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Generalized optimal controller design for all pole systems using standard forms

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In this work, a generalized controller design method for the systems with all pole transfer function has been given. The method basically uses the standard forms, which have been introduced firstly in 50's and improved recent years. In the proposed method, a PI controller, which is in the feed forward and a polynomial controller, which is in the feedback path, are used. Degree of the polynomial controller can be determined according to the degree of the all pole system. Parameters of the controllers can be easily found using the standard forms and the proposed simple mathematical operations. The method directly targets the step response. In the paper, the advantages of the proposed method with the results of some well known controller design methods have been given for comparison.

Key words: Standard forms, controller design, PI, optimization.

INTRODUCTION

Today, despite many proposed modern controller design methods, the use of classical controller design methods is still popular. Amongst the reasons for this, superiority of the classically controlled system's performances and the easiness of the classical controller design methods can be cited. Nevertheless, obtaining the optimum values of the classical fix formed controller parameters is known for a long time. In these methods, the controller parameters are minimized using a known error criteria with respect to the system transfer function. Thus, the optimal values of the controller parameters are obtained. Early works of this field are concentrated on the integral squared error (ISE) criterion since these allowed solutions to be obtained in the s-domain by using Parseval's theorem (Chen, 1994). Again, results of the other criteria, which are given in many textbooks, such as integral absolute error (IAE) and integral time absolute error (ITAE) can be obtained by using extensive computations or by simulations. Therefore, applications of these methods requires expert designer and the process of the method takes long time and needs complex operations. Thus, these methods are not practical. On the other

hand, the use of the closed loop transfer function's standard forms, which is another method for optimal controller design introduced by Graham and Lathrop (1953) in the early 50's, takes an important role for eliminating these disadvantages. This method is based on obtaining system's parameters, which are optimized for the compensated system's closed loop transfer function. For this purpose, coefficients of closed loop transfer functions are obtained for optimal responses. Then the general structure of the standard forms, which is given in Eq.1, can be formed by using these coefficients

$$\frac{C(s)}{R(s)} = \frac{c_k s^k + c_{k-1} s^{k-1} + \dots + c_1 s + c_0}{s^m + d_{m-1} s^{m-1} + \dots + d_1 s + d_0} .$$
 (1)

In this way, controller parameters to compensate the uncompensated system can be directly obtained from the standard forms without using complex and time consuming optimization procedures.

Graham and Lathrop (1953) have used the IAE and ITAE criteria for obtaining these standard forms and they have only considered all pole Standard forms. On the other hand, Dorf and Bishop (1996) have suggested the integral squared error (ISE) criterion, but they did not give the standard form coefficients. Instead, they have obtained the standard form coefficients for the systems

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with one zero using the ITAE criteria for a ramp input. In some textbooks, devoting a separate section for the subject, in the procedure of obtaining the closed loop transfer functions of m poles standard forms with one zero, it is assumed that $c_0=d_0$ and $c_1=d_1$ to have a zero steady-state error for a ramp input (Dorf and Bishop, 1995, 1996). But, this case restricts the independently chosen controller parameters in the standard form based controller design procedures. Again, these restrictions cause very oscillatory step responses for the same systems. Also overshoot of the responses increases. On the other hand, performances of the standard forms, proposed by Atherton and Boz (1998), have almost cured these prob-lems. Basically, in the proposed method, standard forms are obtained for $c_1 \neq d_1$. In the same work, coefficients of the standard forms with all pole and one zero have also been obtained for Integral Squared Time Error (ISTE) and Integral of the Squared Time Error (IST²E) criteria.

As for the use of standard forms in the controller design, many new design methods based on standard forms are cited in the literature (Atherton and Boz, 1998; Atherton and Majhi, 1998; Atherton, 2006; Kaya and Atherton, 2008). Boz and Sari (2008) also proposed a new PI-PD controller design method for third order all pole systems using standard forms. In addition to this, in this work, a generalized version of the controller design method for all pole systems has been studied. The design method uses the standard forms with c1≠d1 optimized for ISTE and IST²E criteria. The proposed controller structure consists of a PI controller in the feed forward path and also a polynomial controller in the feedback path. Degree of the polynomial controller is dependent on the uncompensated system degree (Sari, 2005). The controller parameters can be easily and directly calculated using the obtained basic mathematical expressions. The proposed mathematical expressions, which are generalized for mth degree of all pole systems, will also provide easiness in the applications. The method directly targets the step responses and uses the ISE and IST²E criteria for minimizing the error signal (e(t)). The paper is organized as follows; in the materials and methods section a short description is given for integral performance criteria and Standard forms. The proposed method is explained in the results and discussions section. Advantages and validity of the proposed method over some well known design methods are given with two different examples for comparison in the same section as well. The conclusions of the work are summarized in the conclusions section.

MATERIALS AND METHODS

Integral performance criteria

Performances of dynamical systems are usually defined with their transient responses. On the other hand, the transient response is determined by measuring the system's output in terms of rising time, settling time, overshoot and steady-state error, where a step

or a ramp signal is applied to the system input. Ideally, all of these measurements must be zero, that is, system output must exactly follow the input signal. In practice, however, this is not the case; therefore the output must follow the input as close as possible. If the system does not give desired performance values, then a controller is usually added to the system to achieve the desired responses. There are many controller design methods, which are currently used in practice. Optimization methods can be counted amongst them.

Description of a function, which is called performance index, is usually possible for controller design procedure using parametric optimization. A performance index consists of some performance characteristics, which the system tries to achieve. This function depends on the controller parameters and is optimized numerically. This procedure gives optimal controller parameters which are appropriate for desired response. If the performance index is adjusted to the minimum value according to the system parameters, then the system is called optimum controller system. Performance index is always a positive number or zero. That is to say, ideal system is described as a system which minimizes this index.

The controller is normally required to minimize the error signal, which is the difference between reference input(r(t)), and controlled output signal(c(t)) as given in equation 2

$$e(t) \to 0 \quad t \ge 0 \,. \tag{2}$$

Thus, a criterion suitable to characterize the time response of a system is usually given as an integral function of the error, or its weighted products. A general form of an integral error criterion may be represented as follows

$$J = \int_{0}^{\infty} \Phi[e(t), t] dt \quad . \tag{3}$$

Therefore, an optimum dynamic performance may be taken as the time response which gives a minimum value of J. The integral performance criterion can be expressed in different forms, thus a control system is considered optimal if the selected performance index is minimized by varying the controller parameters. Since the optimal parameters depend directly on the selected criterion, it is important to reexamine some of the well known integral performance criteria. For over forty years many approaches have been used for developing design criteria for optimum transient behavior of a system. Two of the most frequently used criteria, which are the integral squared error, ISE, and the integral absolute error criterion, IAE, were suggested by Graham and Lathrop (1953). The performance indices of the two criteria are given by;

$$J_{ISE} = \int_{0}^{\infty} e^2(t)dt$$
(4)

$$J_{IAE} = \int_{0}^{\infty} |e(t)| dt \,. \tag{5}$$

Time weighted versions of these two criteria have also been introduced in (Zhuang, 1992) for ISE and (Graham and Lathrop, 1953) for IAE. More general representations of these criteria are

$$J_n(\theta) = \int_0^\infty \left[t^n e(\theta, t) \right]^2 dt \tag{6}$$

which is the general time weighted integral squared error criterion, and

Table 1. The minimum ISTE standard forms for a rampinput.

Denominator
s ³ +1.016s ² +4.535s+1
s ⁴ +1.848s ³ +3.235s ² +2.877s+1
s ⁵ +1.289s ⁴ +5.091s ³ +4.013s ² +4.595s+1
s ⁶ +1.813s ⁵ +5.278s ⁴ +6.317s ³ +6.473s ² +3.899s+1

$$J'_{n}(\theta) = \int_{0}^{\infty} t^{n} \left| e(\theta, t) \right| dt$$
⁽⁷⁾

which is the general time weighted integral absolute error criterion where θ denotes variable parameters which are chosen to minimize $J_n(\theta)$. According to the formula (6), the J_0 , J_1 and J_2 are called ISE, ISTE and IST²E respectively.

Standard forms

Another method for the controller design is to use closed loop transfer function's standard forms, which were first introduced by Graham and Lathrop (1953) in early 50's. They obtained these optimum transfer functions using experimental methods, which gave relatively high error rate, in the time domain. In 1995, Dorf and Bishop (1995) introduced optimum step responses of the standard forms for the ISE and IAE criteria. Again in the 1996, Dorf and Bishop (1996) introduced optimum values of closed loop transfer functions for the ITAE criterion in the control handbook, but unfortunately they did not give any explanation of how to obtain these values. In the 1998, Atherton and Boz (1998) have introduced all pole standard forms and Standard forms with a variable zero for the ISTE and IST²E criteria. Generally, the closed loop transfer function of a plant can be represented by

$$\frac{C(s)}{R(s)} = \frac{c_k s^k + c_{k-1} s^{k-1} + \dots + c_1 s + c_0}{s^m + d_{m-1} s^{m-1} + \dots + d_1 s + d_0}$$
(8)

The steady-state error for this system can be shown to be

$$e_{ss} = (c_0 - d_0)r(t) + (c_1 - d_1)\frac{dr(t)}{dt} + (c_2 - d_2)\frac{d^2r(t)}{dt^2} + \dots$$

$$\dots + (c_{k-1} - d_{k-1})\frac{d^{k-1}r(t)}{dt^{k-1}} + (c_k - 1)\frac{d^kr(t)}{dt^k}.$$
(9)

The form of the input r(t) determines the size of the steady-state error. In order to have zero steady-state error with a step function input, the requirement is that $c_0=d_0$. This also means that in a unity feedback control system, the forward transfer function is Type 1 or higher. Since the order of the numerator of C(s)/R(s) can be equal to or less than the order of the denominator, there are many possible forms of C(s)/R(s) for which the steady-state error is zero with a step input.

Steady-state error with a ramp function input become zero when $c_0=d_0$ and $c_1=d_1$. This also means that the system is Type 2 or higher. A study has also been made based on this system by Boz and the results are given in (Boz, 1999).

Standard forms with a zero

With a single zero the transfer function of a standard form may be denoted by $T_{ij}(s)$ with the denominator given as in the all pole form and the numerator by c_1s+1 , which is

$$T_{1j}(s) = \frac{c_1 s + 1}{s^j + d_{j-1} s^{j-1} + \dots + d_1 s + 1}.$$
 (10)

If the closed loop system is required to follow a ramp input with zero steady state error, then c_1 must be equal to d_1 , and the optimal values of the coefficients for the ITAE criterion can be found in (Graham and Lathrop, 1953). As for the ISTE criterion, the standard form coefficients for a ramp input can be expressed as in Table 1 (Atherton and Boz, 1998).

In many cases, contrary to the classical textbooks devoting a chapter for this subject, it is not appropriate to take $c_1 = d_1$ for controller design and minimizing the error index procedures (Atherton and Boz, 1998). In the case of $c_1 \neq d_1$, the optimum values of d coefficients vary with the choice of c_1 . The optimum values of these coefficients for the J₁ and J₂ criteria for T₁₄(s) as a function of c_1 , are given here in Figure 1 (Atherton and Boz, 1998). Step responses of the same system for IST²E criterion are given in Figure 2. As seen in Figure 2, the response is getting faster and the error is reduced while c_1 is increased.

RESULTS AND DISCUSSION

OPTIMAL CONTROLLER DESIGN METHOD FOR N^{TH} DEGREE ALL POLE SYSTEMS

In this section, a new optimal controller application method for nth degree all pole systems has been introduced. In the suggested controller scheme, as shown in Figure 3, a PI controller in the feed forward path and a polynomial controller, which its degree changes according to system degree in the inner feedback path, have been used. In the suggested method, generalized formulae for designing optimal controller parameters have been obtained using the standard forms with a zero. Two comparative examples are given to show the validity of the method.

Generalized optimal controller design method for nth degree all pole systems

nth degree all pole system's transfer function can be represented by

$$G(s) = \frac{a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_2 s^2 + b_1 s + b_0}$$
(11)

This system can be controlled using a PI controller in the feed forward path and a polynomial controller in the inner feedback path as shown in Figure 3. Closed loop transfer function of the inner feedback controller and the system can be represented as,



Figure 1. Optimum values of d_1 , d_2 and d_3 for $T_{14}(s)$.



Figure 2. Step responses for different values of c_1 for $T_{14}(s)$.

$$G'(s) = \frac{a_0}{b_n s^n + b_{n-1} s^{n-1} + (b_{n-2} + a_0 k_{n-2}) s^{n-2} + (b_{n-3} + a_0 k_{n-3}) s^{n-3} + \dots}$$
$$\dots + (b_1 + a_0 k_1) s + a_0 k_0 + b_0$$
(12)

and the resulting closed loop transfer function of G'(s), the PI controller and the unity feedback is given by,

$$T(s) = \frac{l_1 a_0 s + l_0 a_0}{b_n s^{n+1} + b_{n-1} s^n + (b_{n-2} + a_0 k_{n-2}) s^{n-1} + \dots}$$
(13)
$$\dots + (b_1 + a_0 k_1) s^2 + (a_0 k_0 + a_0 l_1 + b_0) s + a_0 l_0$$

which can be normalized to the form



Figure 3. The use of PI controller in the feed forward path for n^{th} degree all pole systems.

$$T(s_m) = \frac{s_m \alpha \frac{l_1}{l_0} + 1}{s_m^{n+1} + s_m^n \alpha^{-1} \frac{b_{n-1}}{b_n} + s_m^{n-1} \alpha^{-2} (\frac{b_{n-2} + a_0 k_{n-2}}{b_n}) + s_m^{n-2} \alpha^{-3} (\frac{b_{n-3} + a_0 k_{n-3}}{b_n}) + \dots}$$
$$\dots + s_m \alpha^{-n} (\frac{a_0 k_0 + a_0 l_1 + b_0}{b_n}) + 1$$
(14)

where
$$\alpha = (\frac{b_n}{a_0 l_0})^{-1/n+1}$$
 and $s_m = s / \alpha = s(b_n / a_0 l_0)^{1/n+1}$,

which means that the response of the system will be faster than the normalized response by a factory of α . To simplify the analysis, numerator and denominator coefficients of the normalized system's closed loop transfer function can be arranged as,

$$d_n = \alpha^{-1} \frac{b_{n-1}}{b_n} \tag{15}$$

$$d_{n-1} = \alpha^{-2} \left(\frac{b_{n-2} + a_0 k_{n-2}}{b_n} \right)$$
(16)

$$d_{n-2} = \alpha^{-3} \left(\frac{b_{n-3} + a_0 k_{n-3}}{b_n} \right) \tag{17}$$

$$d_1 = \alpha^{-n} \left(\frac{a_0 k_0 + a_0 l_1 + b_0}{b_n} \right) \tag{18}$$

$$c_1 = \alpha \frac{l_1}{l_0} \tag{19}$$

Substituting these values into the equations 14 gives the new transfer function of the system, to be

$$T_{1(n+1)}(s_m) = \frac{c_1 s_m + 1}{s_m^{n+1} + d_n s_m^n + d_{n-1} s_m^{n-1} + \dots + d_2 s_m^2 + d_1 s_m + 1}$$
(20)

n + 1 degree standard form with a variable zero can be represented as in equation 20. Using equations 15, 16,

18 and 19 with the transfer function given in equation 20 results in the controller parameters as,

$$l_0 = \alpha^{n+1} \frac{b_n}{a_0} = \frac{b_{n-1}^{n+1}}{a_0 b_n^n} \frac{1}{d_n^{n+1}}$$
(21)

$$l_1 = \alpha^n c_1 \frac{b_n}{a_0} = \frac{b_{n-1}^n}{a_0 b_n^{n-1}} \frac{c_1}{d_n^n}$$
(22)

$$k_0 = \alpha^n \frac{(d_1 - c_1)b_n}{a_0} - \frac{1}{a_0} = \frac{b_{n-1}^n}{a_0 b_n^{n-1}} \frac{(d_1 - c_1)}{d_n^n} - \frac{b_0}{a_0}$$
(23)

$$k_1 = \alpha^{n-1} \frac{d_2 b_n}{a_0} - \frac{b_1}{a_0} = \frac{b_{n-1}^{n-1}}{a_0 b_n^{n-2}} \frac{d_2}{d_n^{n-1}} - \frac{b_1}{a_0}$$
(24)

$$k_2 = \alpha^{n-2} \frac{d_3 b_n}{a_0} - \frac{b_2}{a_0} = \frac{b_{n-1}^{n-2}}{a_0 b_n^{n-3}} \frac{d_3}{d_n^{n-2}} - \frac{b_2}{a_0}$$
(25)

$$k_3 = \alpha^{n-3} \frac{d_4 b_n}{a_0} - \frac{b_3}{a_0} = \frac{b_{n-1}^{n-3}}{a_0 b_n^{n-4}} \frac{d_4}{d_n^{n-3}} - \frac{b_3}{a_0}$$
(26)

$$k_{n-3} = \alpha^3 \frac{d_{n-2}b_n}{a_0} - \frac{b_{n-3}}{a_0} = \frac{b_{n-1}^3}{a_0 b_n^2} \frac{d_{n-2}}{d_n^3} - \frac{b_{n-3}}{a_0}$$
(27)

$$k_{n-2} = \alpha^2 \frac{d_{n-1}b_n}{a_0} - \frac{b_{n-2}}{a_0} = \frac{b_{n-1}^2}{a_0 b_n} \frac{d_{n-1}}{d_n^2} - \frac{b_{n-2}}{a_0}$$
(28)

or generalizing the formula for $\,k\!=\!0,\,1,\,2,\,3,\,4,\,\ldots\,n\!-\!2$, and

$$\sum_{i=0}^{n-2} k_i = \sum_{i=0}^{1} (\alpha^n \frac{(d_{i+1} - c_{i+1})b_n}{a_0} - \frac{b_i}{a_0}) + \sum_{i=1}^{n-2} (\alpha^{n-i} \frac{d_{i+1}b_n}{a_0} - \frac{b_i}{a_0})$$
(29)

$$\sum_{i=0}^{1} l_i = \sum_{i=0}^{1} \alpha^{n-i+1} c_i \frac{b_n}{a_0}$$
(30)



Figure 4. Step responses for example 1.

$$\alpha = \frac{b_{n-1}}{b_n} \frac{1}{d_n} \tag{31}$$

equations can be obtained. An interesting aspect of this method is the use of polynomial feedback controller. The polynomial design method uses feedback from the derivatives of the output, which are simply the system states when the system is represented in controllable canonical form. Since any controllable system can be put in this form by a state transformation, the design approach can be applied to state feedback design for any controllable system, as it is only necessary, in addition, to find the transformation which puts it in controllable canonical form (Atherton, 2006).

Example 1.

Consider the third order transfer function,

$$G(s) = \frac{2}{s^3 + 4s^2 + 6s + 3}.$$
 (32)

Comparing the transfer function of this system with that of Equation 11, gives the following values, n = 3, $a_0 = 2$, $b_3 = 1$, $b_2 = 4$, $b_1 = 6$ and $b_0 = 3$. Then choosing $c_1 = 2$ for ISTE and IST²E criteria and using them in the generalized formulae, which are given in Equations 29 and 30, result in the controller parameters and these data are summarized in Table 2. For the same system, results of some well known PID controller design methods are also obtained. These are Refined Ziegler-Nichols (R.Z-N)

(Hang et al, 1991), Astrom Hagglund(A-H)(Astrom and Hagglund, 1984) and Gain-Phase(Zhuang and Atherton, 1993) controller design methods. Summary of the results obtained from these methods are given in Table 3. Finally, step responses of all design methods together with that of the suggested design method are plotted in the same Figure for comparison (Figure 4).

It is seen from the Figure 4 that, A-H method gives most oscillatory response and longest settling time. On the other hand, result of the suggested design method for IST^2E gives minimum overshoot, settling time and little oscillation. ISTE design also gives relatively less overshoot and short settling time but its response is faster than the IST^2E design.

Example 2.

In this case, consider the fourth order transfer function,

$$G(s) = \frac{6}{s^4 + 4s^3 + 12s^2 + 18s + 9}.$$
 (33)

Coefficients of the transfer function are n = 4, $a_0 = 6$, $b_4 = 1$, $b_3 = 4$, $b_2 = 12$, $b_1 = 18$ and $b_0 = 9$. Again choosing $c_1 = 2$ for ISTE and IST²E criteria from the Figure 1 gives the d coefficients as seen in Table 4. Using these coefficients and equations 13, 14, 15, 16 and 17 result the suggested controller parameters, which are summarized in Table 4. For the same system, the R.Z-N, A-H and Gain-Phase

Table 2. Results of suggested controller design method for Example 1.

	C 1	d ₁	d ₂	d ₃	J	I ₁	I ₀	k 1	k ₀
PI ISTE	2	3.49	4.21	2.1	0.976	6.92	6.6	4.65	3.64
PI IST ² E	2	3.66	4.53	2.65	2.964	3.45	2.6	2.17	1.36

 Table 3. Controller parameters, which are obtained using the R.Z-N, A-H and Gain-Phase methods for example 1.

	Kc	ω	Tc	α	ϕ_m	β	K _p	Ti	T _d
R. Z-N	10.5	2.45	2.57			0.1765	6.3	1.28	0.32
A-H	10.5	2.45	2.57	4	45		7.43	1.97	0.5
Gain-Phase	10.5	2.45	2.57				5.34	4.53	0.31

Table 4. Results of suggested controller design method for example 2.

	C 1	d 1	d_2	d ₃	d_4	J	l ₁	lo	k ₂	k 1	k ₀
PI ISTE	2	4.06	5.74	5.3	2.02	2.35	5.07	5	1.45	4.37	3.72
PI IST ² E	2	4.2	6.51	5.76	2.66	10.47	1.7	1.28	0.17	0.69	0.37

 Table 5. Controller parameters, which are obtained using the R.Z-N, A-H and Gain-Phase methods for example 2.

	Kc	ω	Tc	α	ϕ_{m}	β	Kp	Ti	T _d
R. Z-N	4.13	2.12	2.96			0.569	2.48	1.48	0.37
Å-H	4.13	2.12	2.96	4	45		2.92	2.28	0.57
Gain-Phase	4.13	2.12	2.96				2.1	2.18	0.36



Figure 5. Step responses for Example 2.

methods yield the controller parameters, which are given in Table 5. Step responses of all design methods are also given in Figure 5.

As in the Example 1, A-H method gives biggest overshoot. On the other hand, IST²E and R.Z-N methods provide minimum overshoot. But the settling time of the IST²E method is better than that of the R.Z-N method. Again ISTE method gives relatively fast response with little oscillation.

Conclusion

In this work, a new approach to obtain the optimum controller parameters for nth degree all pole systems has been introduced. The suggested method basically uses the optimized standard forms with a variable zero and directly targets the step response shaping in the time domain. As it is pointed out in the work, choosing $c_1 \neq d_1$ in the standard forms yields relatively better system performances to the step input. By suggesting the generalized formulae for the optimal controller design, the design procedure is simplified. Thus, the need for expert person

person in controller design is eliminated. Again the suggested method contains simple mathematical operations, thus there is no need to optimize the system every time and the time, which is needed for controller design, is shortened. Because of the use of simple mathematical formula, the method can be easily used with a microcontroller. As it is seen from the examples, the method gives superior responses over some well known design methods, thus it can be preferable in designing the optimum controller. The results of the work can also be used in the state feedback design as indicated in the text.

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