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An approximation to the stress distribution analysis for anisotropic clayey soil

M. A. Tekinsoy², T. Taşkıran¹, C. Kayadelen^{3*} and T. Baran⁴

¹Department of Civil Engineering, Dicle University, Diyarbakır, Turkey.

²Department of Civil Engineering, Çukurova University, 01330, Adana, Turkey.

³Department of Civil Engineering, KSU., 51100, Kahramanmaraş, Turkey

⁴Department of Civil Engineering, Çukurova University. 01330, Adana, Turkey.

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Although estimation of a stress distribution in a soil mass assumed as an anisotropic medium gives more realistic results but it is impractical since many elastic parameters are required to be measured. Possibility of more practical way of estimation of a stress distribution, including cross-anisotropy in a different and simpler manner is investigated in this research. In this study a new anisotropic parameter, s , was presented into the theoretical deduction. To see the performance of the proposed method, five independent cross-anisotropic elastic parameters such as E_v , E_H , ν_{vH} , ν_{HH} , G_{vH} and anisotropic parameter, s , of a clayey soil were measured and stress distributions under the center of uniformly distributed circular area were portrayed for the same soil. In this frame, Finite Element Method (with cross-anisotropic linear elastic material model), the proposed method, Boussinesq and Westergaard solutions were carried out and results were compared. It is found that the stresses obtained by the proposed way are compatible with the other solutions and also the stresses agreed with that of FEM analysis beneath a certain depth.

Key words: Anisotropic elastic parameters, anisotropic parameter, stress distribution.

INTRODUCTION

A load-acting on a soil surface, creates stress within the soil mass. Estimation of the stress induced in a soil mass, at a point of a particular depth, is one of the major problems in the geotechnical analysis.

Limitations and assumptions involved with the use of an elastic model for soils are well known. When the stresses are lower than failure value, the stress-strain relationship for most soils can be assumed to be linear.

According to many experimental studies, the soils are accepted as a cross-anisotropic material (Barden, 1963; Hoque et al., 1996; Gazetas, 1981). The fabric of the natural clay deposits are formed by sedimentation and by one-dimensional consolidation over long periods of time. This process causes the fabric units of soils and soil particles to be positioned in horizontal arrangement. It also causes the soils to have dispersed structures (Gazetas, 1982). This preferred orientation and the resul-

ting electrochemical bonds among the clay particles is the cause of cross-anisotropic deformational behavior in clays.

Determination of distribution of stress in cross-anisotropic elastic media induced by various loadings has occupied researchers for many years. In addition to the analytical solutions, FEM solutions are also presented recently for different type of loading and foundation type for anisotropic elastic media (Gerrard and Harrison, 1970; Gerrard, 1977; Gerrard, 1982; Graham and Houlsby, 1983; Keskin, 2004; Kirkgard and Lade, 1991; Lekhnitskii, 1963; Ling et al., 2004; Nayak, 1973).

Five independent elastic parameters given below are required to fully describe cross-anisotropic soil behavior.

E_v = Elastic modulus in z (vertical) direction

E_H = Elastic modulus in x and y direction (horizontal direction)

ν_{vH} = Poisson ratio in horizontal direction induced by vertical direction

ν_{HH} = Poisson ratio in x and y direction induced by stress-

*Corresponding author. E-mail: ckayadelen@ksu.edu.tr.

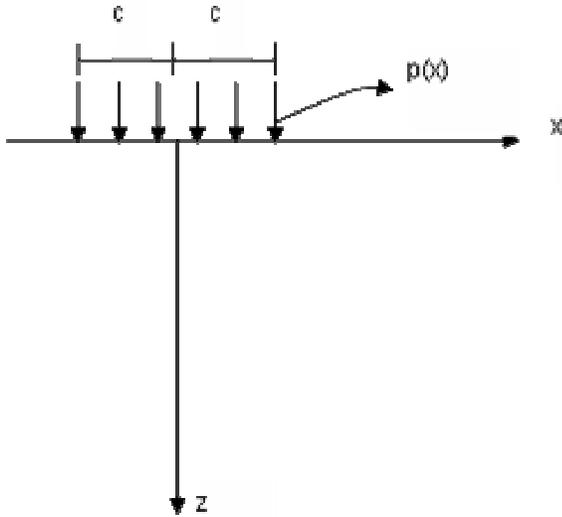


Figure 1. Coordinate axes and concentrated load acted over an infinitely small distance c .

ses on x-y plane

G_{VH} = Shear modulus in x and y direction

Measurement of these elastic parameters is a basic problem when dealing with the cross-anisotropic soils. Investigation for obtaining those five elastic parameters with different equipments or techniques is still lasting (Gareau et al., 2005; Schmertmann, 2005a; Tekinsoy and Laman, 2000; Timashenko and Goodier, 1951; Tsyvich, 1976).

On the other hand, elastic solutions do not predict any dependence to the soil properties (porosity, compacity, etc.) or initial stress state conditions. There are some studies devoted to that subject recently. Dependence of stresses to density (via vibration effect) was studied by Zhu and Clark (1994). They performed laboratory study on the lateral stress in vibrated sand and concluded that lateral stress in sand increases with vibration time. The increase in lateral stress is related to the sand density and vibration time (Wang and Cheung, 2001). Schmertmann (2005) performed a series of plate bearing load tests in a large sand box with boundary lateral stress control showed, by measurements using buried stress cells, that the greater the initial horizontal stress the more rapidly the vertical loading from the plate spreads with depth. As K_i increases (as horizontal stress is increased) the ratio of vertical stress increase and the applied vertical stress ($\Delta\sigma_v/q$) decreases (Westergaard, 1938).

In the content of this study, a simpler parameter which reflects the anisotropic character of a clayey soil was introduced and a solution based on this new representation of anisotropy was produced for estimating stress distribution in a soil mass. Although this kind of representation might be thought as partial representation of anisotropic property, but the solution by this way provides improvement in comparison with the solutions that assumes the soils as isotropic material. To achieve this,

the new quantity called anisotropic parameter, s , is defined as the square root of ratio of vertical volumetric compression index (m_{vz}), to the radial compression index (m_{vr})

$$s = \sqrt{\frac{m_{vz}}{m_{vr}}} \quad (1)$$

The method of the solution for the stress distribution, using the parameter s is presented in the following part.

METHOD OF SOLUTION

Since the soil layers extend in quite large areas, the lateral strain in the soil can be assumed as zero and K_0 conditions are fulfilled. As it is known, K_0 condition does not involve failure, it represents a state of elastic equilibrium of the soil.

At the beginning, the problem would be taken as a plane-strain problem and then it generalized to the three-dimensional case using the axial symmetric property of the problem. Solution was carried out for a concentrated load firstly and then analysis for circular area was obtained by the integration of that concentrated load. Coordinate axes and concentrated load which can be accepted as a uniform load, $p(x)$, acts on an infinitely small distance, c , is shown in Figure 1.

Since the soil extends to infinity in lateral direction, lateral displacements are in negligible order. Thus K_0 conditions will be applied for a solution. Here, ϵ_1 and ϵ_2 denotes principle deformations in vertical and horizontal directions, respectively. Thus, lateral displacement $\epsilon_2 = 0$. Assuming the deformations are small enough and linear, the second deformation invariant can be written as in the following (Yong and Silvestri, 1979).

$$\epsilon_x \cdot \epsilon_z - \frac{\gamma_{xz}^2}{4} = \epsilon_1 \epsilon_2 \quad (2.a)$$

$$\epsilon_x \cdot \epsilon_z - \frac{\gamma_{xz}^2}{4} = 0 \quad (2.b)$$

and γ_{xz} can be regarded as

$$\gamma_{xz} = 2\sqrt{\epsilon_x \epsilon_z} \quad (3)$$

The soils have different coefficients of volume compressibilities in both vertical and horizontal directions and they can be given as

$$m_{vz} = \frac{\Delta z}{z \Delta \sigma_z} \quad \text{and} \quad m_{vx} = \frac{\Delta x}{x \Delta \sigma_x} \quad (4)$$

Taking the deformations $\epsilon_x = \Delta x/x$ and $\epsilon_z = \Delta z/z$, stress increments can be expressed as in the followings

$$\epsilon_x = m_{vx} \Delta \sigma_x \text{ and } \epsilon_z = m_{vz} \Delta \sigma_z \quad (5)$$

In an oedometer test, if the initial thicknesses of the same soil for horizontal and vertical samples are taken as x_0 and z_0 then the stress increments can be shown as σ_x and σ_z as well. Consequently deformations might be stated in terms of total stresses.

$$\epsilon_x = m_{vx} \sigma_x \quad (6.a)$$

$$\epsilon_z = m_{vz} \sigma_z \quad (6.b)$$

If the last equations are substituted into Eq. (3) and assuming the simple shear case ($\sigma_x = \sigma_z = \tau_{xz}$), then the following can be written.

$$\gamma_{xz} = 2\tau_{xz} \sqrt{m_{vx} \cdot m_{vz}} \quad (7)$$

On the other hand, the compatibility condition for the current problem is

$$\frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = \frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} \quad (8)$$

If the values of ϵ_x, ϵ_z and γ_{xz} , as were given in Eq. (6) and in Eq. (7), are replaced into this equality the following compatibility equation is obtained in terms of stresses.

$$2 \sqrt{\frac{m_{vz}}{m_{vx}}} \frac{\partial^2 \tau_{xz}}{\partial x \partial z} = m_{vx} \frac{\partial^2 \sigma_x}{\partial z^2} + m_{vz} \frac{\partial^2 \sigma_z}{\partial x^2} \quad (9a)$$

Considering

$$s = \sqrt{\frac{m_{vz}}{m_{vx}}}$$

the following equation is written.

$$2s \frac{\partial^2 \tau_{xz}}{\partial x \partial z} = \frac{\partial^2 \sigma_x}{\partial z^2} + s^2 \frac{\partial^2 \sigma_z}{\partial x^2} \quad (9b)$$

Here, the parameter, s , represents the cross-anisotropic property of soils and also indicates the stratification which can also be related to their geological formation (Yong and Warkentin, 1975).

The stresses can be expressed in terms of Airy stress function ϕ as;

$$\sigma_x = \frac{\partial^2 \phi}{\partial z^2} \quad \tau_{xz} = -\frac{\partial^2 \phi}{\partial x \partial z} \quad \sigma_z = \frac{\partial^2 \phi}{\partial x^2} \quad (10)$$

If these stress values are substituted into (9b), the following differential equation is obtained.

$$s^2 \frac{\partial^4 \phi}{\partial x^4} + 2s \frac{\partial^4 \phi}{\partial x^2 \partial z^2} + \frac{\partial^4 \phi}{\partial z^4} = 0 \quad (11.a)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{s} \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{s} \frac{\partial^2 \phi}{\partial z^2} \right) = 0 \quad (11.b)$$

Index and mechanical properties of soils remain approximately the same in all lateral directions, which arises from the geological formation of soils. Therefore, the coefficients of volume compressibility can also be taken equal for two perpendicular horizontal directions ($m_{vx} = m_{vy}$) (Zhu and Clark, 1994). On the other hand, since the problem has an axial-symmetry (Figure 1) bi-harmonic equation for three dimensional cases can be written as in the following form

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{s} \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{s} \frac{\partial^2 \phi}{\partial z^2} \right) = 0 \quad (12)$$

where x and y represent the lateral coordinate axes. Again, it will be easier to work in cylindrical coordinates when the axial-symmetric condition presents. Therefore, the following differential equation can also be given for the stress distribution.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{s} \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{s} \frac{\partial^2 \phi}{\partial z^2} \right) = 0 \quad (13)$$

If $Z = \sqrt{s}z$ transformation is used, parameter s is eliminated and a biharmonic equation is obtained.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial Z^2} \right) \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial Z^2} \right) = 0 \quad (14)$$

The solution of Eq. (14) will give the Airy-stress function ϕ as follows (Westergaard, 1938; Yong et al., 1979).

$$\phi = A(r^2 + Z^2)^{1/2} \quad (15)$$

If the stresses are taken as

$$\sigma_z = \frac{1}{s} \frac{\partial}{\partial z} \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} \right) \quad (16a)$$

$$\tau_{rz} = \frac{\partial}{\partial r} \left(\frac{\partial^2 \phi}{\partial r^2} \right) \quad (16b)$$

$$\sigma_r = \frac{\partial}{\partial z} \left(\frac{1}{s} \frac{\partial^2 \phi}{\partial z^2} \right) + \left(\frac{1}{z} \frac{\partial^2 \phi}{r^2} \right) \quad (16c)$$

and the necessary derivations of Eq. (15) are substituted into Eqs 16, then the following stress expressions are obtained as follows

$$\sigma_z = -3As \frac{z^3}{(r^2 + sz^2)^{5/2}} \quad (17a)$$

$$\tau_{rz} = -3As \frac{rz^2}{(r^2 + sz^2)^{5/2}} \quad (17.b)$$

$$\sigma_r = -2A \frac{z}{(r^2 + sz^2)^{3/2}} + 3A \frac{s^2 z^3}{(r^2 + sz^2)^{5/2}} \quad (17.c)$$

In order to find the integration constant A, the following boundary condition can be used (Westergaard, 1938; Yong et al., 1979).

$$P = \int_0^{\infty} 2\pi r \sigma_z dr \quad (18)$$

A is obtained from Eq. (18) as $A = \frac{\sqrt{sP}}{2\pi}$ In this case, the

vertical stress σ_z is found as

$$\sigma_z = \frac{3s^{3/2}P}{2\pi} \frac{z^3}{(r^2 + sz^2)^{5/2}} \quad (19)$$

The found stress σ_z for a concentrated load should be integrated in order to find stress expression for a uniformly distributed load on a circular area. If the equality given in Eq.(19) is integrated (assuming it as a Green Function) between zero and the radius, R, an integral can be given

$$d\sigma_z = \frac{3s^{3/2}}{2\pi} dP \frac{z^3}{(r^2 + sz^2)^{5/2}}$$

$$dP = 2\pi r q_0 dr$$

$$\sigma_z = 3s^{3/2} q_0 z^3 \int_{r=0}^{r=R} \frac{r dr}{(r^2 + sz^2)^{5/2}}$$

Thus, the following stress expression for a circular

loading area is obtained

$$\sigma_z = q_0 \left[1 - \frac{s^{3/2} z^3}{(R^2 + sz^2)^{3/2}} \right] \quad (20)$$

On the other hand as it was mentioned previously, elastic solutions do not predict any dependence with the soil properties but as it can be expected, the stress diffusion in porous media is affected by the amount and geometry of pore spaces and contact mechanism of the skeleton. Investigations have indicated that the magnitude of induced stresses in soil mass is related to the soil density. As the density increases, the stresses induced by external loadings in soil mass increases as well (Barden, 1963; Yong and Warkentin, 1975). This phenomenon was explained by the increase in contact mechanism as the pore space decreases. Under the light of previous investigations, it can be deduced that the "stress transmission" in soils has relation with the compacity, C, and an increasing relationship exists between stress and compacity. However, the sort of relation of "stress" with the compacity can simply be considered as directly proportional. In this respect, the following form of the Equation (20) has seen to have estimated sufficiently proper results and is also used for the present study.

$$\sigma_z = \frac{\gamma_k}{\gamma_s} q_0 \left[1 - \frac{s^{3/2} z^3}{(R^2 + sz^2)^{3/2}} \right] \quad (21)$$

SOIL TESTING PROGRAMME FOR ELASTIC PARAMETERS

In this part of the study, two objectives were aimed. Firstly, it is aimed to measure five independent cross-anisotropic elastic parameters of clay to be able to portray the stress distribution by finite element method assuming the soil as a cross-anisotropic elastic medium. Secondly, to determine anisotropic parameter, s, of the same clay to perform stress distribution by the above proposed method.

Samples employed in this study were obtained from a trench excavation at a site located at the south part of Yenice district in Tarsus city in Turkey. The site is characterized by an alluvium called Çukurova deposited by Seyhan and Tarsus rivers, from Quaternary up to present. The deposit is mainly formed by composition of clay, silt and sand in varying proportions. Underground water table level is at about 1.5 m. Representative geotechnical properties for the samples taken from the trench are summarized in Table 1.

The type of clay is CH and it has greenish-brown color. Cylindrical block samples, with the thin wall steel tube having 2.5 mm thickness and 25 cm diameter and length, were taken from the depth of 2.1 m utilizing hydraulic

Table 1. Soil classification data.

Moisture content, %	28-32
Liquid limit, %	60-67
Plasticity index, %	41-45.
Undrained shear strength, S_u , kPa	47-75
No:200 Passing, %	98-99
fraction of fines (clay+silt), %	92



Figure 2. Photograph of test apparatus. 1-Volume change transducer, 2-Constant pressure supplier, 3-Displacement transducer (LVDT), 4-Load cell, 5-Pore pressure transducer, 6-Cell pressure transducer, 7-Soil specimen, 8-Data logger.

jack. Prior to sampling, the excavation level was lowered by approximately 10 cm. using hand tools to avoid any disturbed soils at the bottom. Sample tubes were then transferred to the Soil Mechanics laboratory at Çukurova University and placed in the humidity room. Block tube samples were taken out by hydraulic Jack and specimens in 50 mm diameter, 100 mm high were trimmed from block samples for tri axial tests.

The soil samples were firstly saturated while the specimens were in nearly-saturated condition initially and average pore pressure parameters, B , was measured about 0.93 at the beginning of the tests. Following saturation, the samples were first K_0 consolidated to their estimated in-situ stress according to standard specified in JGS 0525-2000 (Zhu and Clark, 1994). K_0 value were taken as 0.65 from the earlier performed thin wall oedometer tests, for the same clay. Samples were then loaded in load controlled tests along approximately straight stress path in p' - q space. In two tests, the followed stresses ratios $\Delta q/\Delta p$ were 0.249 and 0.80, respectively. Sets of data entered in least square solution were given in Table 2. Preconsolidation pressure was obtained as 110 kPa from one dimensional oedometer test using Cassagrande method. Expressions of the symbols given in Table 2 have

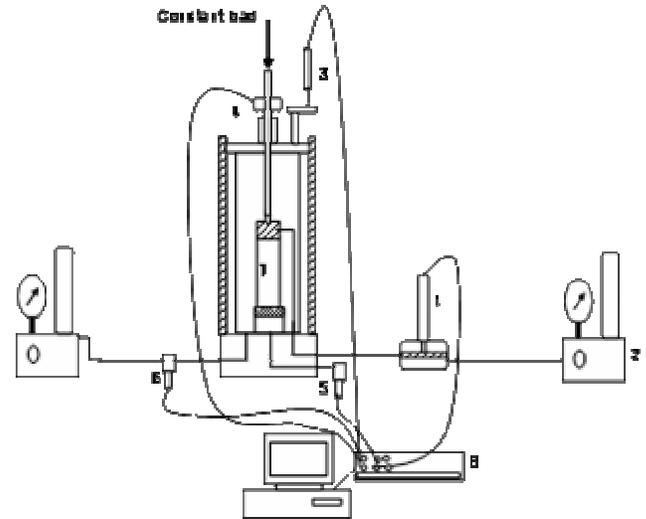


Figure 3. Sketch of test apparatus. 1-Volume change transducer, 2-Constant pressure supplier, 3-Displacement transducer (LVDT), 4-Load cell, 5-Pore pressure transducer, 6-Cell pressure transducer 7-Soil specimen, 8-Data logger.

been briefly summarized in notations.

A modified test apparatus was designed for the load controlled tests. Pore pressures and cell pressures were measured by pressure transducers. Vertical displacement was determined by displacement transducer (LVDT). Volume change was measured by volume change transducer with the accuracy of 0.01 cm^3 and load was measured by load cell. All data were automatically recorded by a Data Logger to a computer. The photograph and the sketch of the test apparatus are given in Figure 2 and 3.

Cross-anisotropic elastic five parameters were calculated by the method proposed by Graham and Houlsby (1983). The reason for choosing the mentioned method for the five elastic parameters was the soil properties are quite suitable for applying the method. α^2 show the value of the ratio of horizontal to vertical stiffness in Table 3. Yenice clay appears to be approximately 1.20 times stiffer in horizontal direction. On the other hand, anisotropic parameter, s , should be determined in order to use Eq. (21) for the stresses distribution calculations. To do this, vertical and horizontal samples for oedometer tests were taken from the same sample tube which was previously used for the tri-axial test samples.

Conventional oedometer tests were carried out according to the standards specified in TS- 1900. The soil samples were initially in nearly saturated condition, as it was known from the tri-axial tests. At the beginning of the tests specimens were loaded to small pressure of 10 kPa to eliminate any disconnection of loading system or probable discontinuities between loading cap and soil sample.

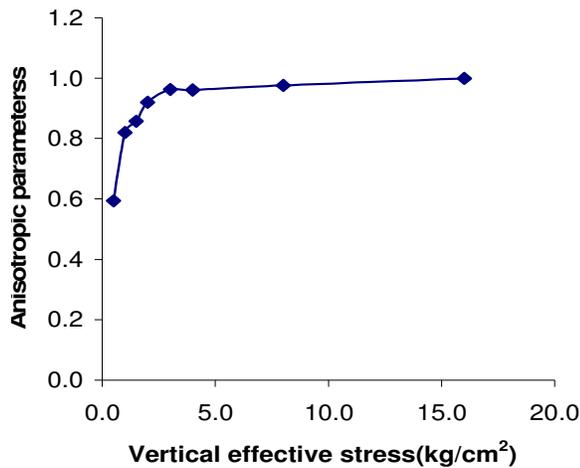
Then, conventional oedometer tests were performed. Vertical and horizontal coefficients of volume compressibilities are obtained from the test results and the aniso-

Table 2. Stress and strain increments data.

Test No	σ_{vc} (kPa)	$\delta p'$ (kPa)	δq (kPa)	δv (%)	$\delta \epsilon$ (%)
Test 1	110	32.06	8	2.7	0.335
Test 2	110	31.04	25	2.34	0.873

Table 3. Results for anisotropic elastic moduli.

K^*/P_c	G^*/P_c	J^*/P_c	ν^*	α^2	E_v (kPa)	E_H (kPa)	ν_{vH}	ν_{HH}	G_{vH} (kPa)	G_{HH} (kPa)
11	7.70	0.62	0.20	1.20	2187	2638	0.182	0.20	1000	1116

**Figure 4.** Relationship between vertical effective stress and anisotropic parameter.

tropic parameter, s , were calculated by Eq. (1). Relationship between vertical effective stress and s values was presented in Figure 4. As it is seen from Figure 4, the value of s increases and it approaches to 1. As the load increases, the difference between vertical and horizontal volume compressibilities decreases. This phenomenon cause to increase in ratio of compressibilities. After a certain loading step, the coefficient of compressibilities becomes nearly the same and their ratio goes to 1, for the Yenice clay. That means, loading increments for an oedometer conditions cause s value to approach to one. It might be useful to note that the previous studies were indicated that there were two types of soils. When the soil is stiffer in horizontal direction than in vertical $s < 1$ and for the case, the soil is stiffer in vertical direction, $s > 1$. For both cases, s approaches to one by loading increments. Table 4 presents vertical and horizontal coefficient of volume compressibilities and their corresponding s values.

Preconsolidation stress, P_c , can be regarded as a yield point of a soil for oedometer conditions and s values prior to the P_c can be considered as pre-yield s values of an particular soil. Since the value of P_c was obtained as 110

kPa, the first two value of s reflects pre-yield behavior of the present soil. In this study, the smallest first s value was used for the stress distribution calculations ($s = 0.594$). The reason for the use of smallest value of s is that, it is the best reflection of pre-yield behavior of the soil. Thus, entering s values, belonging to pre-yield region, results in solution by Eq. (21) to become more comparable with the other linear elastic solutions.

ESTIMATION OF STRESS DISTRIBUTION

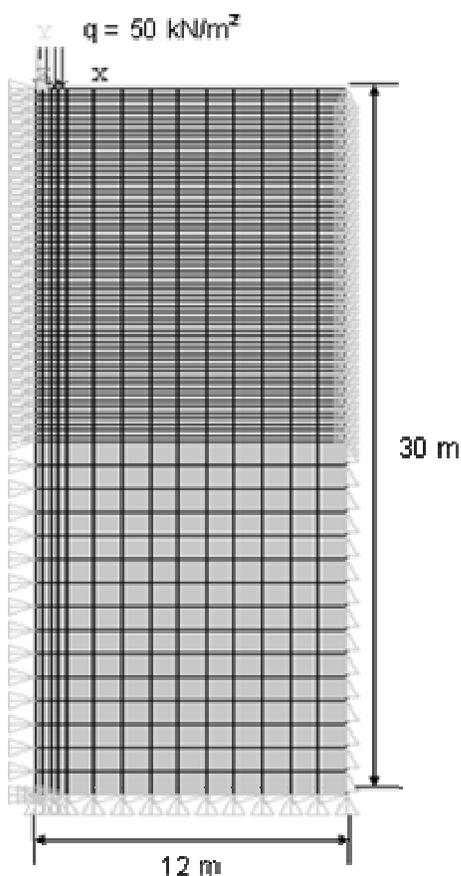
The change of stress under the center of uniformly distributed circular area was investigated by several methods and results were compared with the counter-parts. FEM analysis (taking the soil as a cross-anisotropic linear elastic medium), Eq. (21), Boussinesq and Westergaard methods were applied to the same problem. Solutions were performed for $B = 0.5, 1, 1.5$ and 2 m diameters in order to evaluate the influence of changing radius of loading area on the stress distribution.

Stress changes through the depth (solution was performed for at least $z \geq 2B$) were obtained by FEM analysis using the cross-anisotropic elastic parameters given in the Table 3. Geometry of the problem and meshes for FEM solution is presented in Figure 5. Numerical model was designed and plane 42 solid elements were used which can give an opportunity to perform an axial-symmetric solution. Plane 42 is a solid element having 4 joints and 2 degree of freedom at each joint, u_x and u_y . Material was defined as linear elastic orthotropic. The same elastic parameters, for two perpendicular directions in the horizontal plane, were entered to the program in order to satisfy cross-anisotropic material condition and the solution was performed for axially-symmetric loading. All FEM solutions were done using "Ansys package program". In order to obtain more accurate stress values, meshes are increased in number beneath the loading area.

Boundary conditions: At the axis of symmetry of the model ($x = 0$), the boundary conditions satisfying the symmetry were used ($u_x = 0, u_y \neq 0$). Here, x axis is vertical, y and z axis are two horizontal and the perpendicular axes. The bottom side of model ($y = -30$) were

Table 4. Coefficient of volume compressibilities and corresponding s values.

P vertical effective stresses, kN/m ²	e void ratios (%)	M_{vx} vertical volumetric compressibility m ² /kN	M_{vh} horizontal volumetric compressibility m ² /kN	s anisotropic parameter
0-50	0.6929	0.0144	0.0409	0.5940
50-100	0.6688	0.0285	0.0424	0.8196
100-150	0.6456	0.0279	0.0379	0.8574
150-200	0.6232	0.0272	0.0297	0.9201
200-300	0.5853	0.0233	0.0263	0.9623
300-400	0.5612	0.0152	0.0171	0.9604
400-800	0.4949	0.0106	0.0117	0.9761
800-1600	0.4260	0.0058	0.0056	0.9992

**Figure 5.** Geometry of problem and meshes for FEM analysis.

= 10 m) was defined as $u_x = 0$ and $u_y = 0$.

The stress distribution through the depth was also calculated by means of Eq. (21). In the calculations, the smallest value of s was used.

In order to be able to make a more general comparison, the Boussinesq and Westergaard solutions were also performed for the same problem.

Table 5. Distribution of vertical stresses by various methods, for $B = 0.5$ m.

Depth, z(m)	Boussinesq	Eq. (25)	FEM soln.	Westergaard
0.25	32.32	23.33	25.63	21.13
0.50	14.22	12.37	10.21	9.18
0.75	7.31	6.85	4.68	4.69
1.00	4.35	4.21	3.18	2.86
1.25	2.86	2.81	2.35	1.89
1.50	2.01	2.00	1.79	1.33
1.75	1.49	1.49	1.40	0.99
2.00	1.15	1.15	1.11	0.76
2.25	0.91	0.92	0.90	0.61
2.50	0.74	0.75	0.74	0.49
2.75	0.61	0.62	0.61	0.41
3.00	0.52	0.52	0.52	0.34
3.25	0.44	0.45	0.44	0.29
3.50	0.38	0.38	0.38	0.25
3.75	0.33	0.34	0.33	0.22
4.00	0.29	0.30	0.29	0.20

$B = 0.5$ m, $s = 0.594$.

Solutions were performed for varying diameter (B) of loaded area. Stress distribution up to 4 m depth for $B = 0.5, 1, 1.5$ m and $B = 2$ m diameters were presented in Tables 5, 6, 7, 8 and in Figures 6, 7, 8, 9. Soil compacity was calculated as average 0,604 for Yenice clay, from the samples used for the tests.

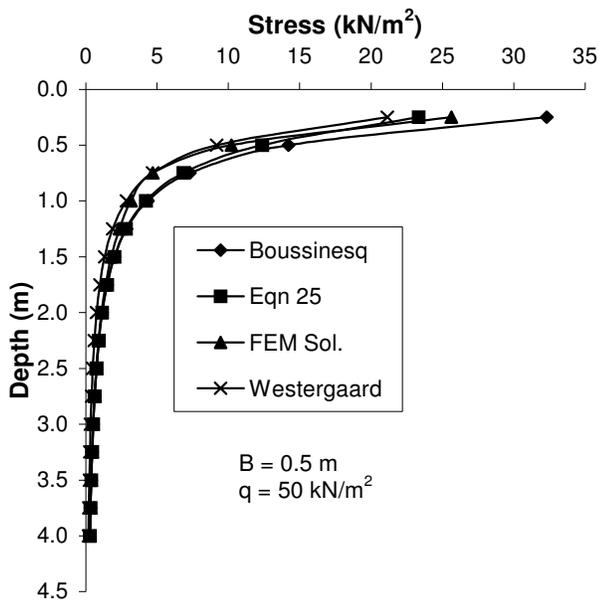
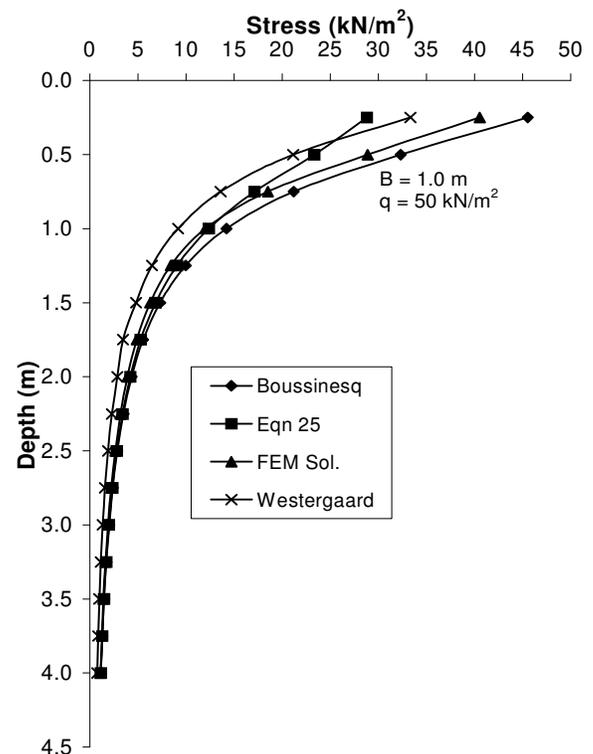
CONCLUSION

In this study, cross-anisotropic character of a clayey soil was reflected by a new parameter called "anisotropic parameter", s , which can simply be obtained by oedometer tests. This kind of representation may not wholly cover the cross-anisotropic character, as it is described in elasticity theory, but introducing this simple way of representation and following solution with Eq. (21) provides a

Table 6. Distribution of vertical stresses by various methods, for $B = 1$ m.

Depth, z (m)	Boussinesq	Eq. (25)	FEM soln.	Westergaard
0.25	45.53	28.8	40.54	33.33
0.50	32.32	23.33	28.89	21.13
0.75	21.20	17.13	18.50	13.6
1.00	14.22	12.37	12.02	9.18
1.25	9.98	9.08	8.44	6.48
1.50	7.31	6.85	6.30	4.81
1.75	5.55	5.30	4.89	3.46
2.00	4.35	4.21	3.91	2.86
2.25	3.49	3.41	3.19	2.30
2.50	2.86	2.81	2.65	1.89
2.75	2.38	2.36	2.23	1.58
3.00	2.01	2.00	1.89	1.33
3.25	1.72	1.72	1.63	1.14
3.50	1.49	1.49	1.41	0.99
3.75	1.30	1.31	1.28	0.87
4.00	1.15	1.15	1.09	0.76

$B = 1$ m $s = 0.594$.

**Figure 6.** Distribution of vertical stresses by various methods.**Figure 7.** Distribution of vertical stresses by various methods.

practical anisotropic elastic solution and it brings improvement to the stresses which are more consistent with cross-anisotropic linear elastic solution, when comparing with other widely used elastic isotropic solutions.

To see the performance of the proposed method by Eq. (21), cross-anisotropic five elastic parameters of Yenice/Tarsus clay were determined by the procedure proposed by Graham and Houlsby (1983). Finite Element Method, Eq. (21), Boussinesq and Westergaard solutions

were carried out for uniformly distributed circular area and results were compared. Stresses obtained by Eq. (21) for the depths $z < 0.5B$ were found small and closer to that of Westergaard's. As it can be seen from the Figures and

Table 7. Distribution of vertical stresses by various methods, for $B = 1.5$ m.

Depth, z(m)	Boussinesq	Eq. (25)	FEM soln.	Westergaard
0.25	48.40	29.73	45.593	38.53
0.50	41.47	27.32	38.87	28.68
0.75	32.30	23.33	30.45	21.13
1.00	24.40	19.08	22.71	15.70
1.25	18.40	15.36	16.91	11.88
1.50	14.20	12.37	12.91	9.18
1.75	11.74	10.04	10.12	7.24
2.00	8.96	8.24	8.13	5.83
2.25	7.31	6.85	6.66	4.77
2.50	6.06	5.76	5.55	3.97
2.75	5.10	4.90	4.70	3.35
3.00	4.35	4.21	4.02	2.86
3.25	3.74	3.65	3.47	2.47
3.50	3.26	3.19	3.03	2.15
3.75	2.86	2.81	2.66	1.89
4.00	2.53	2.50	2.36	1.67

$B = 1.5$ m, $s = 0.594$.

Table 8. Distribution of vertical stresses by various methods, for $B = 2$ m.

Depth, z(m)	Boussinesq	Eq.(25)	FEM soln.	Westergaard
0.25	49.30	30	47.42	41.3
0.50	45.50	28.8	43.43	33.33
0.75	39.20	26.41	37.72	26.58
1.00	32.30	23.33	31.16	21.13
1.25	26.20	20.11	25.02	16.89
1.50	21.20	17.13	20.05	13.60
1.75	17.30	14.55	16.21	11.11
2.00	14.20	12.37	13.28	9.18
2.25	11.80	10.57	11.02	7.67
2.50	9.90	9.08	9.27	6.48
2.75	8.50	7.86	7.89	5.54
3.00	7.30	6.85	6.79	4.81
3.25	6.30	6.01	5.89	4.15
3.50	5.60	5.3	5.16	3.64
3.75	4.90	4.71	4.55	3.22
4.00	4.30	4.21	4.04	2.86

$B = 2$ m, $s = 0.594$.

Tables 5, 6, 7 and 8 the agreement with FEM solution, for all diameters, is better than the linear elastic isotrop solutions (Boussinesq and Westergaard) for all points beneath the depths starting from a point located between $0 < z < B$. It should be noted that, the solution pro-

posed herein is not an alternative method to the cross-anisotropic elastic solutions as it is presented in theory of elasticity, but it can be considered as an improved practical anisotropic elastic solution as compared with widely used isotropic elastic solutions.

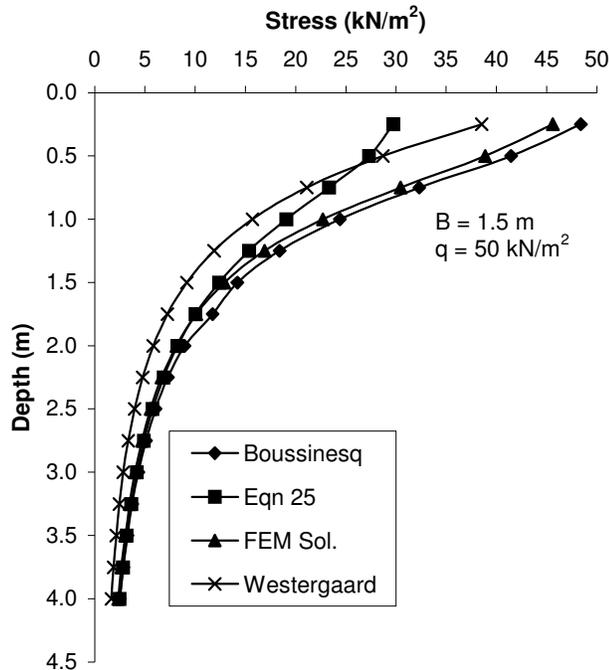


Figure 8. Distribution of vertical stresses by various methods, for $B = 1.5$ m.

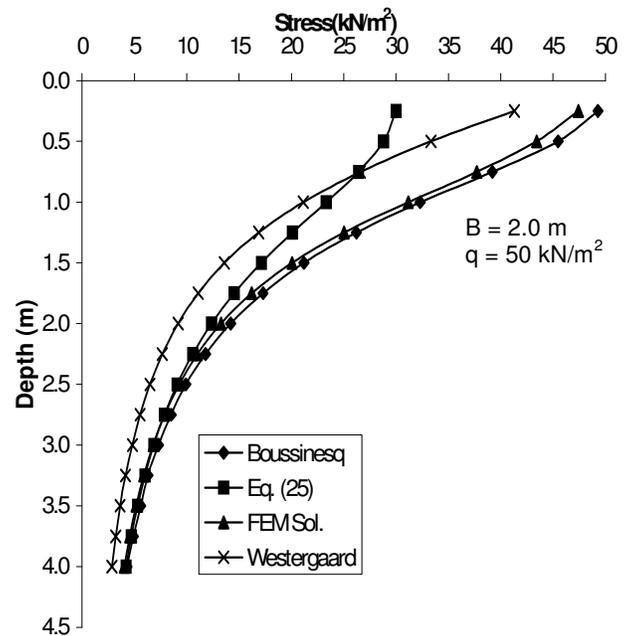


Figure 9. Distribution of vertical stresses by various methods, for $B = 2.0$ m.

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