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Full Length Research Paper

Flight PID controller design for a UAV quadrotor

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This paper presents the modeling of a four rotor vertical take-off and landing (VTOL) unmanned air vehicle known as the quad rotor aircraft. The paper presents a new model design method for the flight control of an autonomous quad rotor. The paper describes the controller architecture for the quad rotor as well. The dynamic model of the quad-rotor, which is an under actuated aircraft with fixed four pitch angle rotors was described. The Modeling of a quad rotor vehicle is not an easy task because of its complex structure. The aim is to develop a model of the vehicle as realistic as possible. The model is used to design a stable and accurate controller. This paper explains the developments of a PID (proportional-integral-derivative) control method to obtain stability in flying the Quad-rotor flying object. The model has four input forces which are basically the thrust provided by each propeller connected to each rotor with fixed angle. Forward (backward) motion is maintained by increasing (decreasing) speed of front (rear) rotor speed while decreasing (increasing) rear (front) rotor speed simultaneously which means changing the pitch angle. Left and right motion is accomplished by changing roll angle by the same way. The front and rear motors rotate counter-clockwise while other motors rotate clockwise so that the yaw command is derived by increasing (decreasing) counter-clockwise motors speed while decreasing (increasing) clockwise motor speeds.

Key words: Quadrotor, proportional-integral-derivative (PID) controller, vertical take-off and landing (VTOL), unmanned aerial vehicles (UAV), MATLAB / Simulink.

INTRODUCTION

UAVs or ‘Unmanned Aerial Vehicles,’ are defined as aircrafts without the onboard presence of pilots (Gene et al., 1997). UAVs have been used to perform intelligence, surveillance, and reconnaissance missions. The technological promise of UAVs is to serve across the full range of missions. UAVs have several basic advantages over manned systems including increased maneuverability, reduced cost, reduced radar signatures, longer endurance, and less risk to crews. Vertical take-off and landing type UAVs exhibit even further maneuverability features. Such vehicles are to require little human intervention from take-off to landing. Unmanned aerial vehicles (UAVs) have potential for full-filling many civil and military applications including surveillance, intervention in hostile environments, air pollution monitoring, and area mapping (Castillo et al., 2005).

Unmanned aerial vehicles (UAV) have shown a growing interest thanks to recent technological projections, especially those related to instrumentation. They made possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost.

In this paper, we studied the behavior of the quadrotor. This flying robot presents the main advantage of having quite simple dynamic features. Indeed, the quadrotor is a small vehicle with four propellers placed around a main body.

The main body includes power source and control hardware. The four rotors are used in controlling the vehicle. The rotational speeds of the four rotors are

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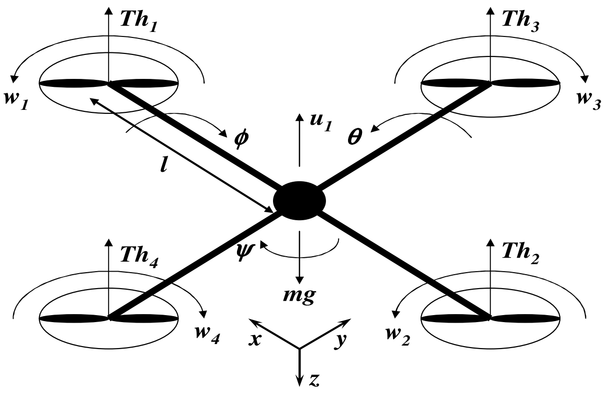

Figure 1. The quadrotor schematic.

independent. Thanks to this independence, it is possible to control the pitch, roll and yaw attitude of the vehicle. Then, its displacement is produced by the total thrust of the four rotors whose direction varies according to the attitude of the quadrotor. The vehicle motion can thus be controlled.

There have been numerous projects involving quadrotors to date, with the first known hover occurring in October, 1922 (Lambermont, 1958). Recent interest in the quadrotor concept had been sparked by commercial remote control versions, such as the DraganFlyer IV (Srikanth et al., 2009). Many groups (Pounds et al., 2002; Altug et al., 2003; Bouabdallah et al., 2004; Dzul et al., 2004) have seen significant success in developing autonomous quadrotor vehicles.

Nowadays, the mini-drones invade several application domains (Hamel et al., 2002). Safety (monitoring of the airspace, urban and interurban traffic); natural risk management (monitoring of volcano activities); environmental protection (measurement of air pollution and forest monitoring); intervention in hostile sites (radioactive workspace and mine clearance), management of the large infrastructures (dams, high-tension lines and pipelines), agriculture and film production (aerial shooting)

In contrast to terrestrial mobile robots for which it is often possible to limit the model to kinematics, the control of aerial robots (quadrotor) requires dynamics in order to account for gravity effects and aerodynamic forces (Guenard et al., 2004). Since Quad-rotor Flying robots use four rotors instead of one rotor to provide thrust to the robot, it has four input forces and six output coordinates; Thus the payload capacity is larger compare to conventional helicopter. This enables the quad-rotor flying robot to carry heavier weights (Lozano et al., 2002).

Most of the quad-rotor flying robots change its direction by manipulation of the individual rotor's speed and does not require cyclic and collective pitch control; the mechanical design is hence simpler and consequently reduces the production cost of the flying robot (Nelson, 1997), however, there are a few shortcomings. Space requirement for the four rotors and the power consumption are the main drawbacks of this design. Four rotors with cross frame configuration are definitely consuming more space compare to the conventional UAVs which have only one rotor (Padfield, 1996).

High power consumption is due to the use of four motors as actuators of the flying robot. Although the minimal cross-coupling simplifies the quad-rotor dynamics, the dynamics of the quad-rotor and specifically its low rate damping can make the vehicle difficult to control. The challenge of controlling the vehicle can be even more difficult for a small, low cost flying vehicle (Stone, 2002).

In general, existing quadrotor dynamic models are developed on the hypothesis of a unique rigid body which is a restrictive hypothesis that does not account for the fact that the system is composed of five rigid bodies: Four rotors and a crossing body frame. This makes the explanation of several aspects, like gyroscopic effects, very difficult. Additionally, simplification hypotheses are generally introduced early in the model development and leads in general to misleading interpretations.

MATHEMATICAL MODELLING

A quadrotor is an under actuated aircraft with fixed pitch angle four rotors as shown in (Figure 1). Modeling a vehicle such as a quadrotor is not an easy task because of its complex structure. The aim is to develop a model of the vehicle as realistically as possible.

In the quadrotor, there are four rotors with fixed angles which represent four input forces that are basically the thrust generated by each propeller as shown in Figure 1. The collective input (u_1) is the sum of the thrusts of each motor. Pitch movement is obtained by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. The roll movement is obtained similarly by increasing (reducing) the speed of the right motor while reducing (increasing) the speed of the left motor. The yaw movement is obtained by increasing (decreasing) the speed of the front and rear motors together while decreasing (increasing) the speed of the lateral motors together. This should be done while keeping the total thrust constant. Each of the controller inputs affects certain side of the quadrotor model; u_2 here affects the rotation in the roll angle while u_3 affect the pitch angle and u_4 control the yaw angle during the flying process and u_1 affect the altitude (z-axis) for this model.

Each rotor produces moments as well as vertical forces. These moments have been experimentally observed to be linearly dependent on the forces for low speeds. There are four input forces and six output states (x, y, z, θ, ψ, φ). Therefore, the quadrotor is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counterclockwise, in order to balance the moments and produce yaw motions as needed.

The compensation of this torque in the center of gravity is established thanks to the use of contra rotating rotors 1 to 3 and 2 to 4. Recall that rotors 2 and 4 turn counterclockwise while rotors 1 and 3 turn clockwise.

In order to move the quadrotor model from the earth to a fixed point in the space, the mathematical design should depend on the direction cosine matrix as in Equation (1)

$$Rzxy = \begin{bmatrix} C_\varphi C_\theta & C_\varphi S_\theta S_\psi - S_\varphi C_\psi & C_\varphi S_\theta C_\psi + S_\varphi S_\psi \\ C_\varphi S_\theta & S_\varphi S_\theta S_\psi + C_\varphi C_\psi & S_\varphi S_\theta C_\psi - C_\varphi S_\psi \\ -S_\theta & C_\theta S_\psi & C_\theta C_\psi \end{bmatrix} \quad (1)$$

where $S_\theta = \text{Sin}(\theta)$, $C_\psi = \text{Cos}(\psi)$, etc., and R is the matrix transformation.

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The dynamic model of the quadrotor helicopter can be obtained via a Lagrange approach and a simplified model is given (Altug et al., 2002).

The equations of motion can be written using the force and moment balance [Equation (2)].

$$\left.\begin{aligned} \ddot{x} &= u_1(Cos\varphi Sin\theta Cos\psi + Sin\varphi Sin\psi) - K_1\dot{x}/m \\ \ddot{y} &= u_1(Sin\varphi Sin\psi Cos\psi - Cos\varphi Sin\psi) - K_2\dot{y}/m \\ \ddot{z} &= u_1(Cos\varphi Cos\psi) - g - K_3\dot{z}/m \end{aligned}\right\} \quad (2)$$

The K_i's given above are the drag coefficients. In the following, we assume the drag is zero, since drag is negligible at low speeds.

As the center of gravity moves up (or down) d units, the angular acceleration becomes less sensitive to the forces, therefore stability is increased. Stability can also be increased by tilting the rotor forces towards the center. This will decrease the roll and pitch moments as well as the total vertical thrust.

For convenience, we defined the inputs as shown in Equation (3):

$$\left.\begin{aligned} U_1 &= (\, Th_1 + Th_2 + Th_3 + Th_4 \,)\,/\,m \\ U_2 &= l\,(\, -Th_1 - Th_2 + Th_3 + Th_4 \,)\,/\,I_1 \\ U_3 &= l\,(\, -Th_1 + Th_2 + Th_3 - Th_4 \,)\,/\,I_2 \\ U_4 &= C\,(\, Th_1 + Th_2 + Th_3 + Th_4 \,)\,/\,I_3 \end{aligned}\right\} \quad (3)$$

Where *Th$_i$*'s are thrusts generated by four rotors and can be considered as the real control inputs to the system, C the force to moment scaling factor, and I_i's are the moment of inertia with respect to the axes.

Therefore the equations of Euler angles become:

$$\left.\begin{aligned} \ddot{\theta} &= u_2 - lK_4\dot{\theta}/I_1 \\ \ddot{\psi} &= u_3 - lK_5\dot{\psi}/I_2 \\ \ddot{\varphi} &= u_1 - K_6\dot{\varphi}/I_3 \end{aligned}\right\} \quad (4)$$

where (x, y, z) are three positions; θ, φ ,ψ three Euler angles representing pitch, roll and yaw, respectively; g the acceleration of gravity; l the half length of the helicopter; m the total mass of the helicopter; I_i's the moments of inertia with respect to the axes, and K_i's, the drag coefficients.

This quadrotor helicopter model has six outputs (x, y, z, θ, ψ, φ) while it only has four independent inputs, therefore the quadrotor is an under-actuated system. We are not able to control all of the states at the same time. A possible combination of controlled outputs can be x, y, z and φ in order to track the desired positions, move to an arbitrary heading and stabilize the other two angles, which introduces stable zero dynamics into the system (Altug et al., 2002; Pounds et al., 2002). A good controller should be able to reach a desired position and a desired yaw angle while keeping the pitch and roll angles constant.

By applying Pythagoras theorem and implementing some assumptions and cancellations as follows:

1. The quadrotor structure is symmetrical and rigid.
2. The Inertia matrix (I) of the vehicle is very small and to be neglected.
3. The center of mass and o' coincides.
4. The propellers are rigid.
5. Thrust and drag are proportional to the square of the propellers speed.

These above equations have been established assuming that the structure is rigid and the gyroscopic effect resulting from the propellers rotation had been neglected.

The Phi (φ_d) and (ψ_d) can be extracted in the following expressions

$$\theta_d = tan^{-1}\left(\frac{y_d - y}{x_d - x}\right)$$

$$and \quad (5)$$

$$\psi_d = tan^{-1}\left(\frac{z_d - z}{\sqrt{(x_d - x)^2 + (y_d - y)^2}}\right)$$

Figure 2 shows the Pythagoras theorem for Equation (2) By supplying the four motors with the required voltage, the system will be on, the thrust here is directly proportional with these voltages, whenever increasing the voltage, the thrust for the motor increase and vice versa.

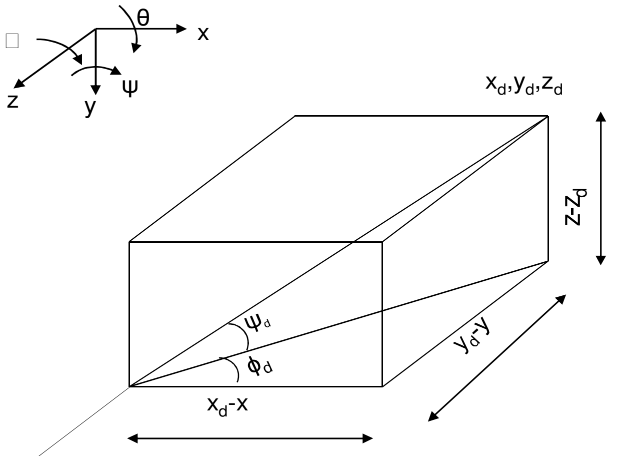

Figure 2. The Quadrotor angles movements.

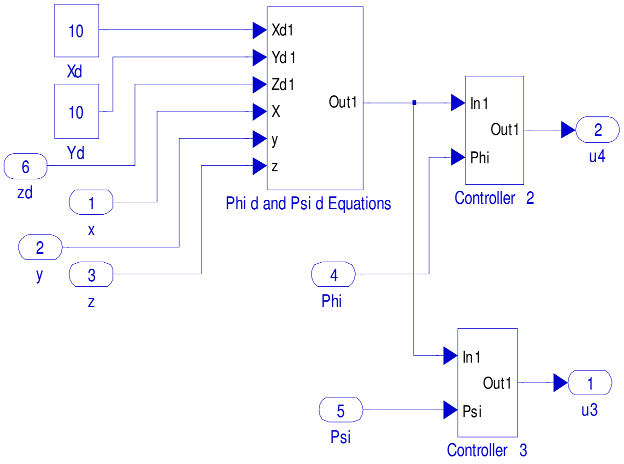

Figure 3. The simulation design for the θd and ψd.

The simulation design for Equation 5 through the MATLAB SIMULINK are shown in Figure 3.

PID CONTROL DESIGN

In this paper, the PID controller for the quadrotor is developed based on the fast response. Using this approach as a recursive algorithm for the control-laws synthesis, all the calculation stages concerning the tracking errors are simplified.

One other aspect of the controller selection depends on the method of control of the UAV. It can be mode-based or non-mode based. For the mode based, controller, independent controllers for each state are needed, and a higher level controller decides how these interact. On the other hand for a non-mode based controller, a single controller controls all of the states together.

However the adopted control strategy is summarized in the control of two subsystems; the first relates to the position control while the second is that of the attitude control.

The quadrotor model above can be divided into two subsystems: A fully-actuated subsystem S1 that provides the dynamics of the vertical position z and the yaw angle (z and ψ). In order to make it possible to design multiple PID controllers for this system, can neglect the gyroscopic effects and thus remove any cross coupling between the parameters (Samir et al., 2004).

$$\begin{bmatrix}\ddot{z}\\ \ddot{\varphi}\end{bmatrix}=\begin{bmatrix}u_1 Cos\varphi Cos\psi)-g\\ u_4\end{bmatrix}+\begin{bmatrix}-K_3\dot{z}/m\\ -K_6\dot{\varphi}/I_3\end{bmatrix}\tag{6}$$

An underactuated subsystem S2 representing the underactuated subsystem which gives the dynamic relation of the horizontal positions (x, y) with the pitch and roll angles as shown down in Equations (7) and (8) respectively.

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}=\begin{bmatrix}u_1 Cos\varphi & u_1 Sin\varphi\\ u_1 Sin\varphi & -u_1 Cos\varphi\end{bmatrix}\begin{bmatrix}Sin\theta Cos\psi\\ Sin\psi\end{bmatrix}+\begin{bmatrix}-K_1\dot{x}/m\\ -K_2\dot{y}/m\end{bmatrix}\tag{7}$$

and

$$\begin{bmatrix}\ddot{\theta}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}u_2\\ u_3\end{bmatrix}+\begin{bmatrix}-lK_4\dot{\theta}/I_1\\ -lK_5\dot{\psi}/I_2\end{bmatrix}\tag{8}$$

Since drag is very small at low speeds, the drag terms in the above equations can be considered as small disturbances to the system so all the nonlinear parts of Equations 6 and 7 are neglected.

The PID control is applied to the equations above with inputs u_1, u_2, u_3, u_4 and outputsϕ, θ, ψ and Z_d. Though these methods were rather successful in local analysis of nonlinear systems affine in control they usually fail to work for a global analysis and nonlinear systems that are non-affine in control (Olfati-Saber, 2001).

For the fully-actuated subsystem, we can construct a rate bounded PID controllers to move states (z,ϕ, θ, ψ) to their desired values. The Zegler Nichols first method was used for tuning of the PID controller (Brian, 2008), as shown in Table 1.

RESULTS AND SIMULATION STUDY

The nominal parameters and the initial conditions of the quadrotor for simulation are shown in Table 2. The proposed control algorithm, as shown in Figure 4 is composed of all controllers, inputs, speed reference and the inner relationships of the thrust.

The quadrotor system is supplied by a step function for

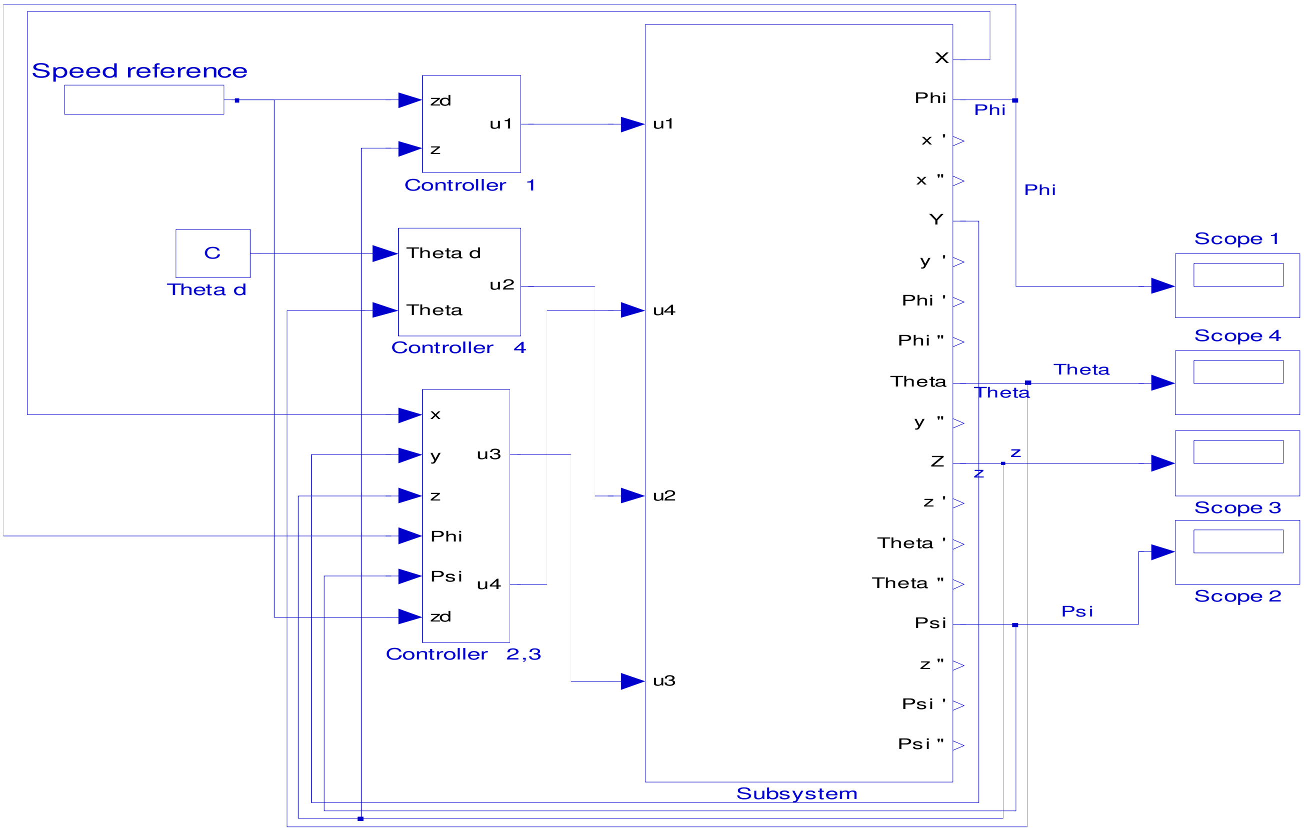

Figure 4. The final simulation model with the PID controllers for the quadrotor.

Table 1. The Ziegler Nichols tuning value.

PID parameter	KP	Kp/Ki	Kd/Kp
P	Time constant/delay time	∞	0
PI	0.9*TC/delay time	Delay time/0.3	0
PID	1.2*TC/delay time	2*delay time	0.5*delay time

the altitude and (z-axis) which is subject to the three step inputs at (3, 10, 20) and the response yields as can be seen in Figure 5 which is contains some transient overshot and another for the Yaw angle (ψ) which is subjected to step input after 5 s as shown in Figure 7 and the roll angle (φ) which respond after 3 s as it can be seen in Figure 6; the pitch angle response is shown in Figure 8 with 5% overshot when subjected to step input. These transient perturbations are due to many reasons such as certain of some mechanical parameters in the design and the simplification of controller design.

The simulation results show that the PID controllers are able to robustly stabilize the quadrotor helicopter and move it to a desired position with a desired yaw angle while keeping the pitch and the roll angles zero, and here in this design, it is easy and with a fast response time, can get the Theta (Pitch angle) to its desired value. The reason for using the PID controllers in this system is to control z, which is sensitive to the changes for the other parameters,

Through using the proposed PID controller method strategy, the good performance can be shown from the speed of response of the quadrotor; although the overshoot in the altitude response was removed, the transient response of the system became faster. The same speed of response can be also seen in the yaw, pitch and roll angles control of Figures 6 to 8.

Conclusion

This paper presented the design of a PID controller algorithm to control the quadrotor system. The model of the vehicle was first modified to simplify the controller

Table 2. The parameters and the initial condition for quadrotor.

Parameter	Value	Unit
I_1	1.25	N_s^2/rad
I_2	1.25	N_s^2/rad
I_3	2.5	N_s^2/rad
K_1	0.010	N_s^2/m
K_2	0.010	N_s^2/m
K_3	0.010	N_s^2/m
K_4	0.012	N_s/rad
K_5	0.012	N_s/rad
K_6	0.012	N_s/rad
m	2	kg
l	0.2	m
G	9.8	m/s^2

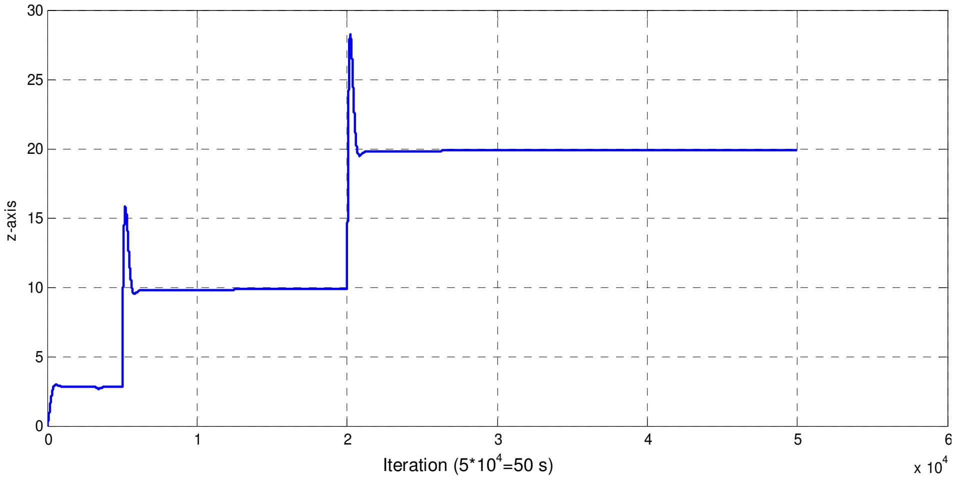

Figure 5. Plot drawing represent the z-axis moving to the desired z-point.

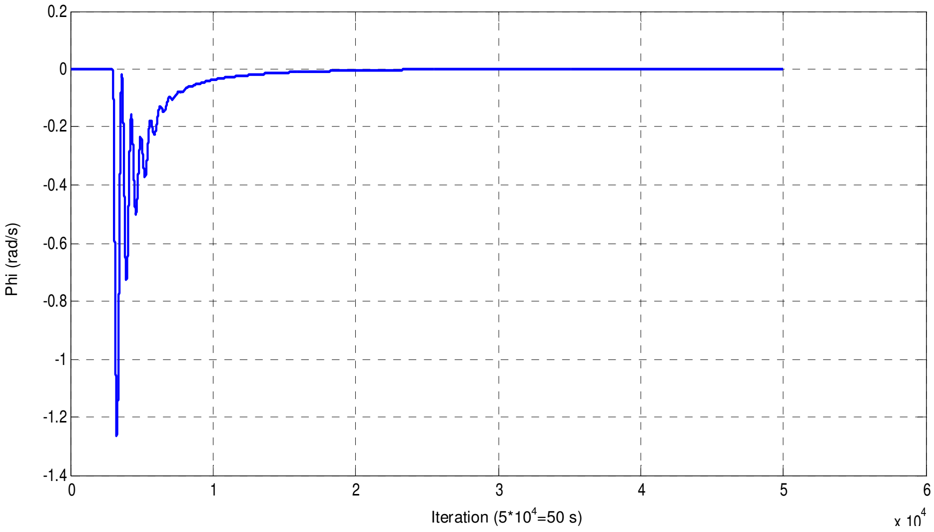

Figure 6. Plot drawing represent the Phi (Roll) angle after 3 seconds to start moving to the desired point.

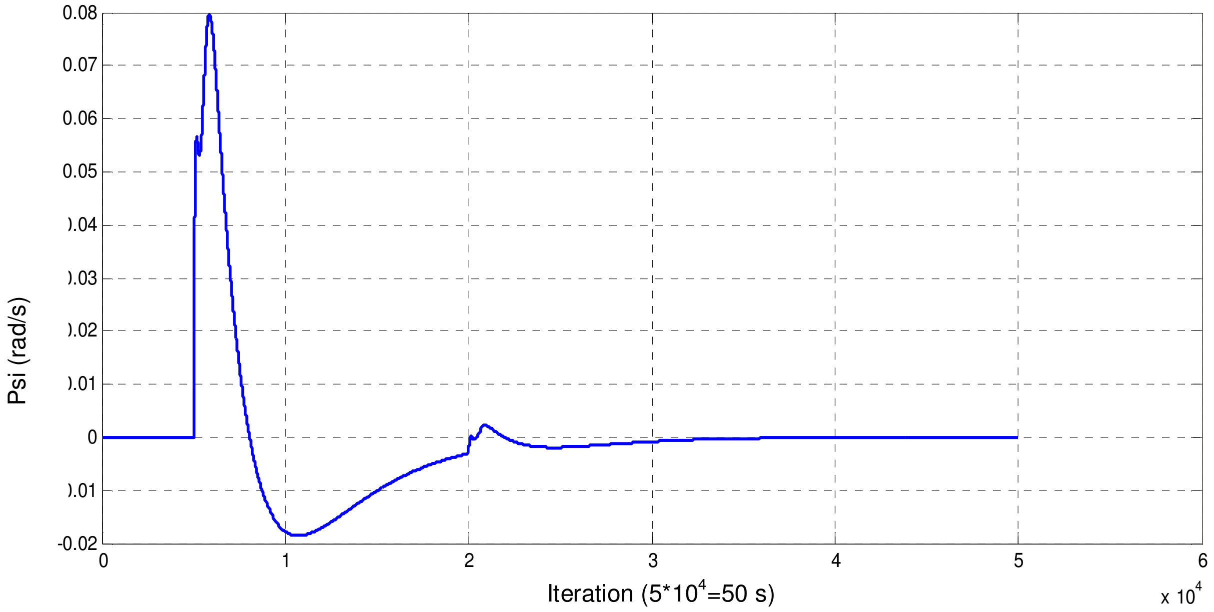

Figure 7. Plot drawing represent the Psi (Yaw) angle after 5 seconds to start moving to the desired point.

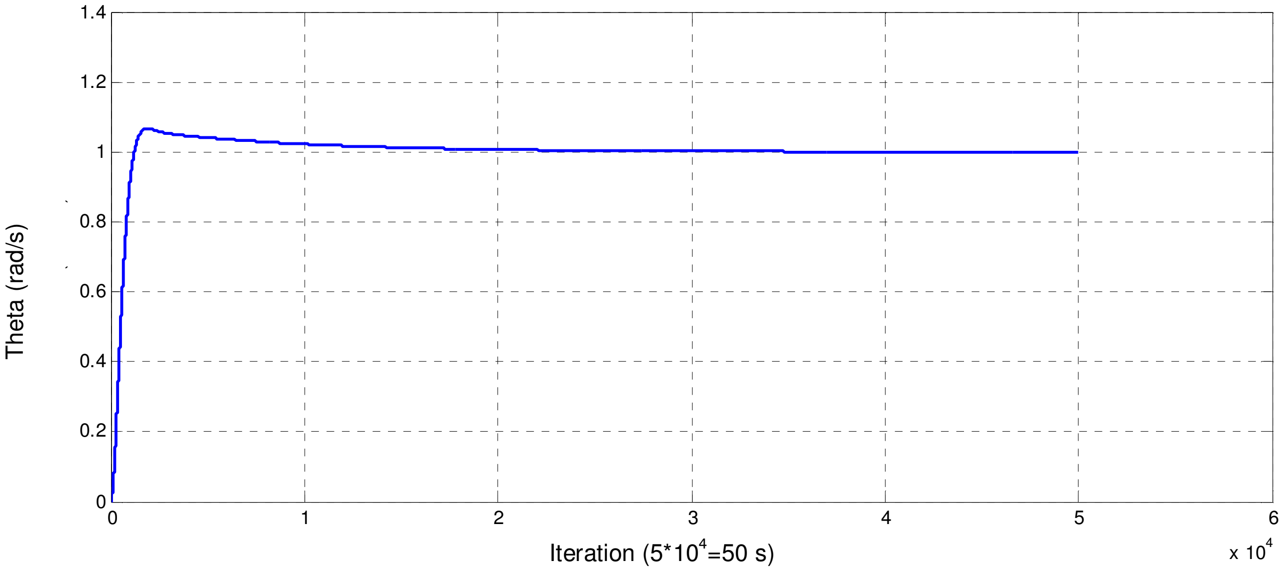

Figure 8. Plot drawing represent the Theta (Pitch) angle start moving to the desired point.

design; a different state space representation was described in the paper.

The resulting system and controller mathematical models were converted to their respective Simulink models for ease of simulations and studies of the system. These resulting Simulink models are ready to be used now by other researchers as the literature does not clearly explain modeling of the quadrotor or supply a working model and controller.

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