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The reliability test assessment of three-parameter Weibull distribution of material life by Bayesian method

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This paper is mainly according to Bayesian formula and using Bayesian method to estimate parameter truth-values of three-parameter Weibull distribution to achieve material reliability evaluation. The parameter values estimated by maximum likelihood method as the prior information of Bayesian estimation, assume that the parameter obeys a certain distribution and let the certain distribution as the prior distribution of parameter, and the three-parameter Weibull distribution is known, so the posterior distribution function of the parameter can be obtained. According to Bayesian estimation, the maximum posteriori estimation is chosen as the truth-value of the estimated parameter, the lifetime data are substituted into the truth-value estimated function of reliability and the theoretical values of the reliability can be got. Case study shows that the Bayesian method has general applicability to the material reliability evaluation.

Key words: Material lifetime, test assessment, three-parameter Weibull distribution, Bayesian method.

INTRODUCTION

Material is the foundation and the forerunner of modern high-new technology and industry. For example, as the aerospace and trains material research and innovation, the space industry and transportation industry etc have rapid development, to ensure that the components are reliable, high demands are put forward about material properties and the use efficiency. The material reliability research has become one of the hot issues in recent years.

There are many results about the material reliability research at home and abroad. For examples, Kasprzyk (2005) mainly discussed to improve the reliability and performance of the non-metallic materials, enhance the utilization of non-metallic materials; Johnsen and Nyhus (2007) adopted the method of reliability test to ensure eligibility and safety use of stainless steel materials; Wang et al. (2008) through research show that coating materials affect the reliability of the sample; Xu et al. (2012) by raising the casting material performance, optimized the casting process, to ensure the reliability of the casting use, which is now in the CRH2, CRH380A type trains of application. The material reliability method was used to research the above, thereby effectively enhancing the applicability and reliability of materials.

Material life is the main factor that influences the reliability of the components used, and according to the material life data, to study the material use reliability is also very necessary. In this paper with three-parameter

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Weibull distribution as the reliability model of the material life, Bayesian method is used to estimate the parameters of three-parameter Weibull model, and then the reliability evaluation of material was carried out. So far, the effective parameter estimation methods mainly have the bootstrap weighted-norm method (Xia et al., 2013), maximum likelihood method (Yan et al., 2005), random number method (El-Adll, 2011), moments and probability weighted moment method Paul-Dario and Thomas, 2011; Deng et al., 2004; Zhao et al., 2010), dual-linear regression method (Zhuang, 1999), correlation coefficient optimization method (Richard and Stepnens, 1993) etc. Because Bayesian method (Mao, 1999) has obvious superiority on the basis of the theory, Sinha and Sloan (1988) and Qin et al. (1988) have introduced application of Bayesian method to the Weibull model. This paper mainly discussed the application of Bayesian method in material reliability evaluation.

THREE-PARAMETER WEIBULL DISTRIBUTION

The three-parameter Weibull distribution is defined as:

$$f(t;\eta,\beta,\tau)=\frac{\beta}{\eta}\left(\frac{t-\tau}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{t-\tau}{\eta}\right)^{\beta}\right),\quad t\ge\tau>0,\ \beta>0,\ \eta>0 \tag{1}$$

where, $f(t;\eta,\beta,\tau)$ is the three-parameter Weibull distribution, t is a stochastic variable of the lifetime, (η,β,τ) are the Weibull parameters, η is the scale parameter, β is the shape parameter, and τ is the location parameter.

The reliability function with the three-parameter Weibull distribution can be expressed as:

$$R(t;\eta,\beta,\tau)=1-\int_{\tau}^{\infty}f(t;\eta,\beta,\tau)dt=\exp(-(\frac{t-\tau}{\eta})^{\beta}) \tag{2}$$

where, $R(t;\eta,\beta,\tau)$ is the reliability function with the three-parameter Weibull distribution.

For many products in engineering practice, the value of the Weibull parameter is unknown and needs to be found with the help of test evaluation. To this end, a lifetime experiment must be conducted. Assume that the lifetime data of a product are obtained by life tests as follows:

$$\mathbf{T}=\{t_i\},\quad t_1\le t_2\le\ldots\le t_i\le\ldots\le t_n,\quad i=1,2,\ldots,n \tag{3}$$

where, $\mathbf{T}$ is the lifetime data vector, t_i is the ith lifetime datum in $\mathbf{T}$, i is the sequence number of t_i, and n is the number of the data in $\mathbf{T}$.

The lifetime data in $\mathbf{T}$ are substituted into Equation (2), and the theoretical value of the reliability can be calculated by

$$\mathbf{R}=\{R(t_i;\eta,\beta,\tau)\} \tag{4}$$

where, $R(t_i;\eta,\beta,\tau)$ is the theoretical value of the reliability.

BAYESIAN METHOD

Maximum likelihood method is a widely used analytical method in parameter estimation. In this paper, let the values of the parameters solved by maximum likelihood method as a priori information of Bayesian estimation, be assumed to obey a certain distribution and let the certain distribution be the prior distribution; meanwhile, the three-parameter Weibull distribution is known, the posterior distribution function of the parameter finally can be obtained (Mao, 1999).

In parameter estimation, using the maximum likelihood method to estimate the truth-values of parameters not only can solve the likelihood equations but also can directly use optimization method to obtain the model parameter values when the logarithmic likelihood function achieve the maximum.

For complete data, the likelihood function of three-parameter Weibull distribution can be written as:

$$L(t_i;\eta,\beta,\tau)=\left(\frac{\beta}{\eta}\right)^{n}\left(\prod_{i=1}^{n}\left(\frac{t_i-\tau}{\eta}\right)\right)^{(\beta-1)}\exp\left[-\sum_{i=1}^{n}\left(\frac{t_i-\tau}{\eta}\right)^{\beta}\right] \tag{5}$$

The logarithm likelihood function can be expressed as:

$$\ln[L(t_i;\eta,\beta,\tau)]=\ln\left(\frac{\beta}{\eta}\right)^{n}+(\beta-1)\sum_{i=1}^{n}\ln\left(\frac{t_i-\tau}{\eta}\right)-\sum_{i=1}^{n}\left(\frac{t_i-\tau}{\eta}\right)^{\beta} \tag{6}$$

Calculate the partial differential of Equation (6) and let the partial differential equal to zero, so the equations can be written as follows:

$$\frac{\partial\ln[L(t_i;\eta,\beta,\tau)]}{\partial\eta}=-\frac{n\beta}{\eta}+\sum_{i=1}^{n}\left(\frac{\beta}{\eta}\left(\frac{t_i-\tau}{\eta}\right)^{\beta}\right)=0 \tag{7}$$

$$\frac{\partial\ln[L(t_i;\eta,\beta,\tau)]}{\partial\beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\ln\left(\frac{t_i-\tau}{\eta}\right)-\sum_{i=1}^{n}\left(\left(\frac{t_i-\tau}{\eta}\right)^{\beta}\ln\left(\frac{t_i-\tau}{\eta}\right)\right)=0 \tag{8}$$

$$\frac{\partial\ln[L(t_i;\eta,\beta,\tau)]}{\partial\tau}=(\beta-1)\sum_{i=1}^{n}\left(-\frac{1}{t_i-\tau}\right)+\sum_{i=1}^{n}\left(\frac{\beta}{\eta}\left(\frac{t_i-\tau}{\eta}\right)^{(\beta-1)}\right)=0 \tag{9}$$

Use the iteration method to solve the Equations (7) to (9), and the estimated truth-values of the three parameters by maximum likelihood estimation can be obtained. The values of the three parameters are also written as (η_1,β_1,τ_1) .

Assumed that $\pi(\theta)$ is the prior distribution and $\theta=(\theta_1,\ldots,\theta_k)$ are the parameters of the model, $t=(t_1,\ldots,t_n)$ are the observations of a sample, and use $\pi(\theta;t)$ as the posterior distribution, $f(t;\theta)$ is the density function of sample, so according to Bayesian rule, the density function of Bayesian formula can be expressed as:

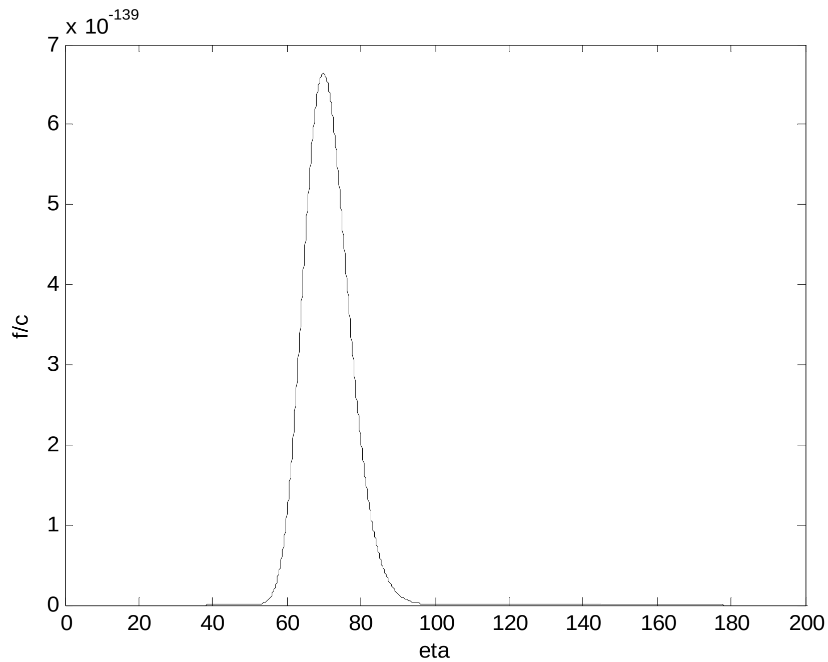

Figure 1. The Distribution of parameter η of the posterior distribution in ceramic material case.

$$\pi(\theta;t)=\frac{f(t;\theta)\pi(\theta)}{m(t)} \quad (10)$$

where, $m(t)=\int_{-\infty}^{+\infty} f(t;\theta)\pi(\theta)d\theta$.

The value θ_{MD} which makes the posterior density function $\pi(\theta;t)$ reach the maximum value called maximum posteriori estimation, is also called Bayesian estimation. In this paper, the maximum posteriori estimation θ_{MD} is the estimated truth-value of the parameter θ. The values of the three parameters by Bayesian method can be written as $\theta=(\eta_2,\beta_2,\tau_2)$.

CERAMIC MATERIAL CASE STUDY AND DISCUSSION

In life tests, Duffy et al. (1993) obtained the lifetime data (h) of aluminum oxide ceramics, as follows (*n*=35):

T= (307 308 322 328 328 329 331 332 335 337 343 345 347 350 352 353 355 356 357 364 371 373 374 375 376 376 381 385 388 395 402 411 413 415 456)

The three parameters truth-values of Weibull distribution estimated by maximum likelihood method are（69.8392,1.9708,300.0086）.

In Bayesian estimation, assume that the value of shape parameter β is 1.9708. It is know that τ is the location parameter. In this case, τ is the minimum lifetime and the scope of τ is $[0,307]$. A suitable value of τ chosed in the scope is 300. Only think that the scale parameter η is a random variable, assume the prior distribution of η is:

$$\pi(\eta)=C'\eta^3 e^{-0.043\eta}$$

where, C' is a constant. When the prior distribution gets maximum value, the value of scale parameter η is 69.8392.

For complete data:

$$f(t|\eta)=\left(\frac{\beta}{\eta}\right)^n\left(\prod_{i=1}^{n}\left(\frac{t_i-\tau}{\eta}\right)\right)^{(\beta-1)}\exp\left[-\sum_{i=1}^{n}\left(\frac{t_i-\tau}{\eta}\right)^{\beta}\right]$$

meanwhile, $m(t)=\int_{-\infty}^{+\infty} f(t;\eta)\pi(\eta)d\eta$. Let

$C=C'\frac{1}{m(t)}\beta^n\left(\prod_{i=1}^{n}(t_i-\tau)\right)^{(\beta-1)}$, the posterior distribution function of parameter to be estimated is formulated as:

$$\pi(\eta|t)=C\eta^{-65.978}\exp\left[-0.043\eta-\sum_{i=1}^{35}\left(\frac{t_i-300}{\eta}\right)^{1.9708}\right]$$

Where, C is a constant that it has no relation with η.

The maximum posteriori estimation of this distribution is $\hat{\eta}=69.8$. The distribution of η is a unimodal shape, as shown in Figure 1.

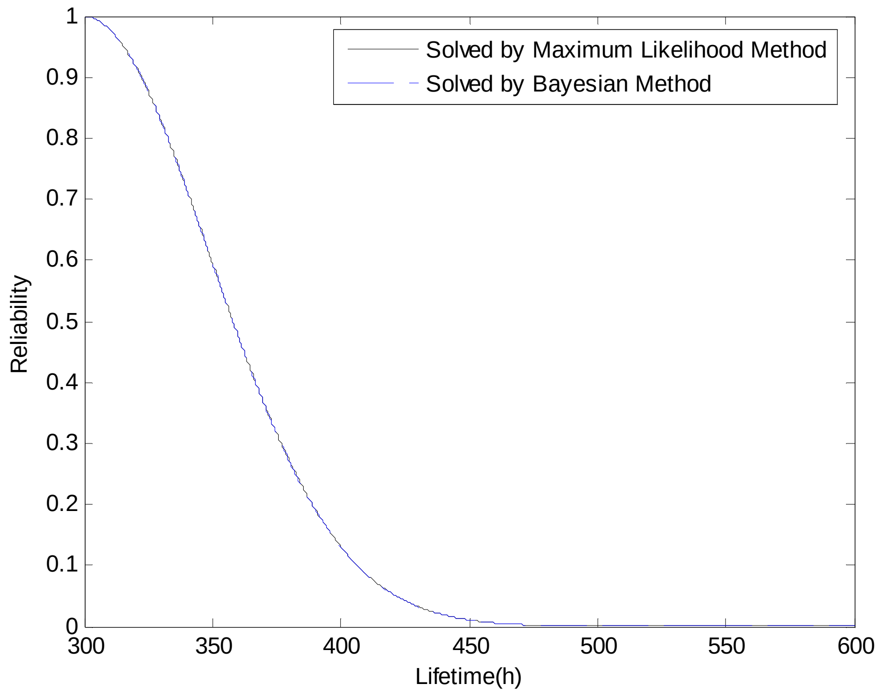

Figure 2. The theoretical value vectors of reliability in ceramic material case.

The three parameters truth-values of Weibull distribution estimated by Bayesian method is (69.8,1.9708,300). Let the values of $(\eta_1, \beta_1, \tau_1)$ and $(\eta_2, \beta_2, \tau_2)$ are substituted into the three-parameter Weibull reliability function respectively, then the reliability truth-value functions can be gotten. The theoretical value vectors of reliability solved by the two methods are shown in Figure 2.

The $K-S$ test values of the reliability fitting models respectively gotten by maximum likelihood method and Bayesian method are 0.0547 and 0.0542. For a given confidence level $\alpha = 0.01$, look-up table can get the critical value $D_c = 0.26896$. The test values of the two methods are less than the critical value D_c that the three-parameter Weibull distribution models fitted by the two methods are appropriate. From Figure 2, we can see that the theoretical values vectors of the reliability $R(t_i;\eta,\beta,\tau)$ are obtained by the two methods are consistent in the overall.

Conclusions

The results of $K-S$ test in the case studied show that three-parameter Weibull distrubution model fitted by Bayesian method is better than the model fitted by maximum likelihood method. And a proper fitting model is more beneficial to assess and improve the reliability of the materials. The two methods to evaluate the reliability of the material are applicable. Although Bayesian method based on the theory has superiority, however, both are specific to a proper prior distribution and the numerical calculation which multiple parameter models will be involved are of considerable complexity.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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REFERENCES

Deng J, Gu DS, Li XB (2004). Parameters and quantile estimation for fatigue life distribution using probability weighted moments. Chin. J. Comput. Mech. 21(5):609-613.

Duffy SF, Palko JL, Gyekenyesi JP (1993). Structural reliability analysis of laminated CMC components. J. Eng. Gas Turb. Power 115(1):103-108. http://dx.doi.org/10.1115/1.2906663

El-Adll ME (2011). Predicting future lifetime based on random number of three parameters Weibull distribution. Math. Comput. Simul. 81(9):1842-1854. http://dx.doi.org/10.1016/j.matcom.2011.02.003

Johnsen R, Nyhus B (2007). New improved method for hisc testing of

stainless steels under cathodic protection. NACE - International Corrosion Conference Series, pp. 074961-0749620.
Kasprzyk MC (2005). The use of non-metallic materials in heat treating. Heat Treating Progress 5(5):40-42.
Qin M, SJ Wu, Peng X (1988). A reliability evaluation method for equipment with a Weibull failure Weibull Distribution. IEEE Trans. Reliab. 37(4):364-368.
Mao SS (1999). Bayesian statistics. Beijing: China Statistics Press.
Paul DTC, Thomas U (2011). General probability weighted moments for the three-parameter Weibull distribution and their application in S-N curves modeling. Int. J. Fatigue 33(12):1533-1538. http://dx.doi.org/10.1016/j.ijfatigue.2011.06.009
Richard AL, Stephens MA (1993). Estimation and test-of-fit for the three parameter Weibull distribution[R]. Technichal Report No.476. Stanford, Califomia.
Sinha SK, Sloan JAB (1988). Estimation of the Parameter Weibull Distribution. IEEE Trans. Reliab. 37(4):364-368. http://dx.doi.org/10.1109/24.9840
Wang M, Wu F, Su YF (2008). Electrochemical characteristics of Y2O3-coated LiCo1/3Ni1/3Mn1/3O2. Acta Physico-Chimica Sinica. 24(7):1175-1179.
Xia XT, Xu YZ, Jin YP, Shang YT, Chen L (2013). Assessment of optimum confidence interval of reliability with three-parameter Weibull distribution using bootstrap weighted-norm method. J. Aerospace Power 28(3);481-488.
Xu GB, Zhu ZF, Cao JF (2012). The research of material technology of the key casting steel of high-speed trains [C]. The Proceedings of China Railway Institute and Vehicle Commission Trains and Coach Academic Communication, pp. 177-181.
Yan XD, Ma X, Zheng RY (2005). Comparison of the Parameters Estimation Methods for 3-Parameter Weibull Distribution [J]. J. Ningbo Univ. (Natl. Sci. Eng. Edition), 18(3):301-305.
Zhao XP, Lv ZZ, Wang WH (2010). A Practical and Effective Method for Estimating Confidence Limits of Population Percentile and Reliability of Three-Parameter Weibull Distribution on Probability Weighted Moment. J. Northwestern Polytech. Univ. 38(3):470-475.
Zhuang WF (1999). The Least Square Assessment of Bilinear Regression for Weibull Parameters Realized by Microcomputer [J]. Electron. Product Reliab. Environ. Test. 5:2-7.