{10}\sin\varphi_{20})b_{11} + (\frac{1}{4\Omega_1}G_3 a_{10}\cos\varphi_{10})\varphi_{11} - (\frac{1}{2\Omega_1}G_2\Omega_2^2 b_{10}^2\cos\varphi_{20})\varphi_{21} \quad (55)$$

$$\varphi_{11}' = (\frac{\sigma_1}{a_{10}} + \frac{1}{2\Omega_1 a_{10}}G_3\cos\varphi_{10} - \frac{1}{\Omega_1 a_{10}}H_3\cos\Omega_1\tau)a_{11} + (\frac{2}{\Omega_1 a_{10}}b_{10}G_2\Omega_2^2\cos\varphi_{20})b_{11} + (\frac{1}{2\Omega_1}G_3\sin\varphi_{10})\varphi_{11} + (\frac{1}{\Omega_1 a_{10}}G_2\Omega_2^2 b_{10}^2\sin\varphi_{20})\varphi_{21} \quad (56)$$

$$b_{11}' = (\frac{1}{4\Omega_2}H_2\Omega_1^2 b_{10}\sin\varphi_{20})a_{11} + (-\frac{1}{2}H_1 + \frac{1}{4\Omega_2}H_2\Omega_1^2 a_{10}\sin\varphi_{20})b_{11} + (\frac{1}{4\Omega_2}H_2\Omega_1^2 a_{10}b_{10}\cos\varphi_{20})\varphi_{21} \quad (57)$$

$$\begin{aligned}\varphi_{21}' = {} & (\frac{1}{2\Omega_2}H_2\Omega_1^2\cos\varphi_{20} - \frac{\sigma_1}{2a_{10}} - \frac{1}{4\Omega_1 a_{10}}G_3\cos\varphi_{10} + \frac{1}{2\Omega_1 a_{10}}H_3\cos\Omega_1\tau)a_{11} \\ & + (\frac{1}{b_{10}}(\frac{\sigma_1}{2} - \sigma_2) + \frac{1}{2\Omega_2 b_{10}}a_{10}H_2\Omega_1^2\cos\varphi_{20} - \frac{1}{\Omega_1 a_{10}}b_{10}G_2\Omega_2^2\cos\varphi_{20})b_{11} \\ & - (\frac{1}{4\Omega_1}G_3\sin\varphi_{10})\varphi_{11} + ((\frac{1}{2\Omega_2}a_{10}H_2\Omega_1^2 - \frac{1}{2\Omega_1 a_{10}}G_2\Omega_2^2 b_{10}^2)\sin\varphi_{20})\varphi_{21}\end{aligned} \quad (58)$$

The stability of a particular fixed point with respect to perturbations proportional to $\exp(\lambda t)$ depends on the real parts of the roots of the matrix. Thus, a fixed point given by Equations (55) to (58) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

For the solution ($a \neq 0$ and $b = 0$), we get:

$$a_{11}' = (-\frac{1}{2}G_1 + \frac{1}{4\Omega_1}G_3\sin\varphi_{10} + \frac{1}{2\Omega_1}H_3\sin\Omega_1\tau)a_{11} + (\frac{1}{4\Omega_1}G_3 a_{10}\cos\varphi_{10})\varphi_{11} \quad (59)$$

$$\varphi_{11}' = (\frac{\sigma_1}{a_{10}} + \frac{1}{2\Omega_1 a_{10}}G_3\cos\varphi_{10} - \frac{1}{\Omega_1 a_{10}}H_3\cos\Omega_1\tau)a_{11} + (\frac{1}{2\Omega_1}G_3\sin\varphi_{10})\varphi_{11} \quad (60)$$

The stability of a given fixed point to a disturbance proportional to $\exp(\lambda t)$ is determined by the roots of:

$$\begin{vmatrix} -\frac{1}{2}G_1 + \frac{1}{4\Omega_1}G_3\sin\varphi_{10} + \frac{1}{2\Omega_1}H_3\sin\Omega_1\tau - \lambda & \frac{1}{4\Omega_1}G_3 a_{10}\cos\varphi_{10} \\ \frac{\sigma_1}{a_{10}} + \frac{1}{2\Omega_1 a_{10}}G_3\cos\varphi_{10} - \frac{1}{\Omega_1 a_{10}}H_3\cos\Omega_1\tau & \frac{1}{2\Omega_1}G_3\sin\varphi_{10} - \lambda \end{vmatrix} = 0 \quad (61)$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix Equation (61) are less than zero.

NUMERICAL RESULTS

Runge-kutta fourth order method has been conducted to determine the numerical solution of the given system. Figure 2 illustrates the response and phase-plane for the non- resonant system at some practical values of the equations parameters without time delay control.

Resonance cases

we see that the amplitude increasing at the resonance cases and the worst case is the simultaneous resonance case when $\Omega = 2\Omega_1,\ \Omega_1 = 2\Omega_2,$ which the amplitudes are increased to about 175% compared with the basic case shown in Figure 3, which means that the system need s to reduced the amplitude of vibration or controlled.

Figure 4 shows the effects of time delayed, from this figure we can see that the delayed is effect at the regions $1.1 \le \tau \le 2.2$, $3.5 \le \tau \le 4.5$ and $5.9 \le \tau \le 6.9$, otherwise not affect. Figure 5, illustrates system at the worst case is the simultaneous resonance case when $\Omega = 2\Omega_1,\ \Omega_1 = 2\Omega_2,$ with time delay control the amplitude of vibrations is very minimum , which means that the time delay control is very effective.

Effect of parameters

The amplitude of the system is monotonic decreasing function of the coefficient gain G_1 more increasing of G_1 leads to saturation phenomena as shown in Figure 6a. The amplitude is a monotonic increasing function of the excitation amplitude of the coefficient G_3. But more increasing of coefficient G_3 leads to very high amplitude and the system becomes unstable as shown in Figure 6b

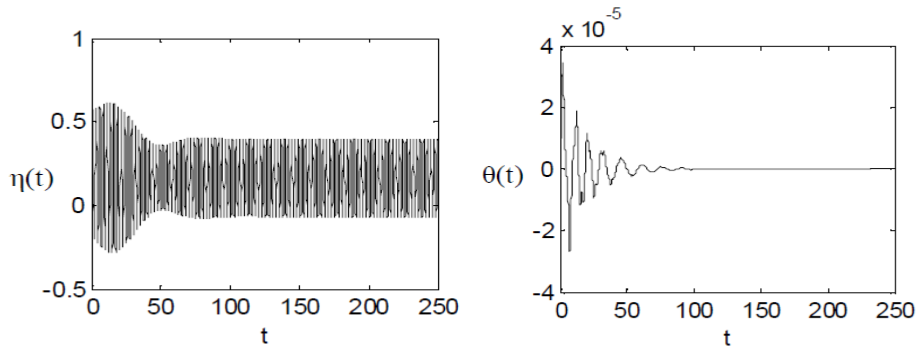

Figure 2. Basic case without control (Ω_1=2.4, Ω=2.5, Ω_2=0.3, H_3=0, G_1=0.1, G_2=0.9, G_3=0.8, H_1=0.1, H_2=0.7).

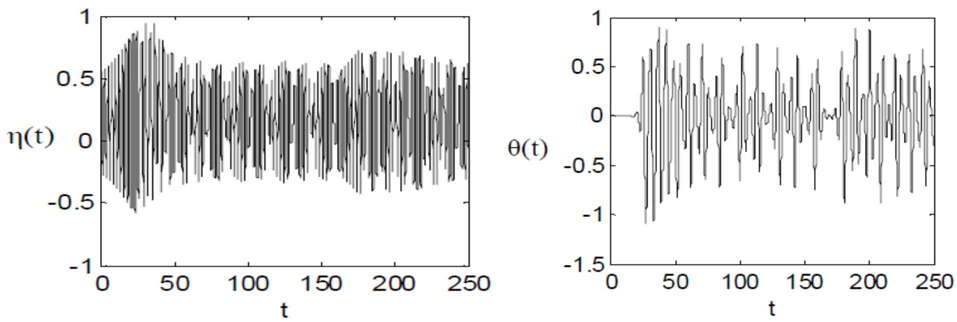

Figure 3. Resonance case without control (Ω_1=2.6, Ω_2=1.3, Ω=5.2, H_3=0, G_1=0.1, G_2=0.9, G_3=0.8, H_1=0.1, H_2=0.7).

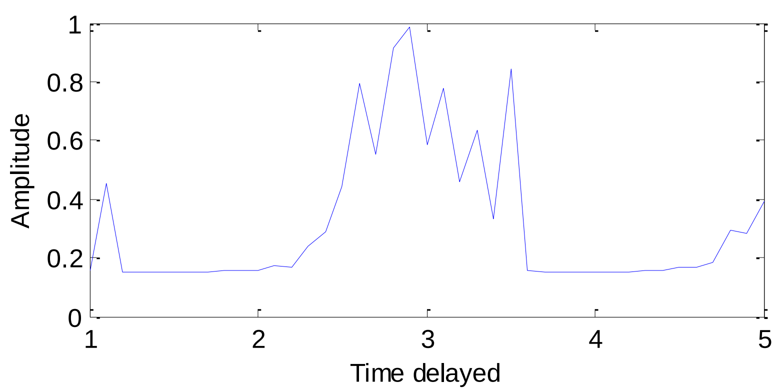

Figure 4. Effects of time delayed (τ) on the system.

Response curves

The frequency response Equations (40) and (41) are nonlinear algebraic equations of a, b. These equations are solved numerically as shown in Figures 7 and 8. From Case 1 where $a \neq 0,\ b \neq 0$: Figure 7, shown that the steady state amplitudes of the system are monotonic decreasing functions in Ω_1, G_1. But the steady state amplitudes of the system are monotonic increasing functions in G_2, G_3, H_3.

Comparison between numerical solution and approximation solution

Now, we get good agreement of the approximate solution obtained from frequency response equation and the

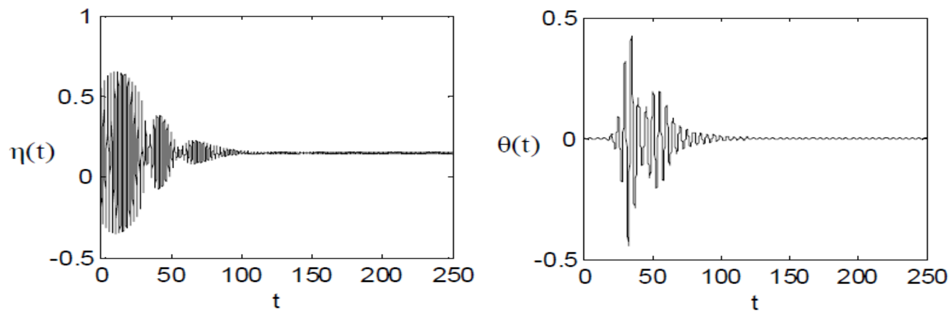

Figure 5. Resonance case with control (Ω_1=2.6, Ω=5.2, Ω_2=1.3, H_3=0.2, G_1=0.1, G_2=0.9, G_3=0.8, H_1=0.1, H_2=0.7, τ=1.3).

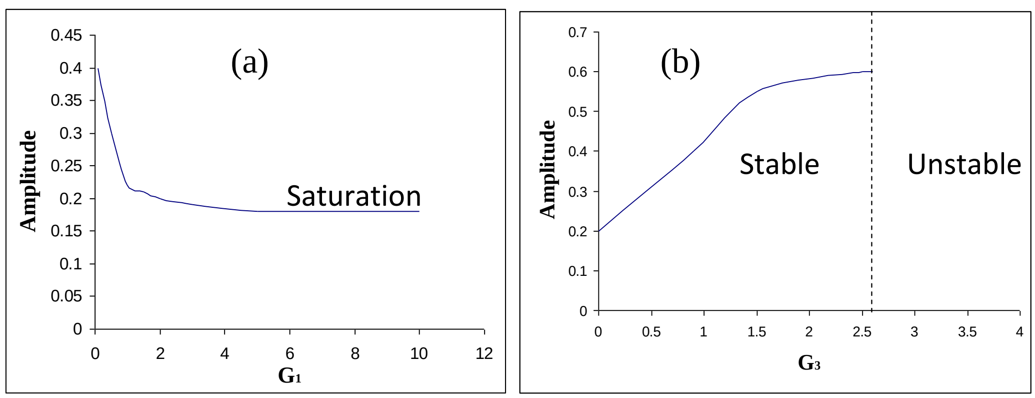

Figure 6. Effect of parameters.

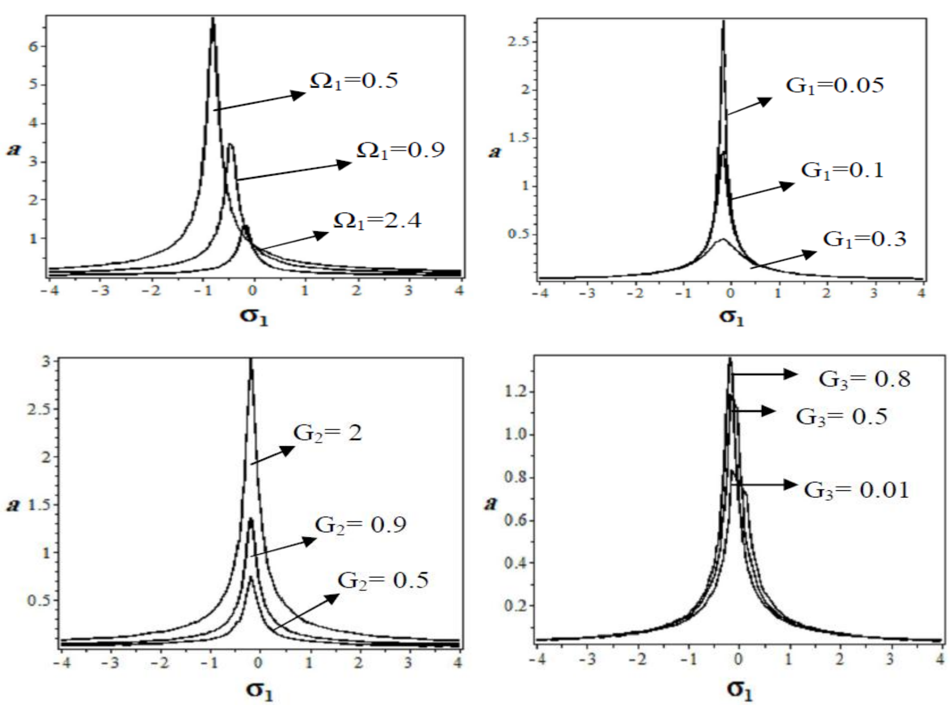

Figure 7. Frequency response curves (a≠0 *and* b ≠0).

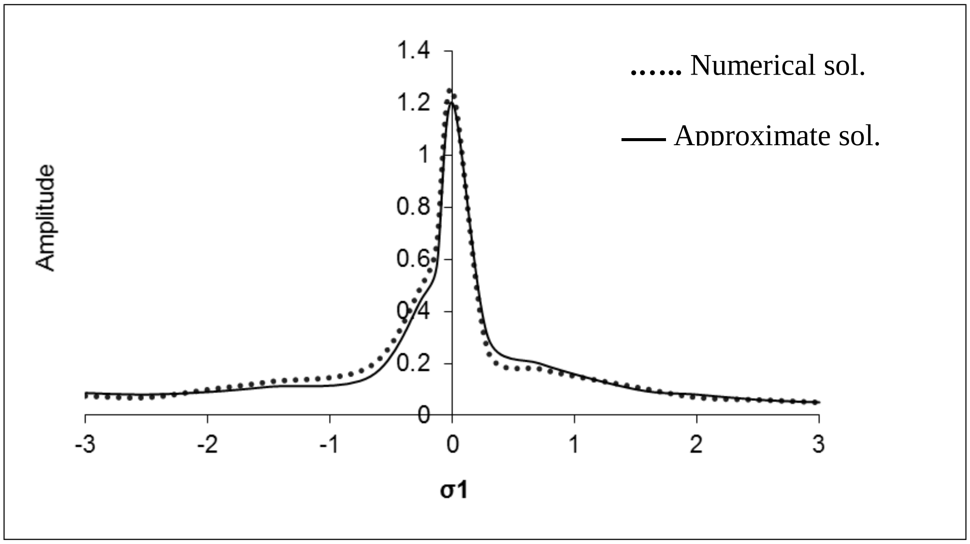

Figure 8. Comparison between numerical solution and approximation solution.

numerical solution obtained by Runge Kutta forth order method as shown in Figure 8.

CONCLUSIONS

The response and stability of the system of coupled non-linear differential equations representing the non-linear dynamical two-degree-of-freedom the system is studied. The time delayed feedback control is applied to suppress the vibration of the system. The equation of motion and the perturbation analysis and the stability of the equilibrium solutions are given. The analytical solutions and numerical simulations of the delayed feedback control are presented. The investigation includes the solutions applying both the perturbation technique, and Runge-Kutta numerical method. Also a comparison with similar published work is reported. Here, the main conclusions of the system are reported briefly.

(1) The worst resonance case is the simultaneous resonance case when $\Omega = 2\Omega_1,\ \Omega_1 = 2\Omega_2,$ which the amplitudes are increased to about 175% compared with the basic case.

(2) The most effective regions of the time delayed is effect are $1.1 \le \tau \le 2.2$, $3.5 \le \tau \le 4.5$ and $5.9 \le \tau \le 6.9$, otherwise not effect.

(3) The time delayed is very powerful technique to reduce the vibration of the system at the simultaneous resonance case.

(4) The steady-state amplitude is monotonic decreasing functions in the gain G_1, and monotonic increasing of the excitation amplitude G_3.pectively.

(5) The approximate solution is good agreement with numerical solution.

Conflict of Interest

The authors have not declared any conflict of interest.

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