

Full Length Research Paper

An alternative approach for calculation/measurement of fundamental powers based on wavelet packet transform

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In order to calculate/measure the fundamental harmonic power values, the wavelet transform based methods have been proposed in the literature. However, the fundamental harmonic of voltage and current signals cannot be decomposed into real and imaginary components in any of these methods based on only wavelet packet transform. It is required to decompose voltage and current signals with harmonics to real and imaginary components in a power system for power quality and control. In this study real and imaginary components are obtained by using the wavelet packet transform. Furthermore, the fundamental active-reactive-apparent powers are determined with two different methods and accuracy of the proposed methods is demonstrated via simulations.

Key words: Wavelet packet transform, fundamental powers, power factor.

INTRODUCTION

The number of nonlinear loads in the energy systems has been increasing rapidly in parallel to developing technology. Nonlinear load characteristics lead to harmonics in voltage and current signals in the power systems. Definitions belonging to the components in power analysis for signals consisting of harmonics are included in IEEE Work Group's source (Arseneau et al., 1996).

In the literature, classical power calculations/measurements based on windowing methods are carried out by means of the Fourier transform. The power analyses in the systems, where the static current and voltage signals are present, are performed with great accuracy in the frequency domain. Power measurement based on the wavelet transform was seen first in the works of W. K. Yoon and M. J. Devaney who used the discrete wavelet transform and 90° phase shift circuits (Yoon and Devaney, 1998, 2000). On the other hand an algorithm was presented which can simultaneously identify all harmonics. In

the first step of this approach, the frequency spectrum of the waveform is decomposed into subbands using discrete wavelet packet transform filter banks. In the second step, continuous wavelet transform is applied to nonzero subbands to evaluate the harmonic contents (Pham and Wong, 1999). Root-mean-square (RMS) and active power calculations by using wavelet packet transform are given in Hamid et al. (2002).

Methods given above requires phase shift circuits and two transforms (discrete wavelet packet and continuous). In this study, the RMS values and phase angle of the fundamental harmonics of the voltage and current signals in the power system are determined based on only the wavelet packet transform and their real and imaginary components, are calculated active-reactive-apparent power values belonging to the fundamental harmonics and the displacement power factor are found with two different methods by using the calculated values. Accuracy and effectiveness of the proposed methods are demonstrated via simulation results for various voltage and current signals with harmonics.

The paper is organized as follows: Wavelet packet transform and Hilbert transform are discussed in Section "Materials and Methods". The proposed methods

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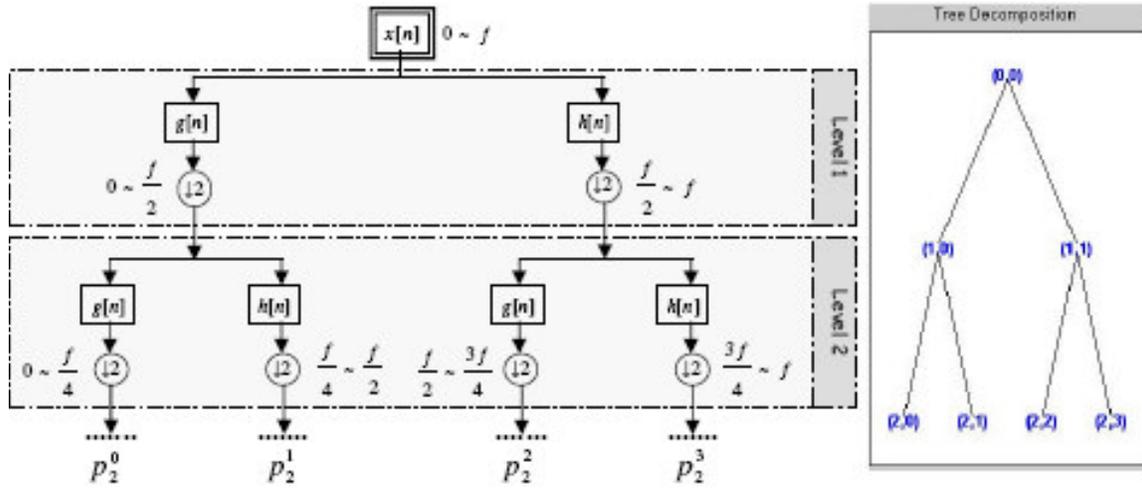


Figure 1. 2-level discrete wavelet packet transform/tree decomposition.

together with simulation results are provided in Section “Results and Discussion”.

MATERIALS AND METHODS

Wavelet packet transform

Wavelet transform has been widely used in various fields for almost twenty years. In general, the discrete wavelet packet transform (DWPT) is given by the following iterations:

$$W_{2n}(t) = \sqrt{2} \sum_{k=0}^{2N-1} g[k]W_n(2t-k) \quad , \quad W_{2n+1}(t) = \sqrt{2} \sum_{k=0}^{2N-1} h[k] \quad (1)$$

In Equation 1, $g[n]$ and $h[n]$ are low-pass and high-pass filters of length $2N$, $W_0(t) = \varphi(t)$ is the scaling function and $W_1(t) = \psi(t)$ is the wavelet function. If a signal $x(t)$ with $N = 2^L$ samples is subjected to the discrete wavelet packet transform up to the level s , we obtain 2^s nodes/packet or frequency bands and each frequency band has 2^{L-s} or $N/2^s$ wavelet packet coefficients (Figure 1). If the wavelet packet coefficients of the s^{th} decomposition level at $2m^{\text{th}}$ node and k^{th} point is denoted by $p_s^{2m}[k]$, the analog signal $x(t)$ can be reconstructed by:

$$x(t) = \sum_{k=1}^{N/2^s} p_s^0[k] \cdot \varphi_{s,k}(t) + \sum_{m=1}^{2^s-1} \sum_{k=1}^{N/2^s} p_s^m[k] \cdot \psi_{s,k}^m(t) \quad (2)$$

In Equation 2, $p_s^m[k]$ are the wavelet packet coefficients of $x[n]$ and $p_s^0[k]$'s are the scaling function coefficients or the coefficients at the 0^{th} node (Wickerhauser, 1994; Goswami and Chan, 1999;

Mathworks, 2000).

Hilbert transform

One of the popular transform in signal processing is the Hilbert transform. For a signal $x(t)$, its Hilbert transform is given by:

$$\mathcal{H}[x(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = x(t) * \frac{1}{\pi t} \quad (3)$$

As a result of this transformation, the amplitude of $x(t)$ doesn't change, the phases of all frequency components are shifted by $\pi/2$.

RESULTS AND DISCUSSION

The proposed calculation/measurement methods

In this study, the proposed calculation/measurement methods are based on our approach in (Vatansever and Ozdemir, 2008). Calculation of RMS and phase angle values of the fundamental harmonics of any voltage or current signal are provided in detail in Vatansever and Ozdemir (2008) and are repeated here for convenience:

$$X_{RMS}^0 = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{1}{N} \sum_{k=1}^{N/2^s} |p_{\mathcal{H},s}^0[k]|^2} \quad (4)$$

$$\theta_0 = \angle p_{\mathcal{H},s}^0[1] - 27.734 + 90^\circ \quad (5)$$

By using these values, real and imaginary components of the fundamental harmonics of the voltage or current signal are calculated as:

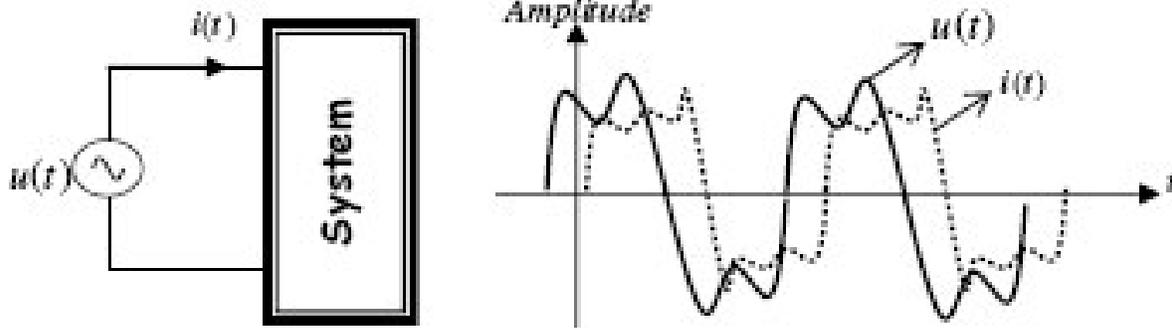


Figure 2. Sample power system and voltage-current graphics.

$$\left. \begin{array}{l} \text{Real part} \rightarrow X_{RMS}^0 \cdot \text{Cos}(\theta_0) \\ \text{Imaginary part} \rightarrow X_{RMS}^0 \cdot \text{Sin}(\theta_0) \end{array} \right\} \quad (6)$$

In this study, power values belonging to the fundamental harmonic are determined with two different ways.

The first method

Voltage and current signals with harmonics for a single phase power system are shown in Figure 2. In this power system, voltage and currents are:

$$\left. \begin{array}{l} u(t) = U_1 \cdot \text{Sin}(\omega_0 t + \alpha_1) + U_2 \cdot \text{Sin}(2\omega_0 t + \alpha_2) + \dots + U_k \cdot \text{Sin}(k\omega_0 t + \alpha_k) \\ i(t) = I_1 \cdot \text{Sin}(\omega_0 t + \beta_1) + I_2 \cdot \text{Sin}(2\omega_0 t + \beta_2) + \dots + I_k \cdot \text{Sin}(k\omega_0 t + \beta_k) \end{array} \right\} \quad (7)$$

From Equation 7, calculation of the fundamental harmonic power expressions are (Arseneau, 1996).

Fundamental active power:

$$P_1 = \frac{U_1 \cdot I_1}{2} \cdot \text{Cos}(\alpha_1 - \beta_1) = \frac{U_1 \cdot I_1}{2} \cdot \text{Cos}(\phi_1) \quad (8)$$

Fundamental reactive power:

$$Q_1 = \frac{U_1 \cdot I_1}{2} \cdot \text{Sin}(\alpha_1 - \beta_1) = \frac{U_1 \cdot I_1}{2} \cdot \text{Sin}(\phi_1) \quad (9)$$

Fundamental apparent power:

$$S_1 = \frac{U_1 \cdot I_1}{2} = \sqrt{P_1^2 + Q_1^2} \quad (10)$$

$$\text{Displacement power factor: } dpf = \frac{P_1}{S_1} \quad (11)$$

It is clear from Equation 8-11 that for calculation of P_1 , Q_1 , S_1 and dpf , we need to determine values of U_1 ,

I_1 , α_1 and β_1 . There exist several classical studies for calculation of these values in the literature. In this study, these values are determined based on the wavelet packet transform as shown in Figure 3. To the best of our knowledge, this type of calculation is an alternative approach.

Simulations based on this method were performed with a graphical user interface (GUI) by using Matlab (Math-works, 2000) and WaveLab v.802 (Donoho, 1999) (Figure 4). In the GUI, it is possible to choose voltage and current signals with harmonics or load them from files. For signals of interest, 128 samples are taken in each pe-riod. Then, samples of the voltage and current signals are subjected to wavelet packet decomposition up to the 5th level. RMS and phase angle values of fundamental harmonics of the signals are computed from obtained wavelet coefficients. The calculated values are shown numerically as well as visually in the GUI. Finally, fundamental powers are calculated by using Equation 8-11. Simulation results are provided in Table 1 from which it can be seen that the true values and those obtained by the proposed method are in good agreement showing the effectiveness of the proposed wavelet packet based approach. Small deviations between the true and calculated values are due to the non-ideal characteristics of the filters used in the wavelet packet decomposition.

The second method

Classical power calculation expressions by using fundamental real and imaginary components of the voltage and current signals with harmonics are given below. Expressions for voltage and current signals with harmonics at the power system, whose block diagram and sample waveform are provided in Figure 2 are

$$\left. \begin{array}{l} u(t) = U_0 + \sum_{k=1}^m \{a_k \cdot \text{Cos}(k\omega_0 t) + b_k \cdot \text{Sin}(k\omega_0 t)\} = U_0 + \sum_{k=1}^m u_k \cdot \text{Sin}(k\omega_0 t + \alpha_k) \\ i(t) = I_0 + \sum_{k=1}^n \{c_k \cdot \text{Cos}(k\omega_0 t) + d_k \cdot \text{Sin}(k\omega_0 t)\} = I_0 + \sum_{k=1}^n i_k \cdot \text{Sin}(k\omega_0 t + \beta_k) \end{array} \right\} \quad (12)$$

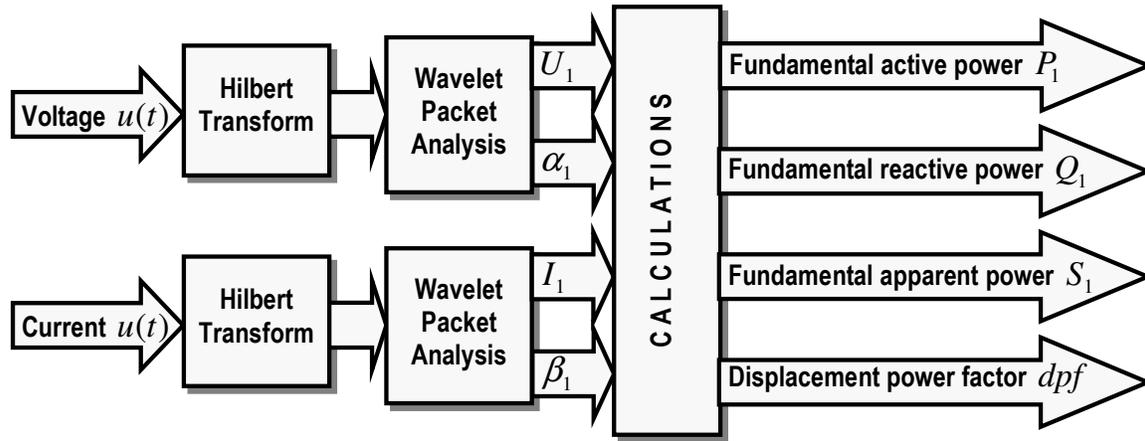


Figure 3. The block diagram of the first proposed method.

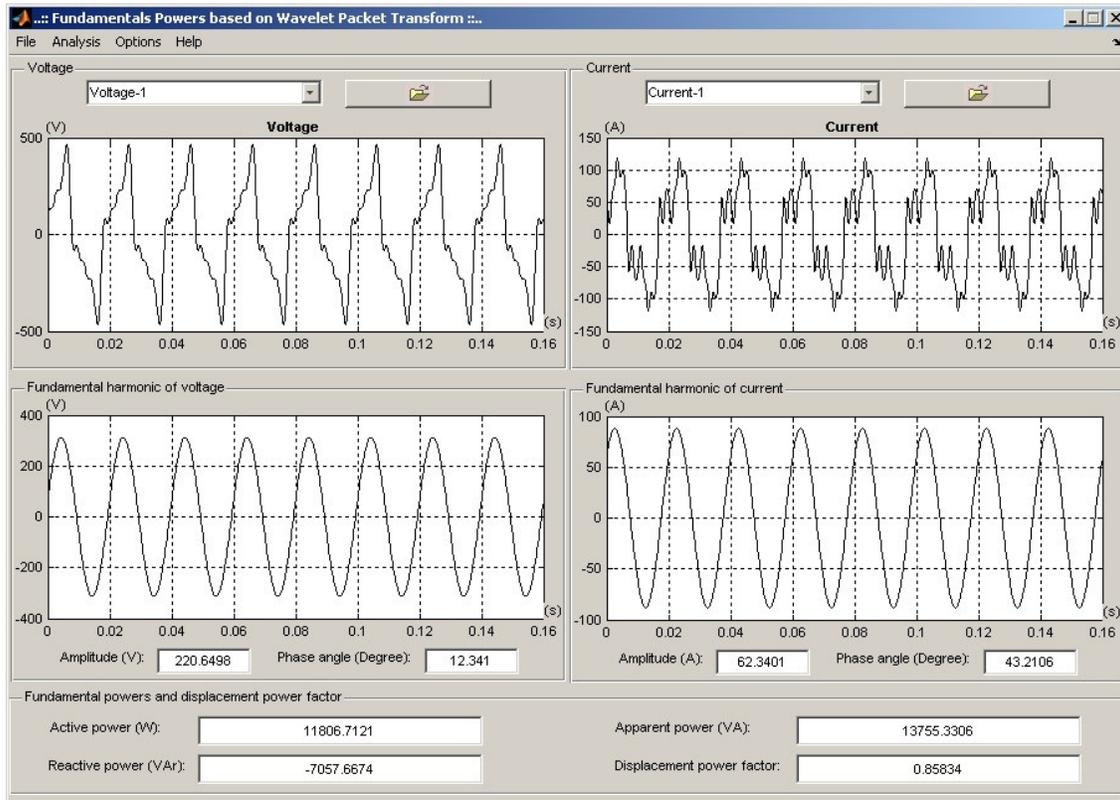


Figure 4. Screenshot of simulation program for the first method.

From Equation 12, the apparent power S , active power P and reactive power Q is given by (Marganitz, 1992).

$$S = U_{RMS} I_{RMS} = \sqrt{U_0^2 + \frac{1}{2} \sum_{k=1}^m (a_k^2 + b_k^2)} \sqrt{I_0^2 + \frac{1}{2} \sum_{k=1}^n (c_k^2 + d_k^2)} \quad (13)$$

$$P = \frac{1}{T_0} \int_0^{T_0} u(t) \cdot i(t) \cdot dt = U_0 \cdot I_0 + \frac{1}{2} \sum_{k=1}^{\min(m,n)} (a_k \cdot c_k + b_k \cdot d_k) \quad (14)$$

$$Q = \frac{1}{T_0} \int_0^{T_0} u(t) \cdot \mathcal{H}\{i(t)\} \cdot dt = \frac{1}{2} \sum_{k=1}^{\min(m,n)} (a_k \cdot d_k - b_k \cdot c_k) \quad (15)$$

Table 1. Comparative simulation results for the first method.

		$\omega = 2\pi 50$	$\omega = 2\pi 60$	
Voltage		$\sqrt{2} \left\{ \begin{array}{l} 220,65 \sin(\omega t + 12,34^\circ) + 80 \sin(3\omega t + 135^\circ) + \\ 50 \sin(5\omega t - 110^\circ) + 25 \sin(7\omega t + 10^\circ) + \\ 10 \sin(9\omega t + 40^\circ) \end{array} \right\}$	$\sqrt{2} \left\{ \begin{array}{l} 220,34 \sin(\omega t - 34,89^\circ) + 75 \sin(5\omega t + 145^\circ) + \\ 30 \sin(7\omega t + 105^\circ) + 10 \sin(11\omega t - 10^\circ) \end{array} \right\}$	
	Current	$\sqrt{2} \left\{ \begin{array}{l} 62,34 \sin(\omega t + 43,21^\circ) + 27 \sin(3\omega t - 153^\circ) + \\ 14 \sin(7\omega t + 112^\circ) + 10 \sin(9\omega t - 155^\circ) + \\ 4 \sin(13\omega t + 28^\circ) \end{array} \right\}$	$\sqrt{2} \left\{ \begin{array}{l} 50,25 \sin(\omega t - 40,56^\circ) + 30 \sin(3\omega t + 123^\circ) + \\ 15 \sin(5\omega t + 162^\circ) + 10 \sin(7\omega t - 175^\circ) + \\ 6 \sin(9\omega t + 15^\circ) \end{array} \right\}$	
FUNDAMENTAL HARMONIC	Amplitude (V)	True	220.650000	220.340000
		Wavelet	220.649812	220.339988
		Error (%)	0.000085	0.000005
	Phase (°)	True	12.340000	-34.890000
		Wavelet	12.341008	-34.889104
		Error (%)	0.008169	0.002568
	Amplitude (V)	True	62.340000	50.250000
		Wavelet	62.340096	50.250117
		Error (%)	0.000154	0.000233
	Phase (°)	True	43.210000	-40.560000
		Wavelet	43.210614	-40.559937
		Error (%)	0.001421	0.000155
Active power (W)	True	11806.655261	11017.914129	
	Wavelet	11806.712063	11017.923211	
	Error (%)	0.000481	0.000082	
Reactive power (VAr)	True	-7057.743786	1093.907895	
	Wavelet	-7057.667444	1094.070523	
	Error (%)	0.001082	0.014867	
Apparent power (VA)	True	13755.321000	11072.085000	
	Wavelet	13755.330585	11072.110106	
	Error (%)	0.000070	0.000227	
Displacement pf	True	0.858334	0.995107	
	Wavelet	0.858337	0.995106	
	Error (%)	0.000350	0.000100	

If the above equations are referred to the fundamental harmonic powers, we obtain

$$S_1 = U_{1_{RMS}} I_{1_{RMS}}, \quad P_1 = \frac{1}{2}(a_1 c_1 + b_1 d_1), \quad Q_1 = \frac{1}{2}(a_1 d_1 - b_1 c_1) \quad (16)$$

As seen in Figure 5, in the second method the real and imaginary components of the fundamental harmonics of the voltage and current signals to be analyzed are the same as those in the first method up to calculations of U_1 , I_1 , α_1 and β_1 . After U_1 , I_1 , α_1 and β_1 are calculated, the real a_1 , c_1 and the imaginary b_1 , d_1 components

are obtained using Equation 6. a_1 , b_1 , c_1 , and d_1 values substituted in Equation 16 in order to calculate the fundamental harmonics powers.

Several simulations were carried out by using the designed GUI. Results are provided in Figure 6 in which instantaneous variations of the real and imaginary components of the fundamental frequency of the voltage and current signals with harmonics are illustrated. In addition, effective values of the real and imaginary components are available numerically. Comparative results are provided in Table 2. As was the case in the first method, the results in Table 2 demonstrate the effectiveness of the proposed method.

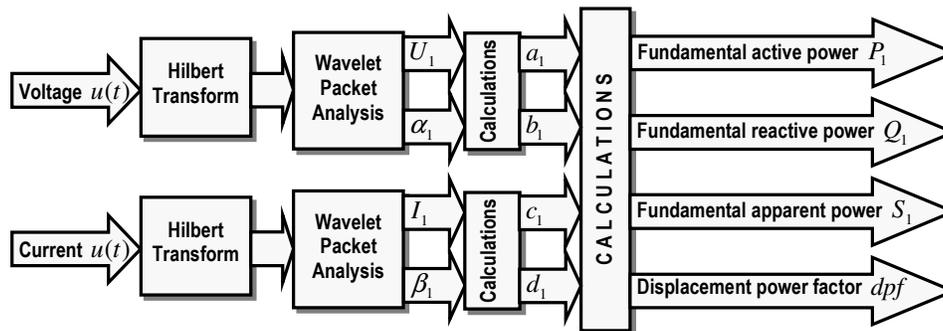


Figure 5. The block diagram of the second proposed method.

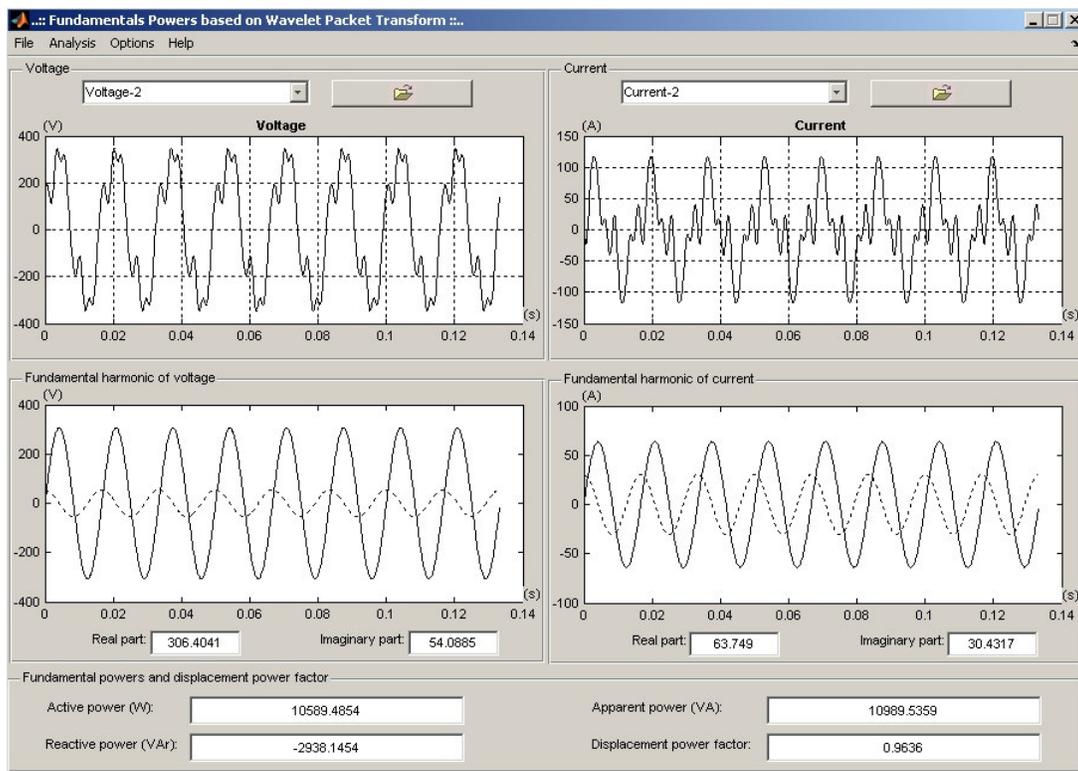


Figure 6. Screenshot of simulation program for the second method.

In Table 3, comparative results of the traditional FFT method, the method used (Yoon and Devaney, 2000) and the proposed methods are shown. The power measurement results shown in Table 3 are obtained by using the following voltage and current signals ($\omega = 2\pi 60$), which are given in (Yoon and Devaney, 2000) (Equation 17-18):

$$u(t) = \sqrt{2} \begin{Bmatrix} 1.0\sin(\omega) + 1.0\sin(5\omega + 15^\circ) + 1.0\sin(11\omega) \\ + 1.0\sin(23\omega - 45^\circ) + 1.0\sin(45\omega + 45^\circ) \end{Bmatrix} \quad (17)$$

$$i(t) = \sqrt{2} \begin{Bmatrix} 1.0\sin(\omega + 60^\circ) + 1.0\sin(5\omega) + 1.0\sin(11\omega) \\ + 1.0\sin(23\omega) + 1.0\sin(45\omega - 45^\circ) \end{Bmatrix} \quad (18)$$

Comparison of the results in Table 3, show that almost all performances of the methods are the same. But in Yoon and Devaney (2000), discrete wavelet transform is used and measurement of the reactive power requires complex filter design for both voltage and current signals.

Again, the results of the traditional FFT method, the

Table 2. Comparative simulation results for the second method.

		$\omega = 2\pi 50$	$\omega = 2\pi 60$		
Voltage		$\sqrt{2} \left\{ 220,12 \sin(\omega t - 34,56^\circ) + 72 \sin(3\omega t + 125^\circ) + 41 \sin(5\omega t - 140^\circ) + 23 \sin(7\omega t + 42^\circ) \right\}$	$\sqrt{2} \left\{ 220,01 \sin(\omega t + 10,01^\circ) + 58 \sin(3\omega t + 111^\circ) + 33 \sin(5\omega t + 97^\circ) + 18 \sin(7\omega t - 52^\circ) \right\}$		
	Current	$\sqrt{2} \left\{ 51,15 \sin(\omega t - 44,44^\circ) + 33 \sin(3\omega t + 103^\circ) + 17 \sin(5\omega t - 12^\circ) + 7 \sin(7\omega t + 67^\circ) \right\}$	$\sqrt{2} \left\{ 49,95 \sin(\omega t + 25,52^\circ) + 28 \sin(3\omega t - 122^\circ) + 17 \sin(5\omega t + 153^\circ) + 9 \sin(7\omega t - 174^\circ) \right\}$		
FUNDAMENTAL HARMONIC	Voltage	Real part	True 256.362971 Wavelet 256.363791 Error (%) 0.000320	306.404758 306.404081 0.000221	
		Imaginary part	True -176.588946 Wavelet -176.586951 Error (%) 0.001130	54.082568 54.088529 0.011022	
			Current	Real part	True 51.647481 Wavelet 51.649002 Error (%) 0.002945
	Imaginary part			True -50.647633 Wavelet -50.646232 Error (%) 0.002766	30.433544 30.431654 0.006210
		Active power (W)		True 11092.156885 Wavelet 11092.198800 Error (%) 0.000378	10589.303651 10589.485407 0.001716
			Reactive power (VAr)	True 1931.901691 Wavelet 1931.660119 Error (%) 0.012504	-2938.664227 -2938.145394 0.017655
	Apparent power (VA)			True 11259.138000 Wavelet 11259.137846 Error (%) 0.000001	10989.499500 10989.535911 0.000331
		Displacement pf		True 0.985169 Wavelet 0.985173 Error (%) 0.000406	0.963584 0.963597 0.001349

Table 3. Comparison of FFT, Ref. 3, first proposed method and second proposed method.

	FFT	(Yoon and Devaney, 2000)	First proposed method	Second proposed method
P_1 (W)	0.500	0.500	0.500	0.500
Q_1 (VAr)	-0.866	-0.866	-0.866	-0.866
S_1 (VA)	1.000	1.000	0.999	0.999
d_{pf}	0.500	-	0.500	0.500

the method used (Hamid et al., 2002) and the proposed methods are given in Table 4 for comparison. The results of the power measurement are obtained using the following voltage and current signals ($\omega = 2\pi 60$) which are given in (Hamid et al., 2002) (Equation 19-20):

$$u(t) = \sqrt{2} \left\{ \begin{aligned} &1.0 \sin(\omega t) + 0.2 \sin(3\omega t + 13^\circ) + 0.2 \sin(5\omega t + 150^\circ) + \\ &0.1 \sin(7\omega t + 140^\circ) + 0.08 \sin(9\omega t + 40^\circ) + \\ &0.1 \sin(11\omega t + 15^\circ) + 0.1 \sin(13\omega t + 150^\circ) \end{aligned} \right\} \quad (19)$$

Table 4. Comparison of FFT, Ref.4, first proposed method and second proposed method.

	FFT	(Hamid et al., 2002)	First proposed method	Second proposed method
P_1 (W)	0.9848	0.9848	0.9848	0.9848
Q_1 (VAr)	-0.1736	-	-0.1736	-0.1736
S_1 (VA)	1.0000	-	1.0000	1.0000
dpf	0.9848	-	0.9848	0.9848

$$i(t) = \sqrt{2} \left\{ \begin{array}{l} 1.0\sin(\omega t + 10^\circ) + 0.1\sin(3\omega t + 150^\circ) + 0.08\sin(5\omega t + 135^\circ) + \\ 0.008\sin(7\omega t - 22.5^\circ) + 0.09\sin(9\omega t + 20^\circ) + \\ 0.07\sin(11\omega t + 45^\circ) + 0.08\sin(13\omega t + 120^\circ) \end{array} \right\} \quad (20)$$

Examination of the results shown in Table 4, reveals that the reactive power of the fundamental component can not be measured with the method given in Hamid et al. (2002). Therefore, in Table 4 just active power and power factor comparisons are made.

Conclusion

In this study, real and imaginary parts of the fundamental component of the voltage and current signals with harmonics are calculated based on only wavelet packet transform. By using these values, active-reactive-apparent powers for the fundamental component and the displacement power factor are determined by using two different methods. The accurateness of the proposed method was demonstrated by using simulations and comparative results. The method can be easily applicable in industrial applications using digital signal processors or microcontroller since presented study does not need trigonometric and complex number calculations.

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