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Full Length Research Paper

Peristalsis in a rotating fluid

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This paper is devoted to the study of peristaltic flow of a viscous fluid in a rotating frame. The governing equations for the flow problem are derived under the long wavelength approximation. The closed form solutions for the stream function and the secondary velocity are obtained. The effects of Taylor number on physical quantities of interest such as the pressure rise per wavelength and the flow rate due to secondary velocity are discussed. The important phenomenon of trapping is also investigated for different values of Taylor number. It is interesting to note that the pressure rise reduces for a rotating fluid in comparison with that of the non-rotating one. Also, increase in rotation reduces the size of trapped bolus and shifts it towards the boundary.

Key words: Peristaltic motion, rotating frame, planar channel.

INTRODUCTION

Flows due to continuous contraction and expansion of flexible walls are known as peristaltic flows. Such types of flow frequently occur in nature particularly in physiology and industry. Typical examples include the urine transport form kidney to the bladder through the ureter, the chime transport in the gastro-intestinal tract, the movement of spermatozoa in the ductus efferentes of the male reproductive tract, etc. Theoretical studies on peristaltic flows have been performed usually for planar channels/tubes. Mathematically, these flows are governed by two nonlinear partial differential equations in two unknown velocity components. These equations can be considerably simplified under the commonly used assumptions of long wavelength and low Reynolds number (Shapiro et al., 1969). In fact, the assumption of long wavelength and low Reynolds number reduces the governing nonlinear partial differential equations to a single, linear, fourth order ordinray differential equation in stream function. This is true for Newtonian as well as many non-Newtonian fluids. However, by using the same assumption, the governing equation of peristaltic flow of some of the non-Newtonian fluids become nonlinear and thus the computations of these flows become immensly difficult. In consequence, it is not always possible to compute an exact solution and therefore, one can predict interesting features with the help of approximate analytic and numerical solutions. There are some recent investigations regarding peristaltic flows of Newtonian and non-Newtonian fluids in the past few decades (Jaffrin and Shapiro, 1971; Raju and Devanathan, 1974; Radhakrishnamacharya, 1982; Srivastava and Srivastava, 1984; Siddiqui and Schwarz, 1993, 1994; Mekheimer et al., 1998; Mekheimer, 2002; Hayat et al., 2002, 2007; Wang et al., 2007; Haroun, 2007a, b; Kothandapani and Srinivas, 2008a, b).

An important area in fluid mechanics is the study of dynamics of fluids when the system is in a state of rigid body rotation. This particular area of fluid dynamics has abundant applications in cosmic and geophysical flows. Furthermore, it has promissing applications in atomospheric science and it helps in understanding the behavior of ocean circulation and the formation of galaxies. In such situations, the governing equations are modeled by incorporating the effects of centrifugal and Coriolis forces in addition to the inertial forces. Undoubtedly, the Coriolis force plays a major role in the ocean circulation and the formation of galaxies. Furthermore, the Coriolis force, caused due to the earth's rotation, strongly affect the flow of the earth's liquid. This area has been widely addressed in studying different features of fluid flows when the system is in a rigid body rotation. A literature survey indicates a number of

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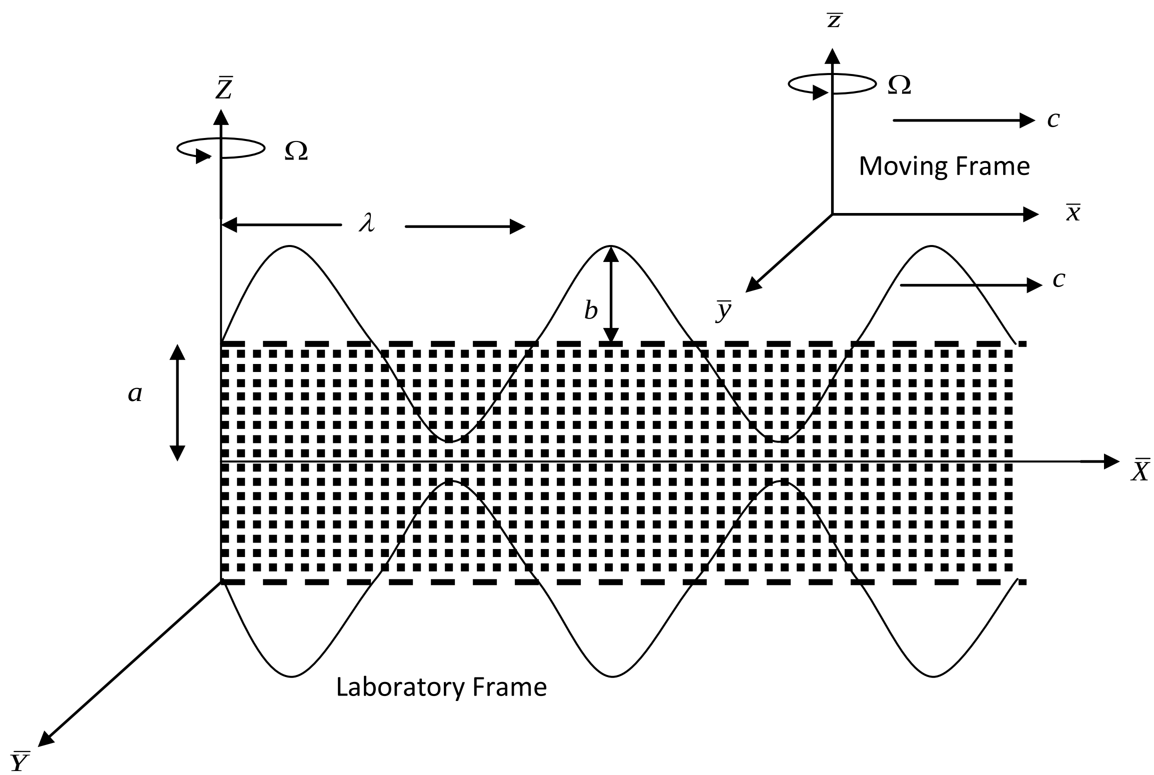

Figure 1. Geometry and coordinate systems.

research articles regarding the flow of fluids in a rotating frame. However, to the best of the authors knowledge, this area has not been addressed for the study of peristaltic flows. It is a known fact that fluid enters a peristatltic pump through a rotating axis and thus theoretical investigations regarding peristaltic flows in a rotating frame is an interesting area, which has not been addressed so far.

In view of the aforementioned facts and discussion, we investigate the peristaltic flow of a Newtonian fluid in a rotating frame of reference under the long wavelength assumption. The resulting equations are solved for an exact solution of the stream function and the secondary flow velocity and some interesting features of peristaltic flow are discussed.

FORMULATION OF PROBLEM

Consider an infinite channel of width $2a$, formed by two infinite walls located at $\bar{Z}=\pm\bar{h}$. The space inside the channel is filled with a homogenous, incompressible Newtonian fluid. An infinite wave train propagates with the speed c along the flexible walls of the channel. The fluid and the channel are assumed to be in a state of rigid body rotation, with a constant angular speed Ω about the $\bar{Z}$-axis. The coordinate system and the geometry of the problem are explained in Figure 1. The equation of the wall surface is given as:

$$\bar{h}\left(\bar{X},\bar{t}\right)=a+b\sin\left[\frac{2\pi}{\lambda}\left(\bar{X}-c\bar{t}\right)\right], \qquad (1)$$

For the flow problem under consideration, we define:

$$\bar{\mathbf{V}}=\left[\bar{U}\left(\bar{X},\bar{Z},\bar{t}\right),\bar{V}\left(\bar{X},\bar{Z},\bar{t}\right),\bar{W}\left(\bar{X},\bar{Z},\bar{t}\right)\right], \qquad (2)$$

where $\bar{U}$, $\bar{V}$, and $\bar{W}$ are the velocity components in $\bar{X}$, $\bar{Y}$, and $\bar{Z}$ directions, respectively. With the help of velocity field, defined by Equation 2, the continuity and the momentum equation for an incompressible fluid yield the following scalar equations:

$$\bar{U}_{\bar{X}}+\bar{W}_{\bar{Z}}=0, \qquad (3)$$

$$\bar{U}_{\bar{t}}+\bar{U}\bar{U}_{\bar{X}}+\bar{W}\bar{U}_{\bar{Z}}-2\Omega\bar{V}=-\frac{\bar{p}_{\bar{X}}}{\rho}+\nu\left[\bar{U}_{\bar{X}\bar{X}}+\bar{U}_{\bar{Z}\bar{Z}}\right], \qquad (4)$$

$$\bar{V}_{\bar{t}}+\bar{U}\bar{V}_{\bar{X}}+\bar{W}\bar{V}_{\bar{Z}}+2\Omega\bar{U}=-\frac{\bar{p}_{\bar{Y}}}{\rho}+\nu\left[\bar{V}_{\bar{X}\bar{X}}+\bar{V}_{\bar{Z}\bar{Z}}\right], \qquad (5)$$

$$\bar{W}_{\bar{t}}+\bar{U}\bar{W}_{\bar{X}}+\bar{W}\bar{W}_{\bar{Z}}=-\frac{\partial\bar{p}_{\bar{Z}}}{\rho}+\nu\left[\bar{W}_{\bar{X}\bar{X}}+\bar{W}_{\bar{Z}\bar{Z}}\right] \qquad (6)$$

where ρ is the density, ν is the kinematic viscosity,

$\bar{p} = \bar{p}^* - \rho\Omega^2\left(\bar{X}^2 + \bar{Y}^2\right)/2$ is the modified pressure, $\bar{p}^*$ is hydrostatic pressure, $\bar{t}$ denotes the time and the subscripts denote differentiation.

The flow phenomenon under consideration is inherently unsteady in the laboratory frame. However, it can be treated as a steady flow in a frame, rotating with the same angular velocity as a laboratory frame, which moves with the speed of the wave. The transformations, relating the coordinates and velocities, of the laboratory frame $\left(\bar{X},\bar{Y},\bar{Z}\right)$ and moving frame $\left(\bar{x},\bar{y},\bar{z}\right)$ are:

$$\bar{x} = \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{z} = \bar{Z},\qquad \bar{u} = \bar{U} - c,\ \ \bar{v} = \bar{V},\ \bar{w} = \bar{W}, \tag{7}$$

where $\bar{u},\ \bar{v},\ \bar{w}$ are the velocity components in the moving frame.

The use of Equation 7 into Equations 3 to 6 eliminates the time derivatives therein and the resulting equations, in new variables, become:

$$\bar{u}_{\bar{x}} + \bar{w}_{\bar{z}} = 0, \tag{8}$$

$$\bar{u}\,\bar{u}_{\bar{x}} + \bar{w}\bar{u}_{\bar{z}} - 2\Omega\bar{v} = -\frac{\bar{p}_{\bar{x}}}{\rho} + \nu\left[u_{\bar{x}\bar{x}} + \bar{u}_{\bar{z}\bar{z}}\right], \tag{9}$$

$$\bar{u}\,\bar{v}_{\bar{x}} + \bar{w}\bar{v}_{\bar{z}} + 2\Omega\left(\bar{u} + c\right) = -\frac{\bar{p}_{\bar{y}}}{\rho} + \nu\left[\bar{v}_{\bar{x}\bar{x}} + v_{\bar{z}\bar{z}}\right], \tag{10}$$

$$\bar{u}\,\bar{w}_{\bar{x}} + \bar{w}\bar{w}_{\bar{z}} = -\frac{\bar{p}_{\bar{z}}}{\rho} + \nu\left[\bar{w}_{\bar{x}\bar{x}} + \bar{w}_{\bar{z}\bar{z}}\right]. \tag{11}$$

We introduce the following non-dimensional variables:

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{\lambda},\ z = \frac{\bar{z}}{a},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c}, w = \frac{\bar{w}}{c},\ h = \frac{\bar{h}}{a},\ p = \frac{a^2\bar{p}}{\lambda\mu c},\ \delta = \frac{a}{\lambda},\ R = \frac{ac}{\nu}. \tag{12}$$

Using these dimensionless variables and the stream function $\psi\left(x,z\right)$ is defined as:

$$u = \psi_z,\ w = -\delta\psi_x, \tag{13}$$

the continuity Equation 8 is identically satisfied and Equations 9 to 11 become:

$$R\left[\delta\psi_z\psi_{xz} - \delta\psi_x\psi_{zz} - \frac{2\Omega a}{c}v\right] = \delta^2\psi_{xxz} + \psi_{zzz} - \frac{\partial p}{\partial x}, \tag{14}$$

$$R\left[\delta\psi_x v_x - \delta\psi_x v_z + \frac{2\Omega a}{c}\left(\psi_z + 1\right)\right] = \delta^2 v_{xx} + v_{zz} - \frac{\partial p}{\partial y}, \tag{15}$$

$$R\delta^3\left[-\psi_z\psi_{xx} + \psi_x\psi_{xz}\right] = -\delta\left(\delta^3\psi_{xxx} + \delta\psi_{xzz}\right) - \frac{\partial p}{\partial z}. \tag{16}$$

For the subsequent analysis, we assume that the wavelength of the wave is large as compared to the half width of the channel, that is, ($\delta << 1$) (Shapiro et al., 1969). Thus, following equations are obtained from Equations 14 to 16:

$$-2Tv = \psi_{zzz} - \frac{\partial p}{\partial x} \tag{17}$$

$$2T\left(\psi_z + 1\right) = v_{zz} - \frac{\partial p}{\partial y} \tag{18}$$

$$\frac{\partial p}{\partial z} = 0, \tag{19}$$

where $T = R\Omega a/c$ is the Taylor's number. Equation 19 suggests that the pressure is not the term containing function of z. By utilizing this fact, we can eliminate the pressure from Equation 17 by cross differentiating. Further, $\partial p/\partial y$ in Equation 18 can be neglected, as the secondary flow is caused by the rotation. In view of the aforementioned facts, we obtain:

$$-2Tv_z = \psi_{zzzz}, \tag{20}$$

$$2T\left(\psi_z + 1\right) = v_{zz}. \tag{21}$$

Equations 20 and 21 are subjected to the following boundary conditions:

$$\left.\begin{aligned}\psi_{zz}\left(0\right) &= 0\\ v_z\left(0\right) &= 0\end{aligned}\right\}\text{symmetry condition} \tag{22}$$

$$\left.\begin{aligned}\psi_z\left(h\right) &= -1\\ v\left(h\right) &= 0\end{aligned}\right\}\text{no-slip condition}, \tag{23}$$

where $h = 1 + \phi\sin\left(2\pi x\right)$ and $\phi = b/a$ is the amplitude ratio.

To obtain the remaining two boundary conditions, we proceed as follows. We define the instantaneous volume flow rate in the laboratory frame in the $\bar{X}$ -direction as:

$$Q_1 = \int_0^{\bar{h}}\bar{U}\left(\bar{X},\bar{Z},t\right)d\bar{Z}. \tag{24}$$

The counterpart of the expession (Equation 23) in the moving frame becomes:

$$q_1 = \int_0^{\bar{h}}\bar{u}\left(\bar{x},\bar{z},t\right)d\bar{z}. \tag{25}$$

The two flow rates can be related through the following equation:

$$Q_1 = q_1 + c\bar{h}. \tag{26}$$

The time-averaged flow over a period T at a fixed position $\overline{X}$ is given by:

$$\bar{Q}_1 = \frac{1}{T}\int_0^T Q_1 dt \cdot \qquad (27)$$

Utilizing the expression of equation 26 in Equation 27, we get:

$$\bar{Q}_1 = q_1 + ac. \qquad (28)$$

If we define $\Theta\left(=\bar{Q}_1/ac\right)$ and $F_1\left(=q_1/ac\right)$ as time-mean flow rates in the laboratory and the moving frames in the direction of peristaltic wave, then we can write Equation 28 as:

$$\Theta = F_1 + 1, \qquad (29)$$

where

$$F_1 = \int_0^h \frac{\partial \psi}{\partial z} dz = \psi(h) - \psi(0). \qquad (30)$$

Equation 30 suggests that we can choose the remaining two boundary conditions as:

$$\left.\begin{aligned} \psi(0) &= 0 \\ \psi(h) &= F_1 \end{aligned}\right\}. \qquad (31)$$

The physical quantities of interest, such as the pressure rise per wavelength Δp and the dimensionless flow rate F_2, due to secondary velocity are defined, respectively, as:

$$\Delta p = \int_0^1 \frac{dp}{dx} dx, \qquad (32)$$

$$F_2 = \int_0^h v dz. \qquad (33)$$

SOLUTION OF THE PROBLEM

The integration of Equations 21 and 22 with respect to z and the utilization of the boundary conditions of Equations 22, 23 and 31 leads to the following unknowns functions ψ and v.

$$\begin{aligned}\psi = &\left\{\sinh\left(2\sqrt{T}h\right) - \sin\left(2\sqrt{T}h\right)\right\}^{-1}\left\{z\left(\sin\left(2\sqrt{T}h\right) - \sinh\left(2\sqrt{T}h\right)\right)\right. \\ &+2\left(F_1+h\right)\left(\sin\left(\sqrt{T}h\right)\sinh\left(\sqrt{T}h\right)\right)\left(\cosh\left(\sqrt{T}z\right)\sin\left(\sqrt{T}z\right) + \cos\left(\sqrt{T}z\right)\right. \\ &\left.\sinh\left(\sqrt{T}z\right)\right) - (1+i)\left(F_1+h\right)\cos\left(\sqrt{T}h\right)\cosh\left(\sqrt{T}h\right)\left(\sin\left((1+i)\sqrt{T}z\right)\right. \\ &\left.\left.-\sinh\left((1+i)\sqrt{T}z\right)\right)\right\},\end{aligned} \qquad (34)$$

$$\begin{aligned}v = &4\sqrt{T}\left(F_1+h\right)\left\{\sin\left(2\sqrt{T}h\right) - \sinh\left(2\sqrt{T}h\right)\right\}^{-1}\left\{\cos^2\left(\sqrt{T}h\right)\left\{\cosh^2\left(\sqrt{T}h\right)\right.\right. \\ &-\cos\left(\sqrt{T}h\right)\cos\left(\sqrt{T}z\right)\cosh\left(\sqrt{T}h\right)\cosh\left(\sqrt{T}z\right) + \sin\left(\sqrt{T}h\right)\sinh\left(\sqrt{T}h\right) \\ &\left.\left.\left(\sin\left(\sqrt{T}h\right)\sinh\left(\sqrt{T}h\right) - \sin\left(\sqrt{T}z\right)\sinh\left(\sqrt{T}z\right)\right)\right\}\right\}.\end{aligned} \qquad (35)$$

With the help of Equations 34 and 35, we obtain the expressions for dp/dx and F_2 as:

$$\frac{dp}{dx} = \frac{4T^{3/2}\left(F+h\right)\left\{\cos\left(2\sqrt{T}h\right) + \cosh\left(2\sqrt{T}h\right)\right\}}{\sin\left(2\sqrt{T}h\right) - \sinh\left(2\sqrt{T}h\right)}, \qquad (36)$$

$$\begin{aligned}F_2 = &\left\{\sin\left(2\sqrt{T}h\right) - \sinh\left(2\sqrt{T}h\right)\right\}^{-1}\left\{\left(F_1+h\right)\left(2\sqrt{T}h\cos\left(2\sqrt{T}h\right)\right.\right. \\ &\left.\left.+2\sqrt{T}h\cosh\left(2\sqrt{T}h\right) - \sin\left(2\sqrt{T}h\right) - \sinh\left(2\sqrt{T}h\right)\right)\right\}.\end{aligned} \qquad (37)$$

It is worth mentioning that in the limiting case, $T \to 0$, we recover the results for the peristaltic flow in non-rotating/fixed frame.

RESULTS

The present study is performed with a view in mind to analyze the effects of rotation on the peristaltic flow of Newtonian fluid in a rotating frame. In the following, we list the main results obtained from the present study.

(1) In contrast to the non-rotating case, the rotation in the present case induces a secondary component of velocity which in turn produces a net flow of fluid in $y-$direction. The magnitude of this net flow induced in $y-$direction increases with an increase in rotation.
(2) The pressure rise per wavelength increases by increasing rotation.
(3) The size of bolus of trapped fluid increases for large values of rotation.

DISCUSSION

Here, we discuss the effects of Taylor number (T) on the pressure rise per wavelength (Δp), the dimensionless flow rate due to the secondary velocity (F_2) and the trapping phenomenon with the help of Figures 2 to 5. Figure 2 presents the variation of Δp with Θ for different values of T. It is observed that the maximum pressure against which peristalsis work as a positive displacement pump (that is, Δp for $\Theta = 0$) decreases by increasing T. We further note that if one keeps increasing T, Δp become negative for all positive values of Θ.

Thus, for a fluid in a frame rotating with large angular

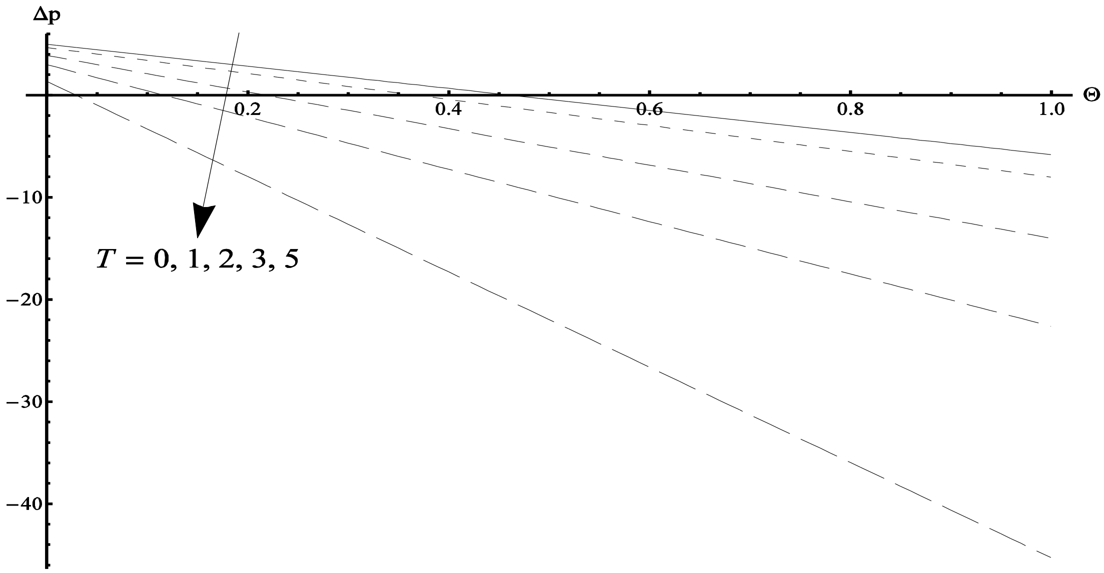

Figure 2. The pressure rise per wavelength Δp plotted against Θ for different values of T with $\phi = 0.6$.

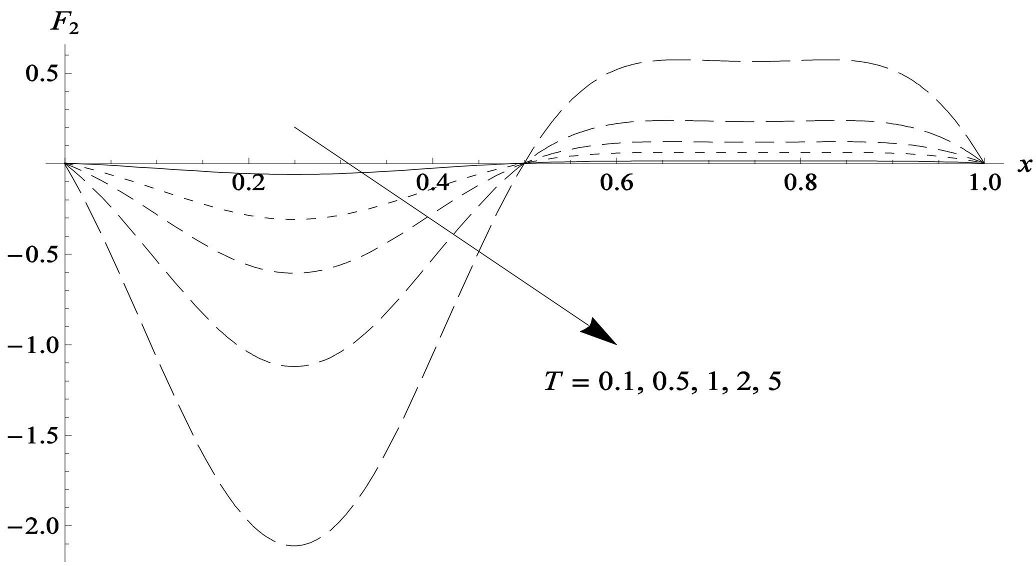

Figure 3. The dimensionless flow rate due to secondary velocity F_2 for different values of T with $\phi = 0.6$ and $F_1 = -1$.

speed pressure does not rise against the direction of the peristaltic wave. It implies that the pressure assists the flow in such a case. In fact, the rotation of the channel produces a negative secondary component of the velocity v, which in turn, is responsible for Coriolis force acting on the fluid in the positive y-direction. This force pulls the fluid outwards and thus, it reduces the pressure rise.

The effects of T on F_2 can be visualized through Figures 3 and 4. From Figure 3, it is evident that when $F_1 = -1$, then F_2 is negative in the wider part of the channel; otherwise, it is positive. However, as F_1 increases F_2 becomes negative over the whole wavelength of the channel (Figure 4). It turns out during computations that the cumulative value of F_2, over one wavelength, remains negative for all values of $F_1 > -1$.

Figure 5 shows the influence of the rotation of the channel on the trapping phenomenon. In the absence

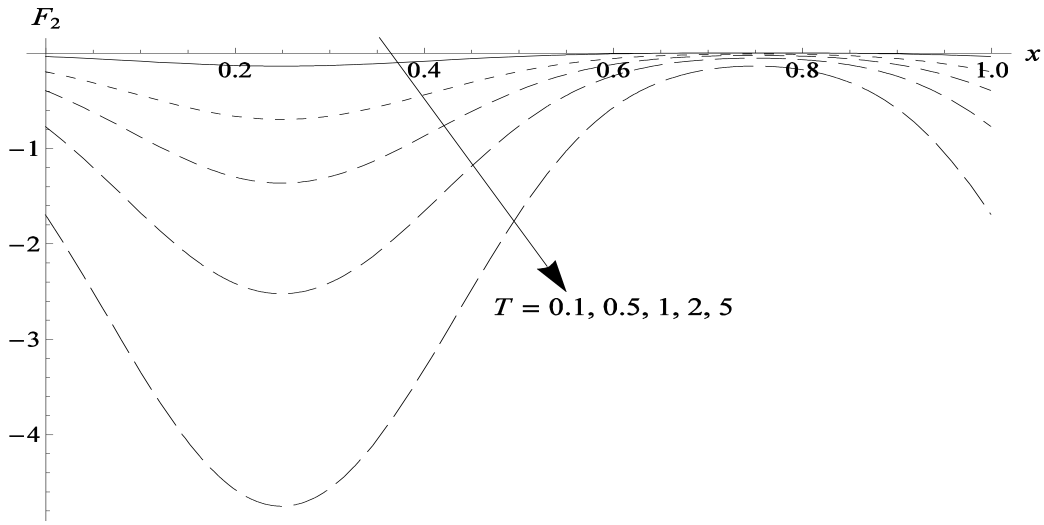

Figure 4. The dimensionless flow rate due to secondary velocity F_2 for different values of T with $\phi = 0.6$ and $F_1 = -0.5$.

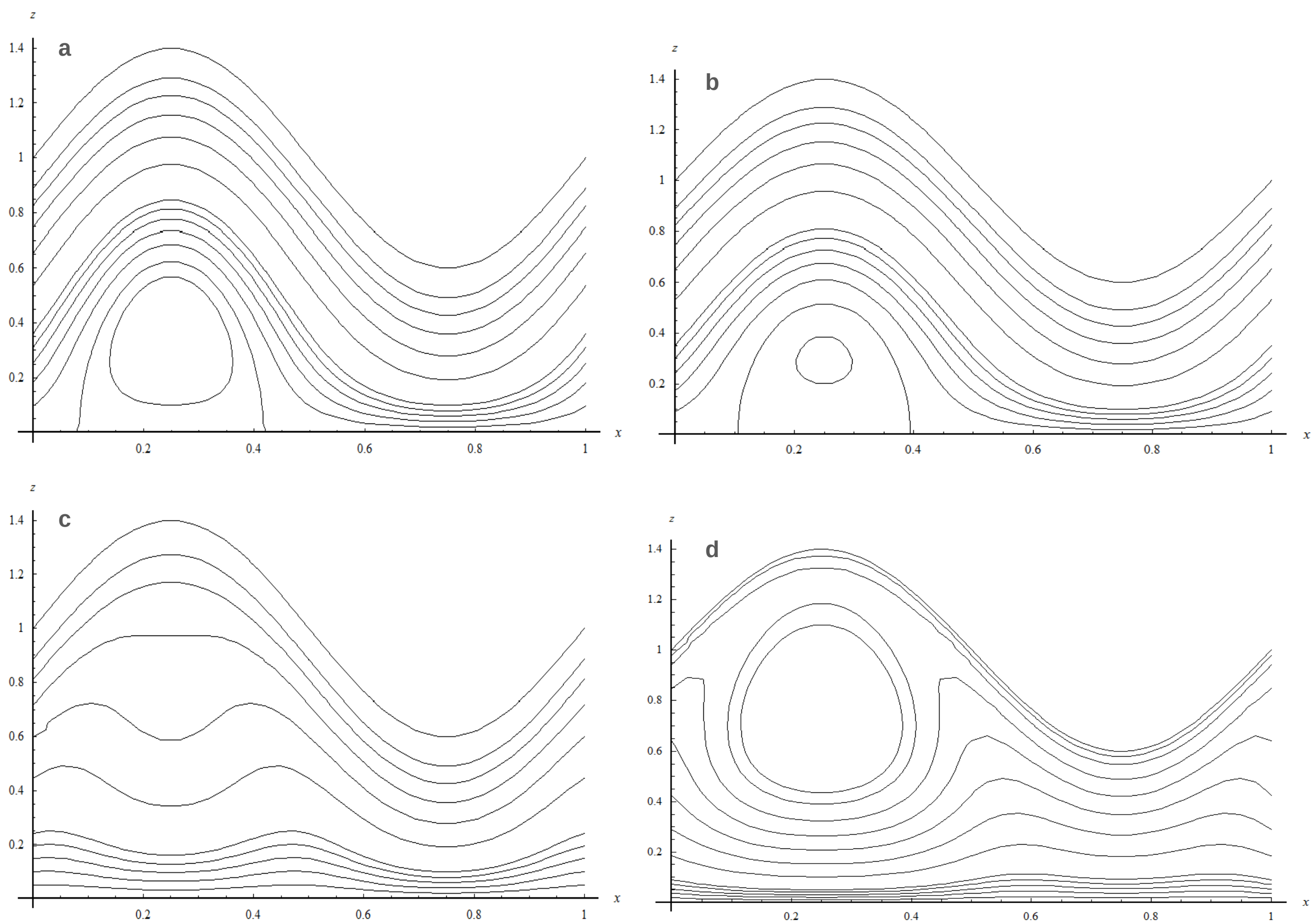

Figure 5. Streamlines for different values of T. Panels (a to c) correspond to $T = 0, 0.5, 2, 5$. The other parameters are $\phi = 0.4$ and $\Theta = 0.6$

of rotation, a trapped bolus exists about the centre streamline. But as T increases, its size reduces and eventually it disappears. A further increase in T confirms its reappearance near the boundary wall.

Conclusions

In this paper, peristaltic flow of a viscous fluid in a rotating frame was studied. Exact expressions for the stream function, the secondary component of velocity, the pressure gradient and the dimensionless flow rate due to the secondary velocity were presented. The effects of the rotation parameter on pumping and trapping phenomena were discussed. It was found that an increase in rotation reduces the pressure rise against the direction of peristaltic wave making it favourable from adverse. Further, an increase in rotation reduces the size of the trapped bolus and shifts it toward the boundary. It is worth mentioning that this type of investigation of peristaltic flow has not been done before.

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REFERENCES

Shapiro AH, Jaffrin MY, Weinberg SL (1969). Peristaltic pumping with long wavelength at low Reynolds number. J. Fluid Mech. 37:799-825.
Jaffrin MY, Shapiro AH (1971). Peristaltic pumping. Ann. Rev. Fluid Mech. 3:13-36.
Raju KK, Devanathan R (1974). Peristaltic motion of a non-Newtonian fluid: Part-II. Rheol. Acta 13:944-948.
Radhakrishnamacharya G (1982). Long wave length approximation to peristaltic motion of a power law fluid. Rheol. Acta 21:30-35.
Srivastava LM, Srivastava VP (1984). Peristaltic transport of blood: Casson model II. J. Biomech. 17:821-829.
Siddiqui AM, Schwarz WH (1993). Peristaltic pumping of a third order fluid in a planer channel. Rheol. Acta 32:47-56.
Siddiqui AM, Schwarz WH (1994). Peristaltic flow of a second order fluid in tubes. J. Non-Newtonian Fluid Mech. 53:257-284.
Mekheimer KS, El-Shehawey EF, Alaw AM (1998). Peristaltic motion of a particle fluid suspension in a planar channel. Int. J. Theor. Phys. 37:2895-2920.
Mekheimer KS (2002). Peristaltic Transport of a couple-stress fluid in uniform and non-uniform channels. Biorheology 39:755-765.
Hayat T, Wang Y, Siddiqui AM, Hutter K, Asghar S (2002). Peristaltic transport of a third order fluid in a circular cylindrical tube. Math. Mod. Meth. Appl. Sci. 12:1691-1706.
Wang Y, Hayat T, Hutter K (2007). Peristaltic flow of a Johnson-Segalman fluid through a deformable tube. Theor. Comput. Fluid Dyn. 21:369-380.
Hayat T, Ali N, Asghar S (2007). Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Phys. Lett. A 363:397-403.
Haroun MH (2007a). Effect of Deborah number and phase difference on peristaltic transport of a third-order fluid in an asymmetric channel. Comm. Non-lin. Sci. Numer. Simm. 8:1464-1480.
Haroun MH (2007b). Non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Comput.. Mat. Sci. 39:324-333.
Kothandapani M, Srinivas S (2008a). Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Int. J. Nonlin. Mech. 43:915-924.
Kothandapani M, Srinivas S (2008b). Non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel through a porous medium. Phys. Lett. A 372:1265-1276.