A back-propagation artificial neural network approach for three-dimensional coordinate transformation

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The European Datum 1950 (ED50) of the Turkish national geodetic network (TNGN) and the World Geodetic System 1984 (WGS84) of the Turkish national fundamental GPS network (TNFGN) are in use as geodetic reference frames in Turkey. According to the use of two reference systems, it is necessary to transform the three-dimensional (3D) coordinate data from ED50 to WGS84 or vice versa. The seven-parameter similarity transformation method is frequently used for 3D coordinate transformation in geodesy. In this study, a back propagation artificial neural network (BPANN) that has been more widely applied in engineering among all other neural network models is evaluated as an alternative 3D coordinate transformation method. BPANN is compared with a popular seven-parameter similarity transformation (Molodensky-Badekas) method over a test area, in terms of root mean square error (RMSE). The results indicated that the employment of BPANN transformed 3D coordinates (X, Y, Z) more accurate than Molodensky-Badekas method and can be useful for 3D coordinate transformation between ED50 and WGS84.

Key words: 3D coordinate transformation, back propagation artificial neural network, seven-parameter similarity transformation, BPANN, Molodensky-Badekas.

INTRODUCTION

The global positioning system (GPS) is frequently used in establishing geodetic networks because GPS provides location and time information with a high accuracy anywhere on the Earth. In geodetic applications, the GPS techniques are widely used for determining three-dimensional (3D) coordinates that is the base in surveying and mapping applications. The World Geodetic System 1984 (WGS84) is used for GPS measurements. To fully utilize WGS84, countries using different datums for their own coordinate basis have to make a datum transformation between their datum and WGS84 or change the datum to WGS84 (Kwon et al., 2005). According to the international trends, the Turkish National Geodetic Network (TNGN) was changed to the Turkish National Fundamental GPS Network (TNFGN) in 2001.

TNGN was established with conventional techniques by the General Command of Mapping between 1934 and 1954. TNGN was based on the European Datum 1950 (ED50) that was defined for the international connection of geodetic networks and it was connected to 8 geodetic control stations of European network for realization. TNGN was established with the ignorance of the tectonic characteristics of Turkey and it did not meet the needs of modern geodetic applications based on GPS.

TNFGN was established in 2001 by the General Command of Mapping. The total number of stations is about 600 and 145 of them were re-surveyed in 2003 and 172 of them in 2004 together with reconnaissance of about 210 points in Western Anatolia for the purpose of improvement and maintenance of TNFGN in 2005. For each station, 3D coordinates and their associated velocities were computed in ITRF2000. Positional accuracies of the stations are about 1-3 cm whereas the relative accuracies are within the range of 0.1 - 0.01 ppm. Besides, the network has been connected to the Turkish Horizontal and Vertical Control Networks through overlapping stations and time-dependent coordinates of all stations are being computed in the context of the maintenance of the network with repeated GPS observations (Caglar, 2005).

In Turkey, maps and coordinate information have been produced in ED50 datum, up to year 2001 and also, the legal (cadastral and property) rights of people based on TNGN. The collected 3D coordinate information in ED50
needs to be transformed to WGS84, due to the reference frame change.

The coordinate transformation has been considered in several studies in Turkey (Kılıçoglu, 1994; Turgut and İnal, 2003; Eren and Uzel, 2006). The artificial neural network (ANN) has been applied in geodetic studies (Miima et al., 2001; Kavzoglu and Saka, 2005; Riedel and Heinert, 2008; Gullu and Yilmaz, 2010) including coordinate transformation and significant progresses were achieved with ANN. Barsi (2001) performed 3D coordinate transformation by ANN and Tierra et al. (2008) put a comparison of the ability of artificial neural networks and official transformation parameters for coordinate transformation and ANN was offered as a suitable tool for transforming between different geodetic reference systems.

The objective of this paper is evaluating a back propagation artificial neural network (BPANN) as an alternative 3D coordinate transformation method between ED50 and WGS84. The 3D coordinates (X, Y, Z) of the control points based on BPANN and a popular seven-parameter similarity transformation (Molodensky-Badekas) method are compared to the known (X, Y, Z) coordinates of the control points over a test area, in terms of root mean square error (RMSE) of the coordinate differences.

### Theoretical review

The 3D coordinate transformation between ED50 and WGS84 can be performed by three steps of coordinate conversions: (1) Geographical to Cartesian; (2) Cartesian to Geodetic; (3) Geodetic to Geographical. The geographical (geodetic) coordinates must first be converted to Cartesian coordinates which is associated with the initial datum. These Cartesian coordinates are then transformed to Cartesian coordinates associated with desired datum. Finally, these transformed Cartesian coordinates are converted to geographical (geodetic) coordinates on the new datum (Featherstone, 1997).

The geodetic coordinates (φ, λ, h) can be converted to Cartesian coordinates (X, Y, Z) by (Heiskanen and Moritz, 1967):

\[
X = (v + h) \cos \phi \cos \lambda \\
Y = (v + h) \cos \phi \sin \lambda \\
Z = [v(1 - e^2 + h)] \sin \phi
\]

(1)

(2)

(3)

where \( \phi \) is the geodetic latitude, \( \lambda \) is the geodetic longitude and \( h \) is the ellipsoidal height. \( v \) represents the radius of curvature in the prime vertical:

\[
v = a\sqrt{1 - e^2 \sin^2 \phi}
\]

(4)

where \( a \) and \( e \) denote the semi-major axis and first eccentricity of the reference ellipsoid.

The inverse conversion from Cartesian coordinates to geodetic coordinates is not straightforward and requires iteration. Several approaches have been developed for this reverse conversion, such as the non-iterative method by Bowring (1985), the iterative method by Borkowski (1989) and the vector method by Pollard (2002). The conversion from (X, Y, Z) to (φ, λ, h) is given in Torge (2001) by:

\[
\phi = \tan^{-1}\left(\frac{Z}{\sqrt{X^2 + Y^2}} \left(1 - e^2\frac{v}{v + h}\right)^{-1}\right)
\]

(5)

\[
\lambda = \tan^{-1}\left(\frac{Y}{X}\right)
\]

(6)

\[
H = \sqrt{X^2 + Y^2} \sec \phi - v
\]

(7)

When the geodetic coordinates have been converted to their Cartesian responses, the seven-parameter similarity transformation (Molodensky-Badekas) method and BPANN can be used to transform these Cartesian coordinates between ED50 and WGS84 for the evaluation process.

### Molodensky-Badekas

The conditions for an efficient 3D coordinate transformation are uniqueness, simplicity and rigor (Collier and Steed, 2001). Fast running, easy performing and highest accuracy can be accepted as criteria for an optimum 3D coordinate transformation method. The seven-parameter similarity transformation is widely used for 3D coordinate transformation in geodesy. The angles are not changed but the position of points may be changed in the seven-parameter similarity transformation. The seven-parameter similarity transformation has three translations of the coordinate origin, one scale factor and three rotation parameters. The 3D coordinates can be transformed into another reference frame by translating the origin, applying rotation in each axis and adjusting the scale. Three common points from two different coordinate systems are enough to estimate those seven transformation parameters, but more common points are used in a least squares adjustment to achieve the highest precision.

The Bursa-Wolf (Bursa, 1962; Wolf, 1963) and Molodensky-Badekas (Molodensky et al., 1962; Badekas, 1969) are the most commonly used methods among seven-parameter similarity transformation methods because of their simplicity for application. Theoretically; the Bursa-Wolf and Molodensky-Badekas models should give the same results when the same data set are used. The only conceptual difference between these models is...
the choice of the point about which the axial rotations and scale factor are applied (Featherstone, 1997). Molodensky-Badekas method removes the high correlation between transformation parameters by relating the parameters to the centroid of the network. For this reason, Molodensky-Badekas method is used for this study.

The Molodensky-Badekas method is a seven-parameter conformal transformation of 3D Cartesian coordinates between datums but is more suited to the transformation between terrestrial and satellite datums (Krakiwsky and Thomson, 1974). This transformation method comprises three shift parameters \((\Delta X, \Delta Y, \Delta Z)\) from the centroid of the terrestrial network \((Xm, Ym, Zm)\), three rotation parameters \((Rx, Ry, Rz)\) and a scale change \((dS)\). The Molodensky-Badekas method, as given by Harvey (1986), is:

\[
\begin{bmatrix}
X \\
Y \\
Z_{WGS84}
\end{bmatrix} = 
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix} + 
\begin{bmatrix}
x_m \\
y_m \\
z_m
\end{bmatrix} + 
\begin{bmatrix}
1 + dS & R_z & -R_y \\
-R_z & 1 + dS & R_x \\
R_y & -R_x & 1 + dS
\end{bmatrix} 
\begin{bmatrix}
X_{ED50} - X_m \\
Y_{ED50} - Y_m \\
Z_{ED50} - Z_m
\end{bmatrix}
\]

\[(8)\]

**Back propagation artificial neural network**

ANNs are simplified models of decision-making processes of a human brain and are formed by interconnected artificial neurons or simply neurons. The input information of the neuron is manipulated by means of synaptic weights that are adjusted during a training process. After the training procedure, an activation function is applied to all neurons for generating the output information (Leandro and Santos, 2007). The multilayer perceptron (MLP) model was selected for this study because MLPs have the ability to learn, operate fast, require small training sets and can be implemented simply among several kinds of ANN models. MLP consists of one input layer with \(N\) inputs, one hidden layer with \(q\) units and one output layer with \(n\) outputs. The output of the model \((y)\) with a single output neuron can be represented by:

\[
y = f\left(\sum_{j=1}^{q} W_{j} f\left(\sum_{l=1}^{N} W_{j,l} x_{l}\right)\right)
\]

\[(9)\]

where \(W\) is the weight between the hidden layer and the output layer, \(w\) is the weight between the input layer and the hidden layer, \(x\) is the input parameter. A sigmoid function is used as activation function for hidden and output layers that is defined by:

\[
f(z) = \frac{1}{1+e^{-z}}
\]

\[(10)\]

where \(z\) denotes the input information of the neuron

In this study BPANN model was selected because it has been more widely applied in engineering among all other ANN applications. BPANN has a feed-forward and supervised learning structure which consists of input layer, one or more hidden layers and one output layer, as shown in Figure 1.

The delta rule based on squared error minimization is used for BPANN training procedure (Haykin, 1999). The training process corresponds to an adjustment of the weights between the hidden layer and the output layer to the data set that is composed of the known input and output parameters. This iterative adjustment updates the weights in order to decrease the difference between the computed output and the actual given output of the neural network. The training procedure consists of feed-forward and back-propagation steps. These steps continue over the training set for several thousand iterations until the network performance reaches an acceptable value.

For determining the performance of the neural network, the mean square error (MSE) can be used that is defined by:

\[
MSE = \frac{1}{N} \sum_{i=1}^{N} (y_{known} - y_{neural})^2
\]

\[(11)\]

where \(N\) is the number of the inputs, \(y_{known}\) denotes the known (target) output value and \(y_{neural}\) denotes the network output value.

**MATERIALS AND METHODOLOGY**

**Test area**

In this study, 3D coordinate transformation was carried out in over test area (approximately 6000 km²) in the province of Eskişehir (Turkey), with latitude boundaries: 39.35°N ≤ φ ≤ 40.00°N and longitude boundaries: 30.50°E ≤ λ ≤ 31.50°E.

**Source data acquisition**

22 points that belong to the Turkish National Triangulation Network were selected with known Cartesian coordinates in ED50 and WGS84 in the test area (Figure 2). The 3D coordinate transformations based on Molodensky-Badekas and BPANN between ED50 and WGS84 were performed with this coordinate data set.

**Applied methodology**

The performances of 3D coordinate transformation methods were evaluated with the differences between the known Cartesian coordinates and the Cartesian coordinates transformed by Molodensky-Badekas and BPANN. These coordinate differences are computed by:

\[
\Delta x, y, z = (X, Y, Z)_{known} - (X, Y, Z)_{transformed}
\]

\[(12)\]
RMSE values of the coordinate residuals (Δx, y, z) were used in the evaluation process of the 3D coordinate transformations based on Molodensky-Badekas and BPANN for investigating the transforming performance of the methods between ED50 and WGS84. The RMSE values of Δx, y, z were calculated by the following equation.

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\Delta_{x,y,z})^2}
\]

Transformation study

The source data set is arbitrary separated into two groups as fundamental data set and control data set. 14 points were selected as fundamental points and 8 points were selected as control points. The fundamental points are used to train BPANN and to determine the transformation parameters of Molodensky-Badekas method. The control points are used to evaluate the performance of the 3D coordinate transformations by comparing the transformed Cartesian coordinates in Molodensky-Badekas and BPANN with the known Cartesian coordinates of the control points. The Cartesian coordinates (X, Y, Z) of the point in ED50 are selected as input layer neurons and the response coordinates (X, Y, Z) in WGS84 are used as output layer neurons in the proposed BPANN. The number of the hidden layer neurons was fixed as 28 by a trial-and-error strategy and the architecture of BPANN was determined as (3:28:3) which gave the smallest network error.

MSE based on the fundamental points is used for analyzing the performance of BPANN in the training process. The statistical values of fundamental data set’s coordinate residuals are summarized in Table 1.

In the evaluating procedure, the Cartesian coordinates (X, Y, Z) of the control points in WGS84 are estimated by the transformation parameters based on the fundamental points for Molodensky-Badekas method and by the trained BPANN. The residuals between the known Cartesian coordinates in WGS84 and the transformed Cartesian coordinates of the control points were estimated. RMSE is used for investigating the Cartesian coordinate...
Table 1. The statistics of fundamental data sets’ BPANN coordinate residuals over the test area (units in m).

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Min</th>
<th>Max</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>(\Delta X)</td>
<td>-0.023</td>
<td>0.018</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(\Delta Y)</td>
<td>-0.017</td>
<td>0.013</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(\Delta Z)</td>
<td>-0.037</td>
<td>0.030</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Table 2. The statistics of control data set’s coordinate residuals from Molodensky-Badekas and BPANN over the test area (units in m).

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molodensky-Badekas</td>
<td>(\Delta X)</td>
<td>-0.022</td>
<td>0.029</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(\Delta Y)</td>
<td>-0.025</td>
<td>0.034</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(\Delta Z)</td>
<td>-0.040</td>
<td>0.052</td>
<td>0.013</td>
</tr>
<tr>
<td>BPANN</td>
<td>(\Delta X)</td>
<td>-0.026</td>
<td>0.020</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(\Delta Y)</td>
<td>-0.017</td>
<td>0.017</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(\Delta Z)</td>
<td>-0.033</td>
<td>0.042</td>
<td>0.008</td>
</tr>
</tbody>
</table>

The differences of the control data set. The statistical values and RMSE value of the control data set’s coordinate residuals are presented in Table 2.

RESULTS AND CONCLUSIONS

The differences between the RMSE values based on the fundamental data set and the control data set that are summarized in Tables 1 and 2 are quite small. It can be considered that the fundamental data set and the control data set are very similar and the fundamental data set represents the test area quite well.

It can be seen from the analysis of the values given in Tables 1 and 2 that, horizontal coordinates \((X, Y)\) of the points are better than the vertical coordinates \((Z)\). It is supposed that this is arisen from different adjustment of horizontal and vertical surveys of the TNGN.

The results presented in Table 2 show that BPANN transformed the Cartesian coordinates \((X, Y, Z)\) of the control points with a better accuracy \((\pm 0.018, \pm 0.014, \text{and} \pm 0.030 \text{m, respectively})\) than Molodensky-Badekas method, in terms of RMSE.

Based on the results of this study, the following conclusions can be drawn:

1. The Cartesian coordinates are transformed with a better accuracy by BPANN than Molodensky-Badekas for 3D coordinate transformation, in terms of RMSE.
2. The employment of a BPANN that is properly structured and trained can be an alternative tool in 3D coordinate transformation.
3. With geographical coverage, more accurate 3D coordinate transformations can be expected from BPANN and also Molodensky-Badekas.

REFERENCES

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