Comparative study on earth pressure distribution behind retaining walls subjected to line loads

F. A. Salman¹*, M. Y. Fattah², S. M. Shirazi¹ and A. Mahrez¹

¹Department of Civil Engineering, Faculty of Engineering, University Malaya, 50603 Kuala Lumpur, Malaysia.
²Building and Construction Engineering Department, University of Technology, Baghdad, Iraq.

Accepted 6 May, 2011

In this paper, the earth pressure distribution generated behind a 20 m high retaining wall was estimated by the finite element method and compared with that obtained from classical earth pressure theories. Soil behavior was assumed to be elasto-plastic with Mohr-Coulomb failure criterion. The concrete retaining wall was represented by linear elastic model. Two-dimensional plane-strain finite element computer program CRISP was utilized after some modification. The results showed that Dubrova’s method gives greater values than Coulomb equation for all modes of wall movements. Whereas, the results obtained from the finite element analysis indicate that the stress distribution is more or less equal to Coulomb equation and ranging at about 90% of the depth for $\Phi=25^\circ$ and 60% for $\Phi=40^\circ$. Below this depth, the pressure distribution becomes much greater than that obtained by Coulomb equation. The finite element analysis shows a clear oscillation in the value of lateral earth pressure caused by line loads in addition to backfill, in the upper half of the wall, this oscillation increases as the line load increases in value and decays as the load goes far away from the wall. In the lower half of the wall height, the lateral earth pressure becomes more close to a linear distribution and its maximum value is at the wall base. The lateral earth pressure distribution will vanish as the position of the line load becomes far away from the wall ($m \geq 0.6 \times D$).

Key words: Earth pressure, finite elements, soil-structure interaction, retaining wall, line loads.

INTRODUCTION

The conventional design techniques provide little information about the distribution and magnitude of lateral earth pressures and wall deformations. The soil mass is assumed to be in the limiting state condition ($K_a$ and $K_p$) which is actually not correct (Potts and Fourie, 1984). The limiting pressures are not mobilized unless sufficient ground and wall movements are developed. A flexible wall is very likely to deform sufficiently in the active pressure case prior to failure. However, a very rigid wall might shear off suddenly without the active pressure being allowed to develop (Bowles, 1997). The finite element method of analysis has been applied to a variety of earth retaining structures and used to calculate stresses and movements for problems involving a wide variety of boundary and loading conditions. Some of the modeling features to be considered in a successful soil-structure interaction analysis are summarized, along with the results from selected soil-structure interaction analyses. Procedures for the finite element analysis of conventional, stable earth retaining structures are well established. They have been successfully applied to the evaluation of the soil-structure interaction for a variety of earth retaining structures during the past decades, including U-frame locks, gravity walls, and basement walls (Ebeling, 1990). An analytical feature used in the Port Allen and Old River study was the inclusion of the Goodman et al. (1968), interface elements between the concrete block walls and the soil backfill. The interface between the backfill and the wall is constrained in previous work so that both move in the same direction by equal magnitude. Clough and Duncan (1969) found that their developed procedures gave results in good agreement with the results of the extensive instrumentation program for Port Allen lock and Old River lock. Later on,
Clough and Duncan (1971) showed that nonlinear incremental finite element procedures could be used to predict lateral earth pressures for conditions ranging from an unmoving wall to limit conditions where the wall being displaced enough to generate active or passive earth pressures. The computed relationships between wall movements and the resultant horizontal earth pressure force were found to be in good agreement with the classical earth pressure theories and the computed deformations were in agreement with those measured by Terzaghi (1934), in his retaining wall tests. The use of interface elements along the soil-to-wall interface was shown to influence the computed earth pressures. Two contributing factors are the incorporation of the compressibility of the foundation in the analysis and the non-uniform loading of the foundation sands. Matsuo et al. (1978) investigated the characteristics of the earth pressure acting on a retaining wall on the basis of the large scale prototype tests in a field. They built a 10 m high concrete wall with silty sand and slugs as backfill materials, in order to study the influence of displacement of the wall on the magnitude and distribution of earth pressure in the vertical direction. Based on the information obtained from the tests, they proposed that a general retaining wall should be designed against the earth pressure at rest. They also compared the measured earth pressures with the analyzed results obtained by the finite element method. They represented the soil as a linear elastic material with triangular elements. They found that the influence of the unit weight ($\gamma$) and Young's modulus (E) on the calculated results are very small. That is, it is enough in the engineering sense to use the rough values of $\gamma$ and E in the calculation of earth pressure at rest, but the calculation is very sensitive to variation in the value of Poisson's ratio. Roth et al. (1979) described the backfill placement analysis of an instrumented, deep basement wall, using the same finite element procedure by Clough and Duncan (1969). The instrumentation measurements after completion of backfilling were compared to the computed results. Good agreement was found between the calculated and the measured lateral earth pressures when interface elements were included along the backfill-to-wall interface. By using interface elements in the finite element analyses of a rigid wall, they were able to simulate the settlement of the backfill adjacent to the wall, resulting in the mobilization of a shear force along the back of the wall. Roth and Crandall (1981) used the hyperbolic elastic finite element techniques for the prediction of elastic earth pressures against these walls. They used silty sand as a backfill material; whereas a corrugated bentonite-filled cardboard and fiberboard was used as an interface material (because bentonite is well known for its low shear strength and high swelling potential). They found that:

(a) Poisson's ratio is the single important parameter affecting the calculated lateral earth pressure.
(b) Changing the modulus of elasticity (E) in the finite element analysis did not significantly change the calculated horizontal wall pressures.
(c) The interface material is one of the possible important factors in governing the earth pressure against the wall.
(d) The effect of soaking the interface material eliminated the cohesion intercept, but did not appear to alter the angle of sliding friction.

Potts and Fourie (1984) carried out a numerical study about the behavior of a propped retaining wall. In their study, the finite element is used to investigate the influence of type of construction (excavation or backfilling) and the initial stress in the soil on the behavior of single propped retaining walls. A linearly elastic-perfectly plastic with a Mohr-Coulomb yield surface is used to model the soil behavior, while the wall is assumed to be linearly elastic and a rigid propped is assumed to act at the top of the wall. The problem was solved as a plane strain condition with eight-noded isotropic elements. They observed that:

(a) The limit equilibrium method used in current design procedures produces reliable estimates for the depth of wall embankment required to maintain stability.
(b) For excavation of walls in soils with a high value of coefficient of earth pressure at rest condition ($K_o$), prop force and wall bending moments greatly exceed those calculated by using the simple limit equilibrium approach.

In addition, large soil and wall movements are experienced even at shallow depths of excavation. The behavior is dominated by the vertical unloading caused by excavation process and large movement still occurs even if the wall is fully restrained from horizontal movement. For backfilled and excavated walls in soils with a low ($K_o$) values, the analyses indicate that the displacements are much smaller in magnitude and that the approximate limit equilibrium calculations produce conservative values of prop force and bending moments. Large zones of failed soils, especially in the front of the wall, are predicted for excavation walls in high ($K_o$) soils, and the lateral wall pressures behind the wall differ substantially from the classic active distribution. Passive conditions in front of the wall are completely mobilized at small excavation depths and before active conditions are approached down the back of the wall. In contrast, excavated walls and backfilled walls in low ($K_o$) soils show lateral pressures which are in agreement with the classical distributions. Potts and Fourie (1986) employed the finite element method to examine the influence of wall movement on the generation of earth pressure. The effects of wall translation, rotation about the top and rotation about the bottom of the wall have been investigated. They found that:

(a) The nature of the wall movement, whether translation
or rotation, has an effect on the equivalent values of $K_a$ and $K_p$ for both rough and smooth walls.

(b) The final values of $K_a$ and $K_p$ are essentially unaffected by the value of $K_o$ or the distribution of Young's modulus in the soil.

(c) The relative displacements necessary to mobilize active and passive conditions depend on the wall, initial $K_o$ value and distribution of Young's modulus.

(d) The mode of wall movement has a considerable effect on the distribution of earth pressure.

Bhatia and Bakeer (1989) performed a finite element analysis of 10 m high instrumented experimental wall resting on a hinged base that was tested by Matsuo et al. (1978) in order to discuss some factors that influence the results of a finite element idealization of the problem of earth pressure behind a gravity wall with dry, cohesionless backfill. Fine elements should be used in the backfill behind the wall-back in region extending horizontally a distance of at least the height of a wall for the active case. Al-Shikhany (2000) investigated the earth pressure distribution on a flexible (propped and cantilever) wall for both excavation and backfilling construction methods. Eight-noded quadrilateral elements were used to represent the soil and the wall whereas the relative displacement between the wall and the soil was simulated by a thin layer interface element. The behavior of the soil and the interface was assumed to be elasto-plastic with Mohr-Coulomb criterion, while the wall was assumed to be elastic material. Al-Shikhany found that the earth pressure depends on the deformation and the movement of the wall, the initial stresses at rest, $K_o$, and existence of prop. It was also found that the results for low $K_o$-values are almost the same for both methods of construction (excavation and backfilling) whereas for high $K_o$-values, the results will be different. A comparison was made between the conventional theories of earth pressure and the finite element method in predicting the distribution of earth pressure behind cantilever retaining walls subjected to line loads.

**ACTIVE EARTH PRESSURE FOR TRANSLATION OF RETAINING WALL**

Under certain circumstances, retaining walls may undergo lateral translation, as shown in Figure (1). A solution to the distribution of active pressure for this case was provided by Dubrova (1963) and was also described by Das (2007). The solution of Dubrova assumes the validity of Coulomb's solution. In order to understand this procedure, let us consider a vertical wall with a horizontal granular failure as shown in Figure (2). For rotation about the top of the wall, the resultant $R$ of the normal and shear forces along the rupture line $AC$ is inclined at an angle $\phi'$ to the normal drawn to $AC$. According to Dubrova, there exists infinite number of quasi-rupture lines such as $A'C'$, $A''C''$, ... for which the resultant force $R$ is inclined at an angle $\psi$, where:

$$\Psi = \frac{\phi' z}{H} \quad (1)$$

Now, refer to Coulomb's active earth pressure equation. For $\beta=90^\circ$ and $\alpha=0$, the relationship for Coulomb's active force can be rewritten as:

$$P_a = \frac{\gamma}{2\cos \delta} \left[ \frac{H}{1 + (\tan^2 \phi + \tan \phi \tan \delta)^{\alpha_{5}}} \right]^2 \quad (2)$$
Figure 2. Quasi-rupture lines behind a retaining wall.

The force against the wall at any \( z \) is then given as:

\[
P_z = \frac{\gamma H}{2 \cos \delta} \left[ \frac{1}{\cos \psi} + (\tan^2 \psi + \tan \psi \tan \delta) \right]^{0.5}
\]  
(3)

The active pressure at any depth \( z \) for wall rotation about the top is:

\[
\sigma_a'(z) = \frac{dP}{dz} = \frac{\gamma z \cos^2 \psi}{\cos \delta (1 + m \sin \psi)^2} \left[ \frac{z \phi \cos^2 \psi - \frac{z^2 \phi}{H(1 + m \sin \psi)}}{(1 + m \sin \psi)} \right]
\]  
(4)

Where \( m = [1 + \frac{\tan \delta}{\tan \psi}]^{0.5} \)  
(5)

For frictionless walls, \( \delta' = 0 \) and Equation (4) simplifies to:

\[
\sigma_a'(z) = \gamma \tan^2 \psi \left( \frac{45 - \psi}{2} \right) \left( z - \frac{z^2 \phi}{H \cos \psi} \right)
\]  
(6)

For translation of the wall, the active pressure can then

\[
\sigma_a'(z)_{\text{translation}} = \frac{1}{2} [\sigma_a'(z)_{\text{rotation top}} + \sigma_a'(z)_{\text{rotation bottom}}]
\]  
(8)

An experimental verification of this procedure was provided by Matsuzawa and Hazazika (1996). The results were obtained from large-scale model tests and are shown in Figure (3). The theory and experimental results show good agreement.

EARTH PRESSURE ARISING FROM SURCHARGES

The ground surface behind a retaining wall may be subjected to surcharges. These may be permanent in character, such as the shallow foundations on adjacent building, or temporary, such as traffic or construction loads caused by plant, storage of materials, etc. The applied surcharges are usually vertical forces, although they may also have a horizontal component, and will in either case result in an increase in the horizontal earth pressure acting on the wall. For a given magnitude and distribution of the surcharge, the horizontal stresses that act on the wall depend on the properties of the soil and the stiffness of the wall and its supports. For surcharges of limited extent, it is convenient to consider the two extreme cases of a completely restrained wall and a wall that moves sufficiently to allow active yield in the soil.

In the case of a rigid wall supporting soil that is not in a state of active yield, it is reasonable to assume that the
soil will behave in an approximately elastic manner. In this case, the stress distribution may be calculated on the basis of elastic theory, using equations developed from the work of Boussinesq (1885). With the simplifying assumptions that the wall is completely restrained and frictionless, it can be shown that the horizontal stresses exerted on the wall are double the horizontal stresses calculated for the same position relative to the surcharge in an elastic half space. If the wall deflects slightly as a result of application of the surcharge, the horizontal stress imposed on the wall because of the soil and surcharge will actually be reduced (IstructE, 1989). Elastic analyses are available for surcharges of limited area and for various geometric arrangements (Poulos and Davis, 1974). There is no theoretical justification for the assumption that earth pressures calculated from linear elastic theory can be used when the ground is in the plastic state of active yield. Although the total stresses will be lower in the active case than for an unyielding wall, the additional active pressure caused by the surcharge may be greater than indicated by elastic calculations in Figure (4). Figure (5) shows an example in

**Figure 3.** Horizontal earth-pressure distribution behind a model rigid retaining wall (Note: sand backfill, $\phi' = 34^\circ$, $\delta' = 2/3 \phi$, $\gamma = 15.4 \text{kN} / \text{m}^3$), (Matsuzawa and Hazazika, 1996).

**Figure 4.** Effect of increasing movement, $\delta$, until wall pressure, $p$, tends to active limit, (IstructE, 1989). $\Delta p_A$ = effect of surcharge at active yield. $\Delta p_B$ = horizontal stress due to the loaded area using the Boussinesq equations.
which a surcharge is imposed behind a frictionless wall supporting a dry soil with an angle of shearing resistance of 30°. It can be seen the wedge solution produces results markedly different from the stress distributions derived from elastic theory. Muhammed-Ali (1987) studied the effect of line loads on instrumented retaining wall. The test results showed that the maximum pressure on the wall is at depth of (0.6 to 0.7 H) from top of the wall (H is the height of the wall). The lateral pressure values become very small at points on the wall lying above the intersection of line joining the load and the wall and make an angle (θ) of less than (35°) with the horizontal. Very little pressure is transferred to the wall when the distance of line load to the wall is greater than (1.3 H). It was also found that the earth pressure resultant acts within a narrow range of about (0.4 - 0.6 H) of the wall depth.

**DESCRIPTION OF THE PROBLEM**

In order to give a meaningful study about the earth pressure distribution, it is decided to analyze the problem adopted by Potts and Fourie (1984), and by Fourie and Potts (1989). The problem geometry and finite element mesh are as shown in Figures (6) and (7) while the material
Figure 7. Finite element mesh.

Table 1. Material properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Soil</th>
<th>Interface</th>
<th>Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>kN/m²</td>
<td>5.5×10⁴</td>
<td>5.5×10⁴</td>
<td>28×10⁴</td>
</tr>
<tr>
<td>ν</td>
<td></td>
<td>0.2</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>c</td>
<td>kN/m²</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Φ</td>
<td>degree</td>
<td>25</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>G</td>
<td>kN/m²</td>
<td>-</td>
<td>250</td>
<td>-</td>
</tr>
<tr>
<td>γ</td>
<td>kN/m³</td>
<td>20</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>t_interface</td>
<td>m</td>
<td>-</td>
<td>0.05</td>
<td>-</td>
</tr>
</tbody>
</table>

properties are given in Table (1). The earth pressure generated behind a retaining wall was studied and compared with that of classical earth pressure theories of Coulomb and Dubrova for three types of wall movements (rotation about the top and about the bottom of the wall and free wall translation).

ANALYSES

Effect of the wall movement on the lateral earth pressure

The effect of mode of wall movement on the stress distribution behind a retaining wall has been investigated using the finite element computer program CRISP. In this study, the computer program CRISP has been developed to perform two-dimensional analysis of soil-wall interaction. The program is primarily based on a program provided by Britto and Grunn (1987), named CRISP (CRitical State Program). The program uses the finite element technique and allows predictions to be made of ground deformations using critical state theories. Some modifications are made on the main finite element computer program (CRISP) to obtain the present computer program (Mod-CRISP) in order to achieve the computations needed in the present study. These include
the addition of eight-noded quadrilateral isoparametric consolidation element with 16-d.o.f. and additional 4-d.o.f. on corner nodes, namely for excess pore water pressure and the addition of thin-layer interface element developed by Desai et al. (1984). The results of modes of motion, such as rotation about the bottom and about the top and free translation are compared with the method proposed by Dubrova in 1963 (Harr, 1966; Das, 2007), and both results with that of Coulomb method. The comparison is carried out using different values of soil friction angle ($\Phi$) and friction angle between the wall and the backfill soil ($\delta$).

Figures (8) and (9) show the earth pressure distribution behind the retaining wall for the case when $\zeta = \zeta$ (free relative movements between the wall and the soil). It is seen that Dubrova’s method gives greater values than Coulomb equation, for all modes of wall movements. Whereas, the results obtained from the finite element analysis indicate that the stress distribution is more or less equal to Coulomb equation and ranging at about 90% of the depth for $\Phi=25^\circ$ and 60% for $\Phi=40^\circ$. Below this depth, the pressure distribution becomes

---

**Figure 8.** Active earth pressure against retaining wall for $\Phi = 25^\circ$ and $\delta = 0.0^\circ$. Note: solid curves refer to present study, while dashed curves refer to Dubrova’s method.

**Figure 9.** Active earth pressure against retaining wall for $\Phi = 40^\circ$ and $\delta = 0.0^\circ$. Note: solid curves refer to present study, while dashed curves refer to Dubrova’s method.
much greater than that obtained by Coulomb equation.

**Effect of line load on the lateral earth pressure**

Figure (10) shows the distribution of lateral pressure against the back face of the wall due to a line load surcharge placed parallel to the crest of the wall of height $D$ (Das, 1990). Based on the theory of elasticity, the horizontal stress $\sigma_z$ at a depth $z$ on a retaining structure can be given as:

For $m > 0.4$

$$\sigma_z = \frac{4q}{\pi D} \frac{m^2n}{(m^2+n^2)^2}$$  \hspace{1cm} (9)

For $m \leq 0.4$

$$\sigma_z = \frac{0.203q}{D} \frac{n}{(0.16+n^2)^2}$$  \hspace{1cm} (10)

where $q =$ load per unit length of the surcharge, and $m$ and $n$ are non-dimensionalized distances as shown in Figure (11).

Figures (11) to (16) show the finite element investigation of the pressure distribution behind the retaining wall (due to earth pressure and line load) and the comparison with Dubrova's method and Coulomb equation. From these figures, the following points can be drawn:

(i) When the initial line load is applied (100 kN/m), the pressure distribution obtained from Coulomb equation is linear whatever is the position of the line load. When the line load increases in value to about (300 to 500 kN/m), the pressure distribution starts to take the shape of a parabola in the upper part of the wall height, this parabola vanishes and the distribution becomes linear towards the wall base. For all studied cases, it was found that the maximum pressure is in the wall base.

(ii) The shape of the pressure distribution obtained by Dubrova's method has a parabolic shape and its maximum value is at about one-third from the wall base, and then starts to decrease in value. While without line load, the maximum value of earth pressure is in the wall base. The value of the lateral earth pressure at the wall base is about (10 to 20%) less than that obtained by Coulomb equation.

(iii) The finite element analysis shows a clear oscillation in the value of lateral earth pressure in the upper half of the wall, this oscillation increases as the line load increases in value and decays as the load goes far away from the wall. In the lower half of the wall height, the lateral earth pressure becomes more close to a linear
distribution and its maximum value is in the wall base. The lateral earth pressure distribution will vanish as the position of the line load becomes far away from the wall ($m \geq 0.6 \times D$).
(iv) The lateral earth pressure computed by the finite element method at the wall base is about (165 to 205%) greater than that calculated by Coulomb equation.

(v) At depths ranging from about (0.00 × D to 0.75 × D), the finite element values oscillate around the Coulomb values, and below these depths the values start to increase.
in very obvious manner. While Dubrova’s method always gives values greater than Coulomb equation for all depths.

(vi) When simulating the interlocking between the backfill material and the retaining wall using the interface element, it was found that there is a clear change in lateral earth pressure distribution especially in the upper two-third of the wall height. In that height, the lateral pressure is less than the case without interface elements and then starts to increase in value towards the wall base. For all values and positions of the line load, the effect of interface elements vanishes in the lower part of the wall.
Figure 14. Effect of combined backfill and line load on the stress distribution behind retaining wall, $\Phi=30^\circ$, $\delta=0.67\Phi$, and $m=0.05D$.

height. This can be due to the mode of motion described by the interface element where slippage between the mass of the wall and the backfill material will occur. This motion will reduce the lateral earth pressure and makes
the results of finite element analysis more close to Coulomb equation that takes in consideration the friction between the wall and the soil.

**CONCLUSIONS**

(1) The value of wall friction angle (when greater than zero) will not affect much the values of pressure distribution behind the retaining wall as the wall friction angle affects mainly the shear stresses between the retained soil and the wall.

(2) The finite element analysis shows a clear oscillation in the value of lateral earth pressure, caused by line loads in addition to backfill, in the upper half of the wall, this oscillation increases as the line load increases in value.

![Figure 14. Effect of combined backfill and line load on the stress distribution behind retaining wall, $\theta=30^\circ$, $\delta=0.67\theta$, and $m=0.05D$.](image-url)
and decays as the load goes far away from the wall. In the lower half of the wall height the lateral earth pressure becomes more close to a linear distribution and its maximum value is in the wall base. The lateral earth pressure distribution will vanish as the position of the line load becomes far away from the wall ($m \geq 0.6 \times D$).
Figure 16. Effect of combined backfill and line load on the stress distribution behind retaining wall, $\theta=30^\circ$, $\delta=0.67^\circ \theta$, and $m=1.0^\circ D$.

(3) When small initial line load is applied (100 kN/m), the pressure distribution obtained from Coulomb equation is linear whatever is the position of the line load. When the line load increases in value to about (300 to 500 kN/m), the pressure distribution starts to take the shape of a parabola in the upper part of the wall height, this parabola
vanishes and the distribution becomes linear towards the wall base. For all studied cases, it was found that the maximum pressure is in the wall base.

(4) The shape of the pressure distribution obtained by Dubrova’s method has a parabolic shape and its maximum value is at about one-third from the wall base, and then starts to decrease in value. While without line load, the maximum value of earth pressure is in the wall base. The value of the lateral earth pressure at the wall base is about (10 to 20%) less than that obtained by Coulomb equation.

ACKNOWLEDGEMENT

The authors wish to thank their colleagues Dr. Yousif J. Al-Shakarchi from the University of Baghdad and Dr. Husain M. Husain from the University of Technology, Baghdad, Iraq, for many valuable discussions and assistance in the interpretation of the data as presented in this paper.

REFERENCES


Clough GW, Duncan JM (1969). Finite Element Analyses of Port Allen and Old River Docks, Contract Report S-69-6, U.S. Army Engineers, Waterways Experiment Station, Vicksburg, MS.


Salman et al.          2267