Full Length Research Paper

Experimental investigation of kinetic energy and momentum correction coefficients in open channels

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In this study, vertical and lateral velocity distribution in a compound channel cross section was measured to investigate kinetic energy and momentum correction coefficients for nine different test discharges. Experiments were carried out for subcritical flow conditions and the Froude number (Fr) and depth ratio (Dr) varied between 0.87 and 0.95, and 0.17 and 0.48 respectively. Kinetic energy and momentum correction coefficients, α and β were computed for each experimental test condition. Results wer Yuan SY, Xiao Ch, Li ZY, Zhu JY (2003). A study on laboratory rearing techniques for *Bactrocera dorsalis* (Hendal). Acta Agr. Univ. Jiangx. 25: 577-580.e compared with findings taken from the experimental work of other researchers carried out in flumes with different cross-sectional shapes. For 37 different experimental test cases, the average values of kinetic energy and momentum correction coefficients, α and β were obtained as 1.094 and 1.034 respectively. An attempt also was made to determine the relation between α and β.

Key words: Velocity distribution, kinetic energy correction coefficient, momentum correction coefficient, open channels.

INTRODUCTION

Kinetic energy and momentum principles often are used in hydraulic problems. Kinetic energy and momentum correction coefficients, α and β, often are assumed to be unity when the energy and momentum principles are used in the computations (example, Chow, 1959; Streeter and Wylie, 1979; French, 1987; Massey, 1989; Roberson and Crowe, 1998). On the other hand, because of the nonuniform distribution of velocities over a channel cross-section, α and β generally are greater than unity.

Kinetic energy and momentum correction coefficients, α and β, are computed using equations [1] and [2] for a single and compound cross-sectional areas of a channel:

$$\alpha = \frac{\int u^3 \, dA}{U^3 A} \approx \frac{\sum u^3 \, \Delta A}{U^3 A} \tag{1}$$

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Symbols: A, Total water area; **Dr**, Depth ratio; **Fr**, Froude number; **h**, Water depth at bankfull level; **H**, Water depth; **Q**, Discharge; **u**, Point velocity; **U**, Cross-sectional mean velocity; α, Kinetic energy correction coefficient; β, Momentum correction coefficient; ΔA, An elementary area in the whole water area.

$$\beta = \frac{\int u^2 \, dA}{U^2 A} \approx \frac{\sum u^2 \, \Delta A}{U^2 A} \tag{2}$$

Where u = point velocity at each point in the cross section, U = cross-sectional mean velocity, A = whole water area, and dA = an elementary area in the whole water area. For a compound cross-sectional area of a channel, α and β are often used in computer models for determination of water surface profiles (example, HEC-2 1991; HEC-RAS 2002).

For the case where the velocities are unidirectional but nonuniform across the section, (Jaeger, 1956) found that

$$\alpha - 1 = 3(\beta - 1) + \frac{1}{A} \int_A \left(\frac{u - U}{U} \right)^3 dA \tag{3}$$

Because of the scarcity of data especially for compound channels, α and β often are assumed to be unity in computations for uniform flow. The study described in this paper was designed to obtain velocity distribution data for investigating the variation of α and β in either compound or single cross-sectional shaped flumes. For this aim, a series of experiments were done in a compound channel flume. Additional velocity distribution data were collected
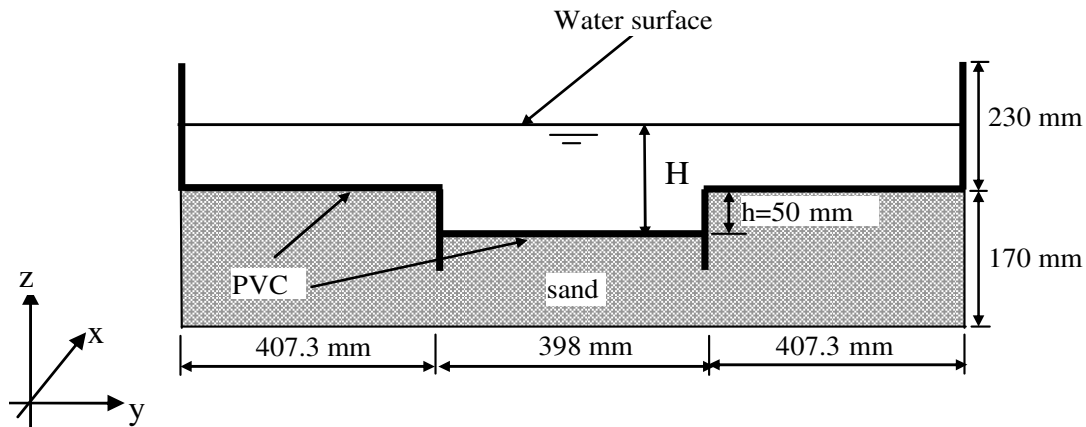
Figure 1. Cross-Section of compound channel flume used in current study.

from Ardiclioglu (1994). Blalock and Sturm (1981, 1983) presented the values of α and β observed in an asymmetrical compound channel flume. In this study, these data were also used in the analysis. In addition, an attempt was made to determine the relation between α and β using the complete data.

MATERIALS AND METHODS

A series of experiments were done in a compound channel flume at the Hydraulic Laboratory of Birmingham University to investigate ki-netic energy and momentum correction coefficients, α and β res-pectively. As can be seen in Figure 1, the flume consists of a main cha-nnel and its two symmetrical floodplains. The main channel and its floodplains have widths of 0.398 m and 0.407 m respectively with a 18 m test length. Both the main channel and its floodplains were made of smooth PVC material. Bed slope of the flume was set to 2.024×10^{-3}. The bed profile was measured by an automatic HR touch sensitive bed profiler. Preliminary experiments were done for nine different test discharges under subcritical flow conditions to produce uniform flows. Discharges were measured by means of an electro-magnetic flow meter, a venturimeter, and a dall tube. Water-surface profiles were measured using a pointer gauge along the flume with 1 m intervals. Depth ratio, Dr ((H-h)/H), varied between 0.17 and 0.48. Froude number, Fr, varied between 0.87 and 0.95.

In order to obtain velocity distribution data for eight different test discharges, vertical velocity profiles were taken at 10 mm depth in-tervals throughout the spanwise section using a Novar 13 mm pro-peller velocity current meter (type 403 of serial No 722 Low Speed). Velocity profiles also were taken at lateral spacings of 20 mm both within the main channel subdivision and on the floodplains for the entire cross section.

Further details of this experimental work are also given by else-where (Seckin et al. 2004; Seckin 2005).

DATA COLLECTION

In order to compare the variation of velocity distribution resulting kinetic energy and momentum correction coefficients, α and β respectively, for flumes with different cross-sectional shapes, addi-tional velocity data were collected from the work of Ardiclioglu (1994) comprising single rectangular channel. The values of α and β presented by Blalock and Sturm (1981, 1983) were also used in

the analysis. The apparatus and procedure used by Ardiclioglu (1994, 1997) and Blalock and Sturm (1981) are summarized herein.

Ardiclioglu (1994) carried out a series of experiments in a single rectangular channel flume at the Hydraulic Laboratory of Cukurova University. The flume built using smooth glass materials was 10 m long, 0.30 m wide and 0.40 m deep. Experiments were done for two different slopes of 0.05 and 0.2%. Tests were done under subcri-tical flow conditions for different test discharges (Q = 19.5, 14.5, 10 and 6 l/s). Froude number (Fr), varied between 0.119 and 0.724. Point velocities were measured using a DISA 55L67 type Laser-Do-ppler anemometer with 20 mm intervals for the lateral direction of the spanwise section, and vertical velocity profiles also were taken at 0.2 0.3 0.5 1, 2, 5 and 10 mm depth intervals as much as possible. More complete details of this experimental arrangement are given elsewhere by Kırkgöz and Ardiclioglu (1997).

Blalock and Sturm (1981) carried out a series of experiments in an asymmetrical compound channel flume consisting of a main cha-nnel and a floodplain. The flume was 1.07 m wide, 0.46 m deep and 24.38 m long, and the main channel was 0.297 m wide and had a bankfull depth of 0.162 m. Experiments were done for eight different slopes of 1.02×10^{-3}, 1.13×10^{-3}, 1.49×10^{-3}, 2.09×10^{-3}, 1.90×10^{-3}, 2.12×10^{-3}, 3.30×10^{-3} and 4.46×10^{-3} respectively. For each slope, a single value of discharge averaged at Q = 0.048 m³/s was used to measure each uniform depth of the flow (H = 0.198, 0.191, 0.183, 0.173, 0.162, 0.152, 0.142 and 0.132 m). The last four depths of flow refer to single channel flow and the rest refers to the compound channel flow. Sufficient point velocity measurements were made across the entire cross section for each depth of the flow at approximately the same discharge. Point velocities were measured with a 1.83 mm outside diameter pitot-static tube opera-ted in conjunction with a differential pressure transducer.

RESULTS AND DISCUSSION

Using the velocity distribution data, the kinetic energy and momentum correction coefficients computed using equa-tions 1 and 2 for the current experimental work and the work of Ardiclioglu (1994). For current study, the values of these coefficients are presented in Table 1. The kinetic energy and momentum correction coefficients for the asymmetrical compound channel flume provided by Bla-lock and Sturm (1981, 1983) were also used as a compa-rison. For all data obtained for the flumes with three different cross-sectional shapes, namely straight

Table 1. Experimental data obtained from the symmetrical compound channel flume.

Test Number	Q (m³/s)	H (m)	α	β	$\dfrac{\alpha - 1}{\beta - 1}$
1	0.0155	0.0596	1.3150	1.1199	2.63
2	0.0239	0.0711	1.2180	1.0746	2.92
3	0.0272	0.0733	1.1655	1.0560	2.95
4	0.0301	0.0761	1.1254	1.0422	2.97
5	0.0344	0.0791	1.1070	1.0362	2.95
6	0.0402	0.0846	1.0714	1.0240	2.98
7	0.0453	0.0886	1.0700	1.0234	2.99
8	0.0502	0.0926	1.0547	1.0185	2.95
9	0.0553	0.0954	1.0512	1.0172	2.97

symmetrical compound, single rectangular and asymmetrical compound channel flumes, the computed values of the kinetic energy and momentum correction coefficients, α and β, are plotted versus cross-sectional mean velocity, U, in Figures 2 and 3 respectively. Figures 2 and 3 indicate that the values of α and β are not strongly related to the mean velocity. The lack of dependence of α and β on the mean velocity is expected. These coefficients depend only on the shape of the velocity distribution and this distribution can be multiplied by a constant to change the mean velocity and the values of α and β will be unchanged.

There is a large difference in velocity distribution between main channel and floodplains for compound channels. That's why, for the asymmetrical and symmetrical compound channels, the values of α and β averaged at 1.156 and 1.056 respectively, while they averaged at 1.0604 and 1.0222 respectively for the single channels. This means that α and β for single channels yield lower averaged values than those of compound channels.

The last term in equation [3] is often small because the integrand always changes sign. Thus it makes sense to seek a linear regression between (α-1) and (β-1). For three different flumes, the values of (α-1) versus the values of (β-1) are plotted in Figure 4. As can be seen in this figure, the slope of four different data sets is in a good agreement for the data sets obtained from different cross-sectional shaped laboratory flumes. However, the relationship between (α-1) and (β-1) for the current study yields to a lower slope value than that of equation [3]. This would be expected because of ignoring the last term in equation [3]. Using all 37 data points without considering the type of cross section yields:

$$\alpha - 1 = 2.7336 \, (\beta - 1) \tag{4}$$

with a determination coefficient of 0.9934. The root-mean-square deviation between the observed and predicted

values of α is 0.0041.

In addition to the relationship between (α-1) and (β-1), the relationship between α and β is also shown in Figure 5, and given by equation [5] as follows:

$$\alpha = 2.6777\beta - 1.6748 \tag{5}$$

The determination coefficient of equation [5] is 0.994.

It should be pointed out that equation [4] was obtained fitting the regression line to a zero intercept by using best-fit linear regression analysis. Therefore, equation [5] should give slightly different results in comparison with the equation [4].

Cobb (1968) using 105 corresponding α and β values from open channels found the following empirical relation:

$$\alpha = 2.66\beta - 1.66 \tag{6}$$

equation [6] is also shown in Figure 5 for a comparison with equation [5]. As can be seen in this figure, equation [5] and [6] are in a good agreement with the 37 data points, although equation [5] gives slightly bigger values.

As a result, for 37 different experimental test cases including different test discharges, bed slopes and cross-sectional shapes, the average values of kinetic energy and momentum correction coefficients, α and β, were determined as 1.094 and 1.034 respectively. For the laboratory flumes, Kolupaila (1956) proposed the average values of α and β as 1.15 and 1.05 respectively. The average values of α and β obtained from compound channels are slightly greater than those given by Kolu-paila (1956). Whereas, they yielded the lower averaged values for the single channels.

The velocity distribution in the flumes was determined mainly by the shape and roughness of the channel. It is well known that the values of α may exceed 2 (example, Chow, 1959; French, 1987) for natural channels where the velocity distribution and roughness effects are often different from those of the laboratory flumes. Jaeger
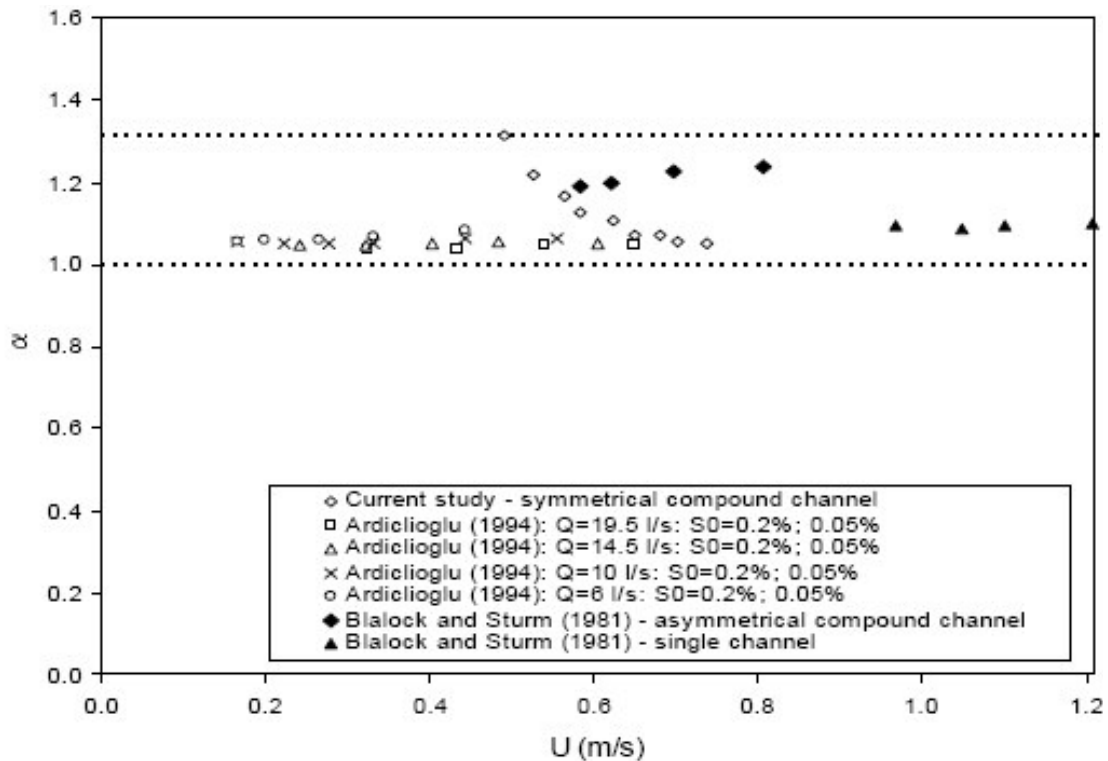
Figure 2. Relation between kinetic energy correction factor (α) and cross-sectional mean velocity (U).
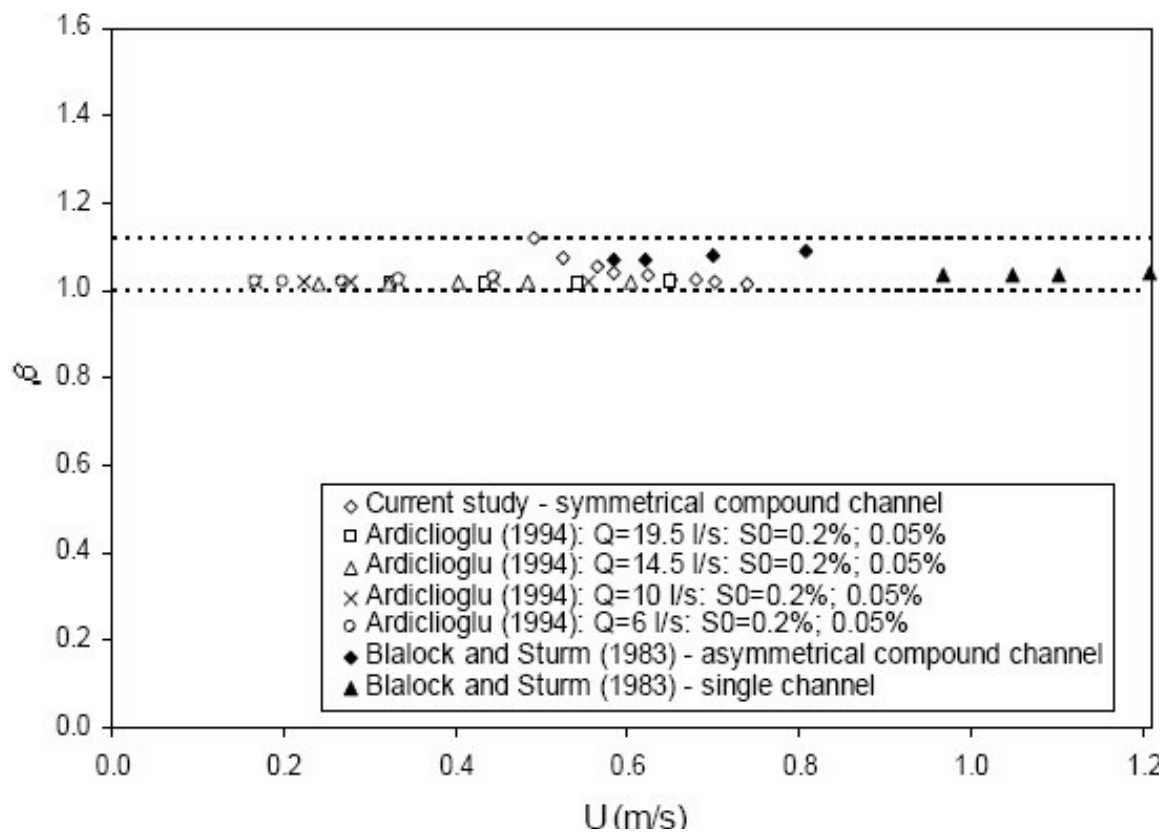

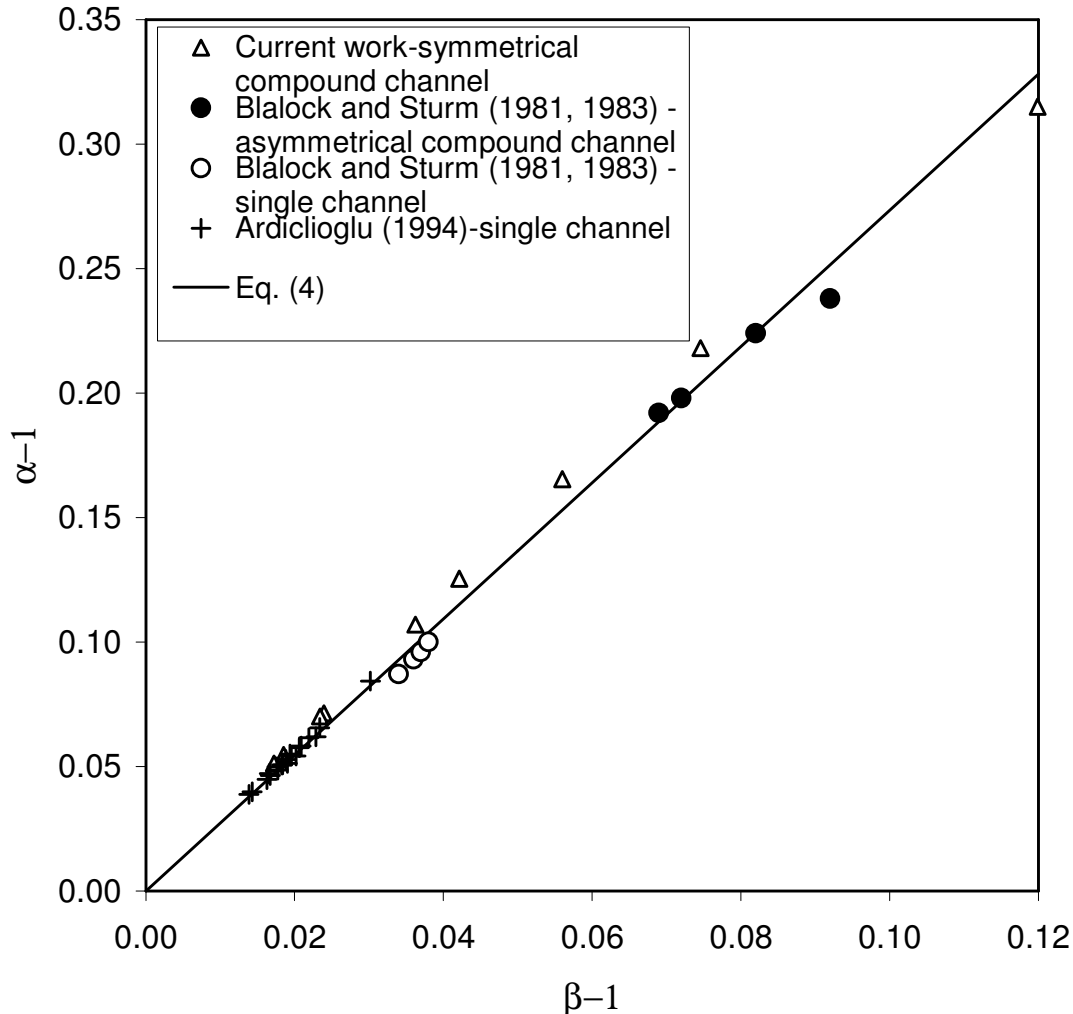**Figure 3.** Relation between momentum correction factor (β) and cross-sectional mean velocity (U).

Figure 4. Relation between (α-1) and (β-1).

(19-68) demonstrated that α depended on the friction coeffi-cient only if uniform flow is considered. Since all data presented herein are limited to the smooth surface flu-mes, the possible effects of the surface roughness were not examined. Thus, the averaged values of α and β proposed herein are recommended for the smooth surface open channels.

Conclusions

Kinetic energy and momentum correction coefficients, α and β, are often used to be unity when the energy and momentum principles are used in the hydraulic computations. However, because of nonuniform distribution of velocities over a channel section, α and β, are generally greater than unity. Lateral momentum transfer between main channel and flood plain is very high in compound channels and the flow velocity distribution is deviated from uniformity due to this property.

Therefore α and β coefficients become high in compound channels. The results showed that α and β coefficients decreases with increases discharge, since the flow become more stable at higher discharge. This study has explored the practical average values of α and β as 1.094 and 1.034 respectively for three different cross-sectional shaped flumes, namely straight symmetrical compound, single rectangular, and straight asymmetrical compound channel flumes. The relation between α and β is also given as α = 2.6777β-1.6748 with a determination coeffi-cient of 0.994.
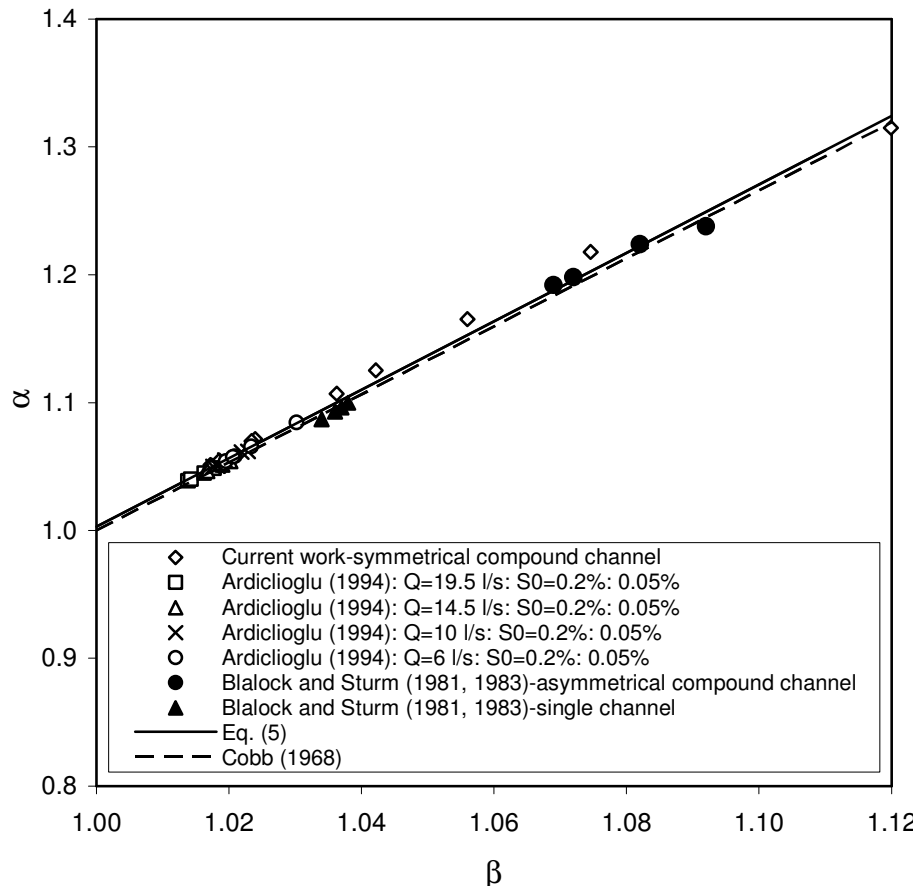
Figure 5. Relation between kinetic energy correction factor (α) and momentum correction factor (β).

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