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Assessing the performance of improved harmony search algorithm (IHSA) for the optimization of unconstrained functions using Taguchi experimental design

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This paper describes the meta-heuristic improved harmony search algorithm (IHSA) and analyzes the performance of IHSA in solving unconstrained function minimization problems. The most important challenge to this algorithm is setting the parameters for various optimization problems. Performance of IHSA is quite sensitive to initial settings. Using Taguchi's experimental design to arrange IHSA parameters, the performance of IHSA was analyzed to find global optima of Rosenbrock and Wood-Colville function.

Key words: Improved harmony search algorithm, Taguchi experimental design, performance of IHSA, unconstrained function.

INTRODUCTION

Over the last four decades, optimization of design and manufacturing parameters has been a major issue for solving real world optimization problems. In order to design and manufacture higher-quality products, current optimization techniques must be improved. Different optimization techniques have been developed for solving various optimization problems (Chen, 2004; Cho and Ahn, 2003; Coello, 2000; Petrović et al., 2011).

Recent advancements in optimization area have introduced new opportunities to achieve better solutions for design and manufacturing optimization problems. Therefore, there is a need to introduce new methods to overcome drawbacks and improve the existing optimization techniques to design and manufacture products economically (Yildiz, 2008).

Real-world optimization problems are very complex in nature and quite difficult to solve using these algorithms.

If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum. When solving problems with high dimensional search space and many local optima, meta-heuristic algorithms are preferable, although they provide near optimal solutions.

Meta-heuristic algorithms imitate natural phenomena, that is, physical annealing in simulated annealing, animal behavior in tabu search, evolution in evolutionary algorithms, etc. There are many meta-heuristic algorithms that are often applied, such as: simulated annealing (Koulamas et al., 1994), tabu search (Glover and Laguna, 1997), genetic algorithm (Konak et al., 2006; Goldberg, 1989, ant colony optimization (Dorigo and Stützle, 2004), shuffled frog leaping (Eusuff et al., 2006), harmony search algorithm (HSA) (Geem et al., 2001), scatter search (Martí et al., 2006), bees algorithm (Baykosoglu et al., 2007). In the paper of Teodorović (2008), a classification and analysis of the results achieved using swarm intelligence was presented.

One of the newer techniques is the HSA developed by

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Geem et al. (2001). This approach is based on the musical performance process that takes place when a musician searches for a better state of harmony. The HSA has been successfully applied to various benchmark and real world optimization problems (Geem et al., 2001; Geem et al., 2002; Lee and Geem, 2004; Fesanghary et al., 2008; Abdolvahhab et al., 2011). However, like other meta-heuristic algorithms, the HSA suffers from some drawbacks. One of these drawbacks is that its capabilities are quite sensitive to initial parameter settings. This problem is due to similar harmonies in the final generations. Over the course of repeated generations, the diversity of harmony gradually reduces, creating a similar harmony. As compared to other meta-heuristics algorithms, this drawback can easily be solved by setting suitable parameters (Sarvari and Zamanifar, 2011). As such, the most important challenge with respect to this algorithm is setting the parameters for various optimization problems (Orman and Mahdavi, 2008; Jaberipour and Khorram, 2010). Some previous studies addressed the issue of dealing with algorithm parameter settings; as such, they offer different settings for these parameters based on different benchmarks. Most of these studies involve problems with low dimensions and have achieved good results in this context.

This paper attempts to analyze the performance of improved harmony search algorithm (IHSA) (Mahdavi et al., 2007) in solving unconstrained function minimization problems. Using Taguchi's experimental design to arrange IHSA parameters, the performance of IHSA was analyzed to find global optima of Rosenbrock and Wood-Colville function.

MATERIALS AND METHODS

Harmony search algorithm

The HSA is a meta-heuristic optimization algorithm, simple in concept, few in parameters and easy in implementation. The steps in the procedure of harmony search are as follows (Geem et al., 2001):

- Step 1: Initialize the problem and algorithm parameters.
 - Step 2: Initialize the harmony memory (*HM*).
 - Step 3: Improvise a new harmony.
 - Step 4: Update the *HM*.
 - Step 5: Check the stopping criterion.
- These steps are described in the next five subsections.

Step 1: Parameters initialization

In this step, the optimization problem is specified as shown in Equation 1:

$$\text{Minimize } f(x), \quad (1)$$

subject to: $X_i \in X_i$, $i = 1, 2, \dots, N$, where $f(x)$ is the objective function, x_i is the set of each design variable, X_i is the set of the

possible range of values for each design variable, that is, $X_i = \{x_i(1), x_i(2), \dots, x_i(K)\}$, N is the number of design variables and K is the number of candidate values for the discrete decision variables. The HSA parameters are also specified in this step: harmony memory size (*HMS*), harmony memory considering rate (*HMCR*), pitch adjusting rate (*PAR*), and the maximum number of improvisations – maximal number of searches (*NI*). These algorithm parameters are explained in the following steps.

Step 2: Initialize the HM

In this step, the *HM* matrix, as shown in Equation 2, is filled with randomly generated solution vectors and sorted by the values of the objective function $f(x)$.

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_N^2 \\ \vdots & \vdots & & \vdots \\ x_1^{HMS} & x_2^{HMS} & \dots & x_N^{HMS} \end{bmatrix}. \quad (2)$$

Step 3: Improvise a new harmony from the HM

In this step, a new harmony vector $X' = X'_1, X'_2, \dots, X'_N$ is improvised by following three rules: HM consideration, pitch adjustment and random selection. As a musician plays any pitch out of the preferred pitches in his/her memory, the value of decision variable is chosen from any pitches stored in HM ($\{X_i^1, X_i^2, \dots, X_i^{HMS}\}$) with a probability of *HMCR* ($0 \leq HMCR \leq 1$) while $(1-HMCR)$ is the rate of randomly selecting one value from the possible range of values.

$$x'_i \leftarrow \begin{cases} X_i^j \in \{X_i^1, X_i^2, \dots, X_i^{HMS}\} & \text{with probability } HMCR \\ X_i & \text{with probability } (1-HMCR) \end{cases} \quad (3)$$

Every component obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. This operation uses the *PAR* parameter, which is the rate of pitch adjustment, as follows (Mahdavi et al., 2007): Pitch adjusting decision for

$$x'_i \leftarrow \begin{cases} \text{YES} & \text{with probability } PAR \\ \text{NO} & \text{with probability } (1-PAR) \end{cases}. \quad (4)$$

The value of $(1 - PAR)$ sets the rate of doing nothing. If the pitch adjustment decision for X'_i YES, X'_i is replaced as follows:

$$x'_i \leftarrow x'_i \pm rand() \cdot bw, \quad (5)$$

where *bw* is an arbitrary distance bandwidth; *rand()* is a random number between 0 and 1.

In step 3, *HM* consideration, pitch adjustment or random selection is applied to each variable of the new harmony vector in

turn.

Step 4: Update the HM

In this step, if the new harmony vector $X' = X'_1, X'_2, \dots, X'_N$ is better than the worst harmony in the HM in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM. The HM is then sorted by the objective function value.

Step 5: Repeat steps 3 and 4 until the termination criterion is satisfied

This is the final step where the computations are terminated when the termination criterion is satisfied. Otherwise, steps 3 and 4 are repeated.

Settings parameters of HSA

The algorithm requires several parameters, including HMS, NI, HMCR, PAR, bw (Sarvari and Zamanifar, 2011).

The HMCR and PAR parameters introduced in Step 3 help the algorithm find globally and locally improved solutions, respectively (Geem et al., 2001). PAR and bw in HSA are very important parameters in fine-tuning of optimized solution vectors, and can be potentially useful in adjusting convergence rate of algorithm to optimal solution. So, fine adjustment of these parameters is of great interest.

The traditional HSA uses fixed value for both PAR and bw. In the HSA method, PAR and bw values are adjusted in initialization step (step 1) and cannot be changed during new generations. The main drawback of this method appears in the number of iterations the algorithm needs to find an optimal solution.

Mahdavi et al. (2007) offer an improvement of the traditional HSA. The key difference between the improvement of the traditional HSA and traditional HSA is in the way of adjusting PAR and bw. To improve the performance of the HSA and eliminate the drawbacks with fixed values of PAR and bw, the improvement of the traditional HSA which uses variable PAR and bw in the improvisation step was proposed by Mahdavi et al. (2007). In this paper, the IHSA was used.

Taguchi experimental design

The Taguchi technique is a well-known, unique and powerful technique for product/process quality improvement. It has broad application in engineering design (Taguchi, 1986) and can be applied to many aspects, such as optimization, experimental design, sensitivity analysis, parameter estimation, model prediction, etc.

The Taguchi experimental design is a more structured and efficient technique that differs from the classical design of experiment, and, in that sense, it is a relatively simple method. The classical design of experiment (DoE) is sometimes too complex, time-consuming and not easy to use (Montgomery, 2001). A large number of trials have to be carried out when the number of process factors increases. By using highly fractionated factorial designs and other types of fractional designs obtained from orthogonal (balanced) arrays instead of full factorial, the Taguchi experimental design allows for an easy set-up of experiments with the minimum number of trials. Fewer trials imply that time and costs are reduced. In a full factorial design with more factors and several levels of each

factor, the total number of trials (N) can be obtained by using the equality as follows:

$$N = L^k, \quad (6)$$

where L is the number of levels and k is the number of design factors.

For example, for experiment with 4 factors at 3 levels, a full factorial design would require $3^4=81$ trials. Using the Taguchi experimental design, the standard orthogonal array (OA) denoted by the symbol $L_9(3^4)$ requires only 9 trials.

Each row of an OA represents one trial with the levels of different factors in that trial. The number of rows must be at least equal to the total degrees of freedom required for the experiment. Each column of an OA represents one factor and its setting levels in each trial.

Traditionally, data from experiments is used to analyze the mean response. However, in the Taguchi experimental design the mean and the variance of the response (experimental result) at each setting of parameters in OA are combined into a single performance measure known as the signal-to-noise (S/N) ratio. Depending on the criterion for the quality characteristic to be optimized, different S/N ratios can be chosen: smaller-the-better, larger-the-better and nominal-the-best (Phadke, 1989).

The Taguchi method is a robust design technique, which in its essence optimizes the settings of the process factor values as close as possible to the target factor values, with minimum variation. However, the goal of this paper is to identify IHSA parameter combination yielding to global optimum of Rosenbrock and Wood-Colville function. To this aim, the application of the Taguchi method was reduced to using the Taguchi experimental design as experimental matrix where IHSA parameters and their corresponding levels were arranged.

Unconstrained test functions

In mathematical optimization, the Rosenbrock function (Equation 10) is a non-convex function used as a performance test problem for optimization algorithms. It was introduced by Rosenbrock (1960). It is also known as Rosenbrock's valley or Rosenbrock's banana function. Finding the valley is trivial, however, convergence to the global optimum is difficult, hence, this problem has been repeatedly used to assess the performance of optimization algorithms. It is defined by Equation 7:

$$f(x) = 100 \cdot (x_2 - x_1^2)^2 + (1 - x_1)^2. \quad (7)$$

This results in a global minimum at $x^* = (1, 1)$, where $f(x) = 0.0$. A different coefficient of the second term is sometimes given, but this does not affect the position of the global minimum.

The second test function is the fourth-degree Wood-Colville function. It is a particularly good test of convergence criteria and simulates a feature of many physical problems quite well (Lee and Geem, 2004). It is computed using the equation as follows:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1) \quad (8)$$

The global minimum of the function is obtained at the point $x^* = (1, 1, 1, 1)$ with the corresponding objective function value of $f^* = 0.0$. Since the

Table 1. IHSA design parameters and their levels.

Factor	Factor description	Level				
		1	2	3	4	5
A	HMS	5	10	20	40	80
B	HMCR	0.35	0.5	0.65	0.8	0.95
C	PAR _{min}	0.1	0.2	0.3	0.4	0.5
D	PAR _{max}	0.8	0.85	0.9	0.95	0.99
E	bw _{min}	0.00001	0.0001	0.001	0.01	0.1
F	bw _{max}	0.2	0.4	0.6	0.8	1

number of design variables was increased, the optimization became more complex and computationally demanding, hence the number of searches was increased. The IHSA was implemented using the available Matlab's code on desktop computer with a Intel Core2Duo E6550, 2.33 GHz processor and 2 GB RAM.

RESULTS

Application of Taguchi experimental design for assessing IHSA performance

Two optimization problems taken from the literature were used to test the performance of IHSA. By arranging IHSA parameters in Taguchi's L_{25} OA, IHSA was tested in finding global optimum of two unconstrained function minimization examples, namely Rosenbrock and Wood-Colville function.

IHSA parameters considered in the experiment were: HMS , $HMCR$, PAR_{min} , PAR_{max} , bw_{min} and bw_{max} . Each of these design factors can have some levels. The IHSA design parameters and the corresponding levels are shown in Table 1.

If all of the possible combinations of the six design parameters were to be considered (full factorial design), then $5^6 = 15625$ experiment trials would have to be carried out. This was unrealistic, so by using Taguchi's OA this number was significantly reduced. As the total degree of freedom (DoF) for the six design factors was $4 * 6 = 24$, the L_{25} OA was used (Phadke, 1989). This experimental design allowed assessing the performance of IHSA taking five $HMCR$ values at different combinations of other parameters.

For each experiment trial corresponding to each row of the L_{25} OA, three replications were used. Thus, a total of 75 experiment trials were conducted in order to assess the performance of the IHSA for each of the test functions. The performance of IHSA was tested at various numbers of improvisations using the performance measure shown in Equation 9:

$$E(m) = \sum_{i=1}^n (x_i(m) - g)^2, \quad (9)$$

Where, x_i is the resulted value from running IHSA at

given parameter combination settings and g is the global optimum of the test function. The parameter combination that yields a lower value of $E(m)$ has better performance.

In the case of Rosenbrock's function the performance of IHSA was tested at various numbers of improvisations, that is: 5000, 12500, 25000, 50000 and 100000. On the other hand, in the case of Wood-Colville function, the performance of IHSA was tested at 25000, 75000, 150000, 300000 and 600000 improvisations.

When applying the IHSA to the two selected test functions, the design variables, x_i , were set between -10.0 and 10.0 . The L_{25} OA used for experimental design is shown in Table 2.

The performance of IHSA was assessed using Equation 9, and is graphically shown in Figure 1(a) for Rosenbrock and Figure 1(b) for Wood-Colville function. From Figure 1(a), one can see that the global optimum was consistently found 5 times. The IHSA parameter combination yielding to global optimum corresponds to trials 8 and 9 for 50000 and 100000 improvisation and trial 2 for 100000 improvisations. It is also noticeable that with the increasing number of improvisations, the average $E(m)$ decreases. However, this is not valid for particular IHSA parameter combination (trial 15). When IHSA was run at 100000 improvisations, 22 out of 25 parameter combinations yielded average $E(m)$ bellow 0.005 which means that these values are quite close to the global optimal point.

As shown in Figure 1(b), the global optimum of Wood-Colville function was consistently found 2 times. The IHSA parameter combination yielding to global optimum corresponds to trials 15 for 300000 and 600000 improvisations. When IHSA performance is considered, the observations for Rosenbrock function are valid. Generally, the average $E(m)$ is indirectly proportional to the number of improvisations.

The relationship between the number of the global optima found and the number of improvisations for the selected test functions is shown in Figure 2.

The best obtained IHSA parameter combinations were further tested for consistency. The IHSA was rerun with the best identified parameter combinations for 10 times and the performance is shown in Table 3.

For solving the Rosenbrock function, the best IHSA parameters obtained using the Taguchi experimental designs

Table 2. L₂₅ used for experimental design.

Trial no.	Coded design factor						Actual IHSA parameter values					
	A	B	C	D	E	F	HMS	HMCR	PAR _{min}	PAR _{max}	bw _{min}	bw _{max}
1	1	1	1	1	1	1	5	0.35	0.1	0.8	0.00001	0.2
2	1	2	2	2	2	2	5	0.5	0.2	0.85	0.0001	0.4
3	1	3	3	3	3	3	5	0.65	0.3	0.9	0.001	0.6
4	1	4	4	4	4	4	5	0.8	0.4	0.95	0.01	0.8
5	1	5	5	5	5	5	5	0.95	0.5	0.99	0.1	1
6	2	1	2	3	4	5	10	0.35	0.2	0.9	0.01	1
7	2	2	3	4	5	1	10	0.5	0.3	0.95	0.1	0.2
8	2	3	4	5	1	2	10	0.65	0.4	0.99	0.00001	0.4
9	2	4	5	1	2	3	10	0.8	0.5	0.8	0.0001	0.6
10	2	5	1	2	3	4	10	0.95	0.1	0.85	0.001	0.8
11	3	1	3	5	2	4	20	0.35	0.3	0.99	0.0001	0.8
12	3	2	4	1	3	5	20	0.5	0.4	0.8	0.001	1
13	3	3	5	2	4	1	20	0.65	0.5	0.85	0.01	0.2
14	3	4	1	3	5	2	20	0.8	0.1	0.9	0.1	0.4
15	3	5	2	4	1	3	20	0.95	0.2	0.95	0.00001	0.6
16	4	1	4	2	5	3	40	0.35	0.4	0.85	0.1	0.6
17	4	2	5	3	1	4	40	0.5	0.5	0.9	0.00001	0.8
18	4	3	1	4	2	5	40	0.65	0.1	0.95	0.0001	1
19	4	4	2	5	3	1	40	0.8	0.2	0.99	0.001	0.2
20	4	5	3	1	4	2	40	0.95	0.3	0.8	0.01	0.4
21	5	1	5	4	3	2	80	0.35	0.5	0.95	0.001	0.4
22	5	2	1	5	4	3	80	0.5	0.1	0.99	0.01	0.6
23	5	3	2	1	5	4	80	0.65	0.2	0.8	0.1	0.8
24	5	4	3	2	1	5	80	0.8	0.3	0.85	0.00001	1
25	5	5	4	3	2	1	80	0.95	0.4	0.9	0.0001	0.2

are: $HSA=10$; $HMCR=0.95$; $PAR_{min}=0.1$; $PAR_{max}=0.85$; $bw_{min}=0.001$, and $bw_{max}=0.8$ with 50 000 improvisations. A similar proposal of IHSA parameters for solving the Rosenbrock function was offered by Orman and Mahdavi (2008) in their paper: $HMS=5$, $HMCR=0.99$; $PAR_{min}=0.01$; $PAR_{max}=0.9$; $bw_{min}=0.0001$ with 50 000 improvisations.

DISCUSSION

Analyzing Table 3 and Figures 1(a) and (b), one can see that global optima are found using small HMS depending on the number of improvisations. This means that in searching for the global optimum for small scale optimization problems, it is sufficient to use small HMS . Such a small number of HMS corresponds to a problem with 2 and 4 variables, that is, functions with 2 and 4 variables, which are considered in this paper. However, this does not mean that such a small number of HMS will correspond to more complex problems with multiple variables or real problems.

Conclusions

IHSA possesses a potential for obtaining good solutions in solving unconstrained function minimization problems for various combinations of parameters. Using the Taguchi experimental design, one can identify IHSA parameter settings yielding to good solutions with minimum experimentation. However, the optimum calibration of IHSA parameters in solving real world optimization problems needs more experience. Future work will include integrating the Taguchi experimental design with IHSA in solving real optimization problems.

A series of computational experiment trials, planned according to Taguchi's experimental design, was used to systematically identify the best combination of IHSA parameters at various numbers of improvisations in order to analyze the performance of IHSA in solving unconstrained function minimization problems.

Analysis of the results showed that there was direct relationship between the number of improvisations of the IHSA and the number of global optimum found, and that



Figure 1. The performance of IHSA: (a), Rosenbrock function; (b), Wood-Colville function.



Figure 2. Number of improvisations and global optimums found.

Table 3. Testing the best identified IHSA parameter combinations.

Parameter Combination	Rosenbrock function					Wood-Colville function	
	I	II	III	IV	V	I	II
Trial	2	8	8	9	9	15	15
Number of improvisations	10^5	5×10^4	10^5	5×10^4	10^5	3×10^5	6×10^5
Average E	0	0.001639	0	0	0	3.80E-06	0

the number of global optimums found was decreased with an increase in the number of function variables. For solving unconstrained function minimization problems, it was sufficient to use small *HMS* along with increasing the number of improvisations, especially when the number of design variables was increased, and this will be analyzed in the further research. The authors will also test the sensitivity of IHSA parameters to real engineering problems in future research.

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