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$(G'/G, 1/G)$-expansion method for traveling wave solutions of (2+1) dimensional generalized KdV, Sin Gordon and Landau-Ginzburg-Higgs Equations

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The $(G'/G, 1/G)$-expansion method is one of the most direct and effective method for obtaining exact solutions of nonlinear partial differential equations (PDEs). In the present article, we construct the exact travelling wave solutions of nonlinear evolution equations in mathematical physics via (2+1) dimensional generalized KdV, Sin Gordon Equation and Landau-Ginzburg-Higgs Equation by $(G'/G, 1/G)$-expansion method, where $G(\xi)$ satisfies the auxiliary ordinary differential equation (ODE) $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$ where λ and μ are arbitrary constants.

Key words: (2+1) dimensional generalized KdV, Sin Gordon Equation, Landau- Ginzburg-Higgs equation, $(G'/G, 1/G)$-expansion method, auxiliary equation, travelling wave solutions.

INTRODUCTION

Nonlinear evolution equations play a significant role in various scientific and engineering fields, such as, optical fibers, solid state physics, fluid mechanics, plasma physics, chemical kinematics, chemical physics geochemistry, etc. Nonlinear wave phenomena of diffusion, reaction, dispersion, dissipation, and convection are very important in nonlinear wave equations. In recent years, the exact solutions of nonlinear PDEs have been investigated by many researchers who are concerned in nonlinear physical phenomena and many powerful and efficient methods have been offered by them. Among non-integrable nonlinear differential equations, there is a wide class of equations that is referred to as the partially integrable, because these equations become integrable for some values of their parameters. There are many different methods to look for the exact solutions of these equations. The most famous algorithms are the truncated Painleve expansion method (Liu et al., 2001), the Weierstrass elliptic function method (Kudryashov, 2009), the tanh-function method (Abdou, 2007; El-Wakil et al., 2010; Fan, 2000; Wazwaz, 2008a, b; Zhang et al., 2002) and the Jacobi elliptic function expansion method (Chen and Wang, 2005; Liu et al., 2001; Lu, 2005; Wazzan, 2009; Yomba, 2008; Yusufoglu and Bekir, 2008). There are other methods which can be found in (Kawahara, 1972; Wang and Zhang, 2007; Wang et al., 2005). For integrable nonlinear differential equations, the inverse scattering transform method (Ablowitz and Clarkson, 1991), the Hirota method (Hirot, 1971), the truncated Painleve expansion method (Zayed et al., 2007), the Backlund transform method (Miura, 1978; Rogers and Shadwick, 1982) and the Exp- function method (Naher et al., 2011, 2012; He and Wu, 2006; Inan, 2010; Akbar and Ali, 2011a, b; Akbar et al., 2012a) are used for searching the exact solutions. Wazwaz (2008a) introduced a direct and concise method, called the (G'/G)-expansion method

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to look for traveling wave solutions of nonlinear PDEs, where $G = G(\xi)$ satisfies the second order linear ODE $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$; λ and μ are arbitrary constants. For additional references see the articles (Akbar et al., 2012b, c; El-Wakil et al., 2010; Parkes, 2010; Zayed et al., 2004a, b; Akbar and Ali, 2012; Zhang and Xia, 2007, 2008).

In this article, we bring in an alternate approach, called $(G'/G, 1/G)$ –expansion method to find the traveling wave solutions of the, via (2+1) dimensional Generalized KdV, Sine-Gordon Equation and Landau-Ginzburg-Higgs Equation where $G = G(\xi)$ satisfies the auxiliary ODE $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$. λ and μ are arbitrary constants. Recently, El-Wakil et al. (2010) and Parkes (2010) have shown that the extended tanh-function method proposed by Fan (2000) and the basic (G'/G)-expansion method proposed by Wang et al. (2008) are entirely equivalent in as much as they deliver exactly the same set of solutions to a given nonlinear evolution equation. This observation has also been pointed out recently by Kudryashov (2009). In this article, we assert even though the basic (G'/G)-expansion method is equivalent to the extended tanh-function method, the further improved (G'/G)-expansion method presented in this letter is not equivalent to the extended tanh-function method. The method projected in this article is varied to some extent from the extended (G'/G)-expansion method. Further solitary wave solutions are achieved via the $(G'/G, 1/G)-$ expansion method. This approach will play an imperative role in constructing many exact travelling wave solutions for the nonlinear PDEs via (2+1) dimensional Generalized KdV, Sin Gordan and Landau-Ginzburg-Higgs Equations. It is worth mentioning that an exemplary work is made by Yang (2012a, b, 2013) on fractional calculus and its applications.

METHODOLOGY

Here, we describe the main steps of the (*G´*/*G,* 1/*G*)-expansion method for finding travelling wave solutions of nonlinear evolution equations. Suppose a nonlinear equation for $P(x,t)$ is given by

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \dots) = 0, \quad (1)$$

in which both nonlinear term(s) and higher order derivatives of $P(x,t)$ are all involved. In general, the left-hand side of Equation 1 is a polynomial in ψ and its various derivatives. The (*G´*/*G,* 1/*G*)-expansion method for solving Equation 1 proceeds in the following steps:

Step 1: Look for traveling wave solution of Equation 1 by taking

$$P = P(\xi), \qquad \xi = x \pm Vt, \quad (2)$$

where V is nonzero constant, $P(\xi)$ the function of ξ. Substituting Equation 2 into Equation 1 yields an ordinary differential equation (ODE) for $P(\xi)$.

$$Q(u, \pm Vu', u', V^2u'', u'', \dots) = 0. \quad (3)$$

Step 2: If possible, integrate Equation 3 term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) may be set to zero.

Step 3: According to the (*G´*/*G,* 1/*G*)-expansion method supposes that $u(\xi)$ can be expressed by a finite power series of

$$u(\xi) = \sum_{n=0}^{M} a_n \varphi^n + \sum_{n=1}^{M} b_n \varphi^{n-1}\psi, \quad (4)$$

where $a_n (n = 0,1,2,\dots,M)$ and $b_n (n = 1,2,\dots,M)$ are constants to be determine later and $\varphi(\xi)$ and $\psi(\xi)$ are

$$\varphi(\xi) = \left(\frac{G'(\xi)}{G(\xi)}\right), \psi(\xi) = \left(\frac{1}{G(\xi)}\right), \quad (5)$$

which satisfied

$$G''(\xi) + \lambda G(\xi) = \mu. \quad (6)$$

Then Equations 5 and 6 yields

$$\varphi' = -\varphi^2 + \mu\varphi - \lambda, \ \psi' = -\varphi\psi, \quad (7)$$

From the three cases of general solutions of the Equation 6, we have:

Case 1:

When $\lambda > 0$, the general solution of Equation 6 is

$$G(\xi) = A_1 \sinh(\sqrt{-\lambda}\xi) + A_2 \cosh(\sqrt{-\lambda}\xi) + \frac{\mu}{\lambda},$$

we have

$$\psi^2 = -\frac{\lambda}{\lambda^2\sigma + \mu^2}(\varphi^2 - 2\mu\psi + \lambda), \quad (8)$$

where A_1 and A_2 are two arbitrary constants and $\sigma = A_1^2 - A_2^2$.

Case 2:

When $\lambda < 0$, the general solution of Equation 6 is

$$G(\xi) = A_1 \sin(\sqrt{\lambda}\xi) + A_2 \cos(\sqrt{\lambda}\xi) + \frac{\mu}{\lambda},$$

we have

$$\psi^2 = \frac{\lambda}{\lambda^2\sigma - \mu^2}(\varphi^2 - 2\mu\psi + \lambda), \tag{9}$$

where A_1 and A_2 are two arbitrary constants and $\sigma = A_1^2 + A_2^2$.

Step 4: Determine M. This, usually, can be accomplished by balancing the linear term of highest order with the highest order nonlinear term which obtained in Step 2.

Step 5: Substituting Equation 4 into Equation 3 using Equation 7 and Equation 8 will yield a polynomial in φ and ψ, in which the degree of ψ is not larger than 1.Compare the like powers of φ^M and $\varphi^M\psi$ equal to zero, yields a set of algebraic equations for $a_n(n = 0,1,2,\dots,M), \mu, \lambda(\lambda < 0), A_1, A_2$ and V.

Step 6: Solve the system which obtained in step 5 for $a_n(n = 0,1,2,\dots,M), \mu, \lambda(\lambda < 0), A_1, A_2$ and V with the help of Maple 13, to determine these constants. Putting these constant into Equation 4, one can obtain the travelling wave solutions expressed by the hyperbolic functions of Equation 2. We can obtain the more general type and new exact traveling wave solution of the nonlinear partial differential Equation 1.

Step 7: Similarly substituting Equation 4 into Equation 3 using Equation 7 and 9 will yield a polynomial in φ and ψ, in which the degree of ψ is not larger than 1. Compare the like powers of φ^M and $\varphi^M\psi$ equal to zero, yields a set of algebraic equations for $a_n(n = 0,1,2,\dots,M), \mu, \lambda(\lambda < 0), A_1, A_2$ and V. Then we obtain one more solution which expressed by trigonometric functions as proceeding before.

APPLICATIONS

Here, we apply (*G´/G*, 1/*G*)-expansion method to construct traveling wave solution of the (2+1) dimensional Generalized KdV, Sin Gordan Equation and Landau-Ginzburg-Higgs Equation. Numerical results are very encouraging.

(2+1) dimensional Generalized KdV Equation

$$\frac{\partial^2}{\partial t \partial x}u + \frac{\partial}{\partial x}u.\frac{\partial^2}{\partial x^2}u + \frac{\partial^4}{\partial x^4}u + \frac{\partial^2}{\partial y^2}u = 0,$$

Consider the transformation $u(x,t) = u(\xi)$, $\xi = kx + ly + \omega t$ we have

$$\left(\frac{d^2}{d\xi^2}U(\xi)\right)k\,\omega + \left(\frac{d}{d\xi}U(\xi)\right)k^3\left(\frac{d^2}{d\xi^2}U(\xi)\right) + \left(\frac{d^4}{d\xi^4}U(\xi)\right)k^4 + \left(\frac{d^2}{d\xi^2}U(\xi)\right)l^2 = 0, \tag{10}$$

By applying the balancing principle we have $M = 1$. Therefore the trail solution is

$$u = a_1\varphi + b_1(\psi). \tag{11}$$

Putting Equation 11 into Equation 10 with Equation 5, we have

$$\begin{aligned}&-60a_1\varphi^3\psi\mu + 30a_1\psi^2\mu^2\varphi - 36b_1\psi^2\mu\varphi^2 + 28b_1\varphi^2\psi\lambda - 11b_1\psi^2\mu\lambda + 5b_1\psi\lambda^2k^4 - 3k\omega a_1\varphi\psi\mu - 5k^3a_1\varphi^2b_1\psi\lambda\\&+5k^3a_1^2\psi\mu\varphi\lambda + 2k^3a_1\psi^2\mu b_1\lambda + 40a_1\varphi^3\lambda + 24b_1\varphi^4\psi + 16a_1\varphi\lambda^2 + 6b_1\psi^3\mu^2 - 45a_1\varphi\psi\mu\lambda\\&+24a_1\varphi^5 + 2k\omega a_1\varphi\lambda + 2k\omega b_1\varphi^2\psi - k\omega b_1\psi^2\mu + k\omega b_1\psi\lambda + 5k^3a_1^2\varphi^3\psi\mu - 4k^3a_1\varphi^4b_1\psi\\&-3k^3a_1^2\psi^2\mu^2\varphi - k^3a_1\psi^3\mu^2b_1 - k^3a_1\lambda^2b_1\psi + k^3b_1^2\varphi\psi^3\mu - k^3b_1^2\varphi\psi^2\lambda - 3l^2a_1\varphi\psi\mu - 2k^3a_1^2\varphi^5 + 2l^2a_1\varphi^3 + 2k\omega a_1\varphi^3\\&-4k^3a_1^2\varphi^3\lambda - 2k^3a_1^2\lambda^2\varphi - 2k^3b_1^2\varphi^3\psi^2 + 2l^2a_1\varphi\lambda + 2l^2b_1\varphi^2\psi - l^2b_1\psi^2\mu + l^2b_1\psi\lambda + 6k^3a_1\varphi^2b_1\psi^2\mu = 0,\end{aligned}$$

Comparing the like powers of φ and ψ we have system of equations:

$$C_1 = \frac{3k^3a_1^2\lambda^2\mu^2}{\lambda^2\sigma + \mu^2} - \frac{30a_1\lambda^2\mu^2}{\lambda^2\sigma + \mu^2} + \frac{k^3b_1^2\lambda^3}{\lambda^2\sigma + \mu^2} - \frac{2k^3b_1^2\lambda^3\mu^2}{(\lambda^2\sigma + \mu^2)(\lambda^2r + \mu^2)} + 16a_1\lambda^2 + 2k\omega a_1\lambda - 2k^3a_1^2\lambda^2 + 2l^2a_1\lambda = 0,$$

$$C_2 = \frac{l^2 b_1 \lambda \mu}{\lambda^2 \sigma + \mu^2} - \frac{12 b_1 \lambda^2 \mu^3}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} + \frac{47 b_1 \lambda^2 \mu}{\lambda^2 \sigma + \mu^2} + \frac{2 k^3 a_1 \lambda^2 \mu^3 b_1}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} - \frac{8 k^3 a_1 \lambda^2 \mu b_1}{\lambda^2 \sigma + \mu^2} + \frac{k \omega b_1 \lambda \mu}{\lambda^2 \sigma + \mu^2} = 0,$$

$$C_3 = \frac{3 k^3 a_1^2 \lambda \mu^2}{\lambda^2 \sigma + \mu^2} - \frac{30 a_1 \lambda \mu^2}{\lambda^2 \sigma + \mu^2} + \frac{3 k^3 b_1^2 \lambda^2}{\lambda^2 \sigma + \mu^2} - \frac{2 k^3 b_1^2 \lambda^2 \mu^2}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} + 40 a_1 \lambda + 2 l^2 a_1 + 2 k \omega a_1 - 4 k^3 a_1^2 \lambda = 0$$

$$C_4 = \frac{36 b_1 \lambda \mu}{\lambda^2 \sigma + \mu^2} - \frac{6 k^3 a_1 b_1 \lambda \mu}{\lambda^2 \sigma + \mu^2} = 0,$$

$$C_5 = \frac{2 k^3 b_1^2 \lambda}{\lambda^2 \sigma + \mu^2} + 24 a_1 - 2 k^3 a_1^2 = 0,$$

$$C_6 = -\frac{2 k^3 a_1 \lambda^3 \mu b_1}{\lambda^2 \sigma + \mu^2} + \frac{k \omega b_1 \lambda^2 \mu}{\lambda^2 \sigma + \mu^2} + \frac{l^2 b_1 \lambda^2 \mu}{\lambda^2 \sigma + \mu^2} - \frac{12 b_1 \lambda^3 \mu^3}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} + \frac{11 b_1 \lambda^3 \mu}{\lambda^2 \sigma + \mu^2} + \frac{2 k^3 a_1 \lambda^3 \mu^3 b_1}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} = 0,$$

$$C_7 = 5 b_1 \lambda^2 k^4 - \frac{2 l^2 b_1 \lambda \mu^2}{\lambda^2 \sigma + \mu^2} + \frac{24 b_1 \lambda^2 \mu^4}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} - \frac{28 b_1 \lambda^2 \mu^2}{\lambda^2 \sigma + \mu^2} - \frac{4 k^3 a_1 \lambda^2 \mu^4 b_1}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} + \frac{5 k^3 a_1 \lambda^2 \mu^2 b_1}{\lambda^2 \sigma + \mu^2} - \frac{2 k \omega b_1 \lambda \mu^2}{\lambda^2 \sigma + \mu^2}$$
$$+ k \omega b_1 \lambda - k^3 a_1 \lambda^2 b_1 + l^2 b_1 \lambda = 0,$$

$$C_8 = -3 k \omega a_1 \mu + 5 k^3 a_1^2 \mu \lambda + \frac{60 a_1 \lambda \mu^3}{\lambda^2 \sigma + \mu^2} - 45 a_1 \mu \lambda - 3 l^2 a_1 \mu + \frac{4 k^3 b_1^2 \lambda^2 \mu^3}{(\lambda^2 \sigma + \mu^2)(\lambda^2 r + \mu^2)} - \frac{3 k^3 b_1^2 \lambda^2 \mu}{\lambda^2 \sigma + \mu^2} - \frac{6 k^3 a_1^2 \lambda \mu^3}{\lambda^2 \sigma + \mu^2} = 0,$$

$$C_9 = -5 k^3 a_1 b_1 \lambda - \frac{78 b_1 \lambda \mu^2}{\lambda^2 \sigma + \mu^2} + 2 k \omega b_1 + \frac{13 k^3 a_1 \lambda \mu^2 b_1}{\lambda^2 \sigma + \mu^2} + 28 b_1 \lambda + 2 l^2 b_1 = 0$$

$$C_{10} = 5 k^3 a_1^2 \mu - \frac{5 k^3 b_1^2 \lambda \mu}{\lambda^2 \sigma + \mu^2} - 60 a_1 \mu = 0,$$

$$C_{11} = -4 k^3 a_1 b_1 + 24 b_1 = 0$$

Solving the above system, we have one solution set.

1st Solution set

Case 1

When $\lambda > 0$

$$a_1 = -6, b_1 = 6\sqrt{\sigma \omega - \sigma l^2}, \lambda = -\omega + l^2, \mu = 0, k = -1.$$

Which yields

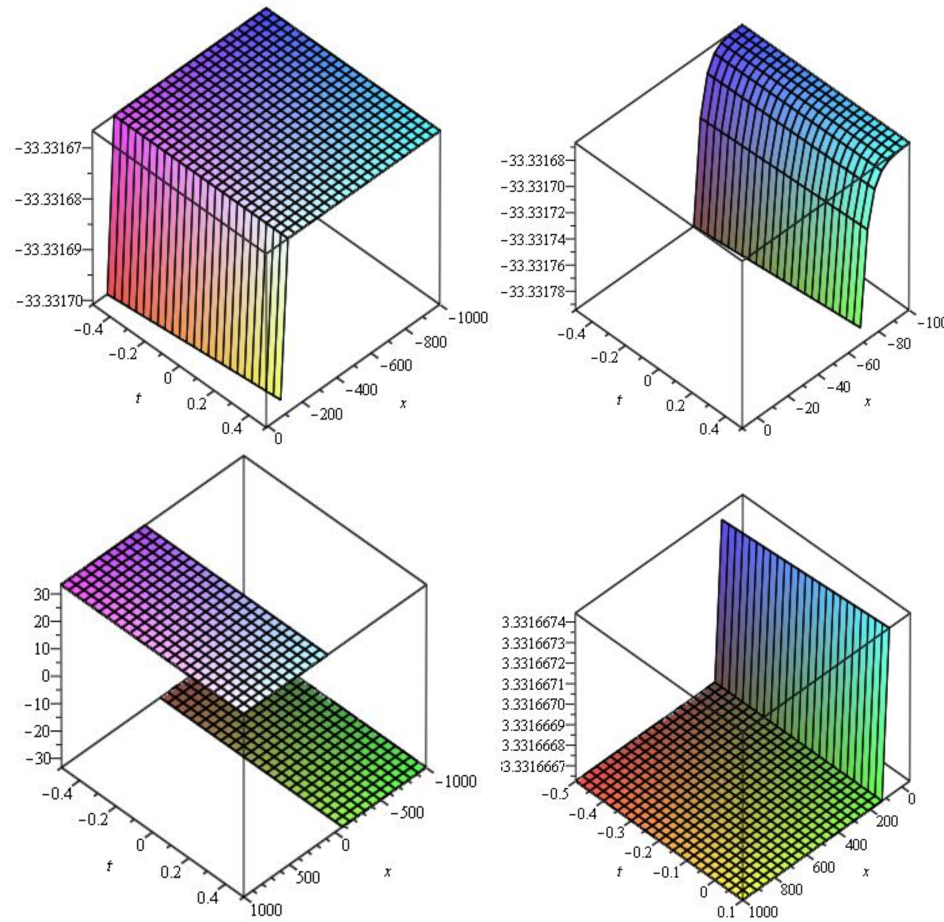

Figure 1. 1st solution set (Case 1 - When $\lambda > 0$ for (2+1) dimensional Generalized KdV Equation.

$$u(x,t)=\frac{6(-l^2+\omega)^{3/2}\left(A_1\cosh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)+A_2\sinh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)\right)}{A_1\sinh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)(-\omega+l^2)+A_2\cosh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)(-\omega+l^2)}+\frac{6\sqrt{\omega(A_1^2-A_2^2)-l^2(A_1^2-A_2^2)}(-\omega+l^2)}{A_1\sinh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)(-\omega+l^2)+A_2\cosh\left(\sqrt{-l^2+\omega}(-x+ly+\omega t)\right)(-\omega+l^2)}$$

as shown in Figure 1.

Case 2

When $\lambda < 0$

$$a_1=-6,\ b_1=6\sqrt{\sigma\omega-\sigma l^2},\ \lambda=-\omega+l^2,\ \mu=0,\ k=-1.$$

Which yield

$$u(x,t)=\frac{6(-l^2+\omega)^{3/2}\left(A_1\cos\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)-A_2\sin\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)\right)}{A_1\sin\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)(-\omega+l^2)+A_2\cos\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)(-\omega+l^2)}+\frac{6\sqrt{\omega(A_1^2+A_2^2)-l^2(A_1^2+A_2^2)}(-\omega+l^2)}{A_1\sin\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)(-\omega+l^2)+A_2\cos\left(\sqrt{l^2-\omega}(-x+ly+\omega t)\right)(-\omega+l^2)}$$

Landau-Ginzburg-Higgs (LGH) Equation

$$\frac{\partial^2}{\partial t^2}u + \frac{\partial^2}{\partial x^2}u + m^2 u + n^2 u^3 = 0,$$

Consider the transformation $u(x,t) = u(\xi), \xi = kx + \omega t$ we have

$$\left(\frac{d^2}{d\xi^2} U(\xi)\right)\omega^2 + \left(\frac{d^2}{d\xi^2} U(\xi)\right)k^2 + m^2 U(\xi) + n^2 U(\xi)^3 = 0 \quad (12)$$

By applying the balancing principle we have $M = 1.$ Therefore the trail solution is

$$u = a_1\varphi + b_1(\psi). \quad (13)$$

Putting Equation 13 into Equation 12 with Equation 5, we have

$$2\omega^2 a_1\varphi^3 - 3\omega^2 a_1\varphi\psi\mu + 2\omega^2 a_1\varphi\lambda + 2\omega^2 b_1\varphi^2\psi - \omega^2 b_1\psi^2\mu + \omega^2 b_1\psi\lambda + 2k^2 a_1\varphi^3 - 3k^2 a_1\varphi\psi\mu + 2k^2 a_1\varphi\lambda + 2k^2 b_1\varphi^2\psi$$
$$- k^2 b_1\psi^2\mu + k^2 b_1\psi\lambda + m^2 a_1\varphi + m^2 b_1\psi + n^2 a_1^3\varphi^3 + 3n^2 a_1^2\varphi^2 b_1\psi + 3n^2 a_1\varphi b_1^2\psi^2 + n^2 b_1^3\psi^3 = 0,$$

Comparing the like powers of φ and ψ we have system of equation

$$C_1 = -\frac{3n^2 a_1 b_1^2\lambda^2}{\lambda^2\sigma + \mu^2} + 2\omega^2 a_1\lambda + 2k^2 a_1\lambda + m^2 a_1 = 0,$$

$$C_2 = \frac{\omega^2 b_1\lambda\mu}{\lambda^2\sigma + \mu^2} + \frac{k^2 b_1\lambda\mu}{\lambda^2\sigma + \mu^2} - \frac{2n^2 b_1^3\lambda^2\mu}{(\lambda^2\sigma + \mu^2)(\lambda^2 r + \mu^2)} = 0,$$

$$C_3 = 2\omega^2 a_1 - \frac{3n^2 a_1 b_1^2\lambda}{\lambda^2\sigma + \mu^2} + 2k^2 a_1 + n^2 a_1^3 = 0,$$

$$C_4 = -\frac{2n^2 b_1^3\lambda^3\mu}{(\lambda^2\sigma + \mu^2)(\lambda^2 r + \mu^2)} + \frac{k^2 b_1\lambda^2\mu}{\lambda^2\sigma + \mu^2} + \frac{\omega^2 b_1\lambda^2\mu}{\lambda^2\sigma + \mu^2} = 0,$$

$$C_5 = -\frac{2\omega^2 b_1\lambda\mu^2}{\lambda^2\sigma + \mu^2} - \frac{2k^2 b_1\lambda\mu^2}{\lambda^2\sigma + \mu^2} + \omega^2 b_1\lambda + k^2 b_1\lambda + \frac{4n^2 b_1^3\lambda^2\mu^2}{(\lambda^2\sigma + \mu^2)(\lambda^2 r + \mu^2)} + m^2 b_1 - \frac{n^2 b_1^3\lambda^2}{\lambda^2\sigma + \mu^2} = 0,$$

$$C_6 = -3\omega^2 a_1\mu - 3k^2 a_1\mu + \frac{6n^2 a_1 b_1^2\lambda\mu}{\lambda^2\sigma + \mu^2} = 0,$$

$$C_7 = -\frac{n^2 b_1^3\lambda}{\lambda^2\sigma + \mu^2} + 2\omega^2 b_1 + 2k^2 b_1 + 3n^2 a_1^2 b_1 = 0.$$

Solving the above system, we have one solution set

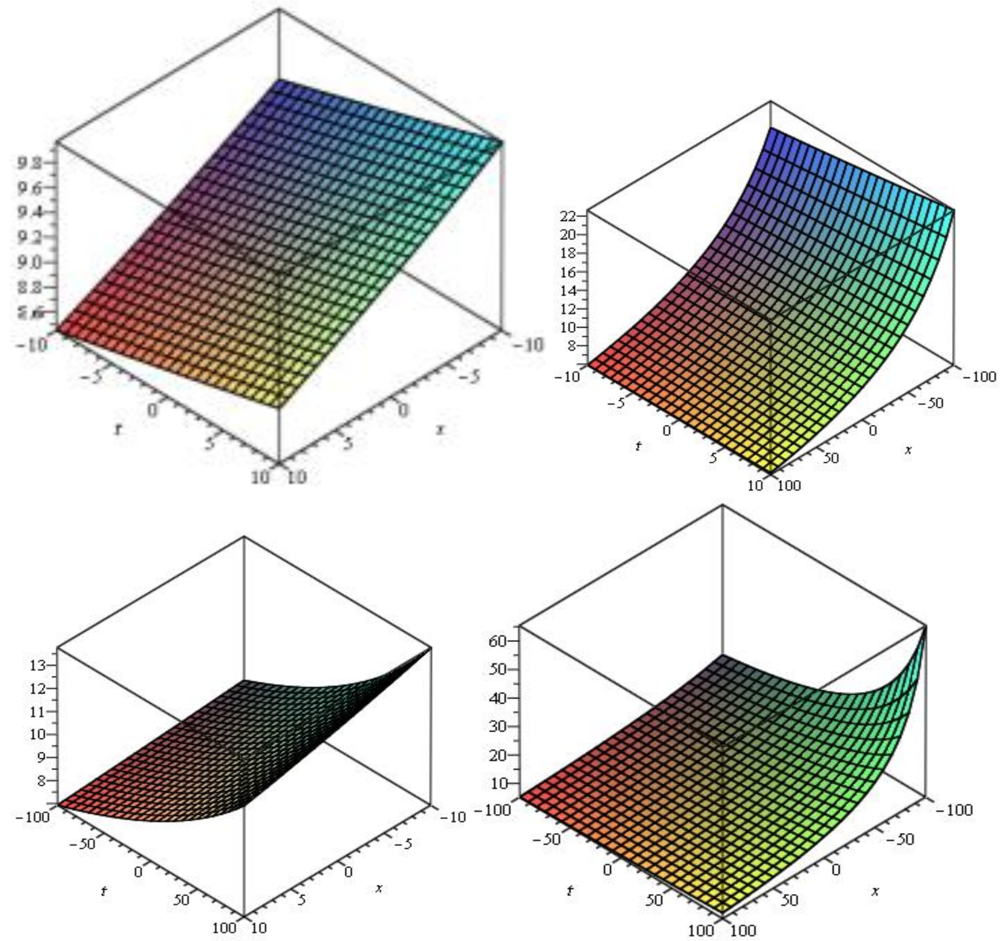

Figure 2. 1st solution set (Case 1 - When $\lambda > 0$ for Landau-Ginzburg-Higgs (LGH) Equation.

1st Solution set

Case 1

When $\lambda > 0$

$$a_1 = \frac{\sqrt{-\frac{1}{2}\,\omega^2 - \frac{1}{2}\,k^2}}{n},\ b_1 = \frac{\sqrt{-\sigma}\ m}{n},\ \lambda = -\frac{2\,m^2}{\omega^2 + k^2},\ \mu = 0$$

which yields

$$u(x,t) = -\left(\frac{\sqrt{-2\omega^2 - 2k^2}\sqrt{2}\left(\frac{m^2}{\omega^2+k^2}\right)^{\frac{3}{2}}\left(A_1\cosh\left(\sqrt{2}\sqrt{\frac{m^2}{\omega^2+k^2}}\,(kx+\omega t)\right) + A_2\sinh\left(\sqrt{2}\sqrt{\frac{m^2}{\omega^2+k^2}}\,(kx+\omega t)\right)\right)}{\left(n\left(-\frac{2A_1\sinh\left(\sqrt{2}\sqrt{\frac{m^2}{\omega^2+k^2}}\,(kx+\omega t)\right)m^2}{\omega^2+k^2} - \frac{2A_2\cosh\left(\sqrt{2}\sqrt{\frac{m^2}{\omega^2+k^2}}\,(kx+\omega t)\right)m^2}{\omega^2+k^2}\right)\right)}\right) + \quad \ldots$$

as shown in Figure 2.

Case 2

When $\lambda < 0$

$$a_1 = \frac{\sqrt{-\frac{1}{2}\omega^2 - \frac{1}{2}k^2}}{n}, b_1 = \frac{\sqrt{-\sigma}\, m}{n}, \lambda = -\frac{2m^2}{\omega^2 + k^2}, \mu = 0$$

which yields

$$u(x,t) = \frac{1}{2}\left(\frac{\sqrt{-2\omega^2-2k^2}\left(\frac{-2m^2}{\omega^2+k^2}\right)^{\frac{3}{2}}\left(A_1\cos\left(\sqrt{\frac{-2m^2}{\omega^2+k^2}}(kx+\omega t)-A_2\sin\left(\sqrt{\frac{-2m^2}{\omega^2+k^2}}(kx+\omega t)\right)\right)\right)}{\left(n\left(\frac{-2A_1\sin(\sqrt{\frac{-2m^2}{\omega^2+k^2}}(kx+\omega t)}{\omega^2+k^2} - \frac{2A_2\cos\left(\sqrt{2}\sqrt{\frac{m^2}{\omega^2+k^2}}(kx+\omega t)\right)m^2}{\omega^2+k^2}\right.\right.}\right. + \cdots,$$

Sin Gordan Equation

$$u_{tt} - u_{xx} + u - \frac{1}{6}u^3 = 0.$$

Consider the transformation $u(x,t) = u(\xi), \xi = kx + \omega t$ we have

$$\omega^2\frac{d^2U}{d\xi^2} - k^2\frac{d^2U}{d\xi^2} + U(\xi) - \frac{1}{6}\left(U(\xi)\right)^3 = 0. \qquad (14)$$

By applying the balancing principle we have $M = 1.$ Therefore the trail solution is

$$u = a_1\varphi + b_1(\psi). \qquad (15)$$

Putting Equation 14 into Equation 15 with Equation 5, we have

$$2\omega^2 a_1\varphi^3 - 3\omega^2 a_1\varphi\psi\mu + 2\omega^2 a_1\varphi\lambda + 2\omega^2 b_1\varphi^2\psi - \omega^2 b_1\psi^2\mu + \omega^2 b_1\psi\lambda - 2k^2 a_1\varphi^3$$
$$+ 3k^2 a_1\varphi\psi\mu - 2k^2 a_1\varphi\lambda - 2k^2 b_1\varphi^2\psi + k^2 b_1\psi^2\mu - k^2 b_1\psi\lambda + a_1\varphi + b_1\psi - \frac{1}{6}a_1^3\varphi^3 - \frac{1}{2}a_1^2\varphi^2 b_1\psi - \frac{1}{2}a_1\varphi b_1^2\psi^2 - \frac{1}{6}b_1^3\psi^3.$$

Comparing the like powers, φ and ψ we have system of equation

$$C_1 = a_1 + 2\omega^2 a_1\lambda - 2k^2 a_1\lambda + \frac{1}{2}\frac{a_1 b_1^2\lambda^2}{\lambda^2\sigma + \mu^2} = 0,$$

$$C_2 = \frac{\omega^2 b_1\lambda\mu}{\lambda^2\sigma + \mu^2} - \frac{k^2 b_1\lambda\mu}{\lambda^2\sigma + \mu^2} + \frac{1}{3}\frac{b_1^3\lambda^2\mu}{(\lambda^2\sigma + \mu^2)(\lambda^2 r + \mu^2)} = 0,$$

$$C_3 = -2k^2 a_1 + 2\omega^2 a_1 - \frac{1}{6}a_1^3 + \frac{1}{2}\frac{a_1 b_1^2\lambda}{\lambda^2\sigma + \mu^2} = 0,$$

$$C_4 = -\frac{2\omega^2 b_1 \lambda \mu^2}{\lambda^2\sigma+\mu^2} + \frac{2k^2 b_1 \lambda \mu^2}{\lambda^2\sigma+\mu^2} - \frac{2}{3}\frac{b_1^3\lambda^2\mu^2}{(\lambda^2\sigma+\mu^2)(\lambda^2 r+\mu^2)} + b_1 + \omega^2 b_1 \lambda - k^2 b_1 \lambda + \frac{1}{6}\frac{b_1^3\lambda^2}{\lambda^2\sigma+\mu^2} = 0,$$

$$C_5 = \frac{\omega^2 b_1 \lambda^2 \mu}{\lambda^2\sigma+\mu^2} - \frac{k^2 b_1 \lambda^2 \mu}{\lambda^2\sigma+\mu^2} + \frac{1}{3}\frac{b_1^3\lambda^3\mu}{(\lambda^2\sigma+\mu^2)(\lambda^2 r+\mu^2)} = 0,$$

$$C_6 = -\frac{a_1 b_1^2 \lambda \mu}{\lambda^2\sigma+\mu^2} - 3\omega^2 a_1 \mu + 3k^2 a_1 \mu = 0,$$

$$C_7 = 2\omega^2 b_1 - 2k^2 b_1 - \frac{1}{2}a_1^2 b_1 + \frac{1}{6}\frac{b_1^3\lambda}{\lambda^2\sigma+\mu^2} = 0.$$

Solving the above system, we have one solution set.

1st Solution set

Case 1

When $\lambda > 0$

$$a_1 = \sqrt{3\omega^2 - 3k^2},\ b_1 = \sqrt{6}\sqrt{\sigma},\ \lambda = \frac{2}{-\omega^2+k^2},\ \mu = 0.$$

Which yields

$$u(x,t) = -\frac{\sqrt{3\omega^2-3k^2}\left(-\frac{2}{-\omega^2+k^2}\right)^{3/2}\left(A_1\cosh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)+A_2\sinh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)\right)}{\frac{2A_1\sinh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}+\frac{2A_2\cosh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}}$$
$$+\frac{2\sqrt{6}\sqrt{A_1^2-A_2^2}}{(-\omega^2+k^2)\left(\frac{2A_1\sinh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}+\frac{2A_2\cosh\left(\sqrt{-\frac{2}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}\right)}.$$

As shown in Figure 3.

Case 2

When $\lambda < 0$

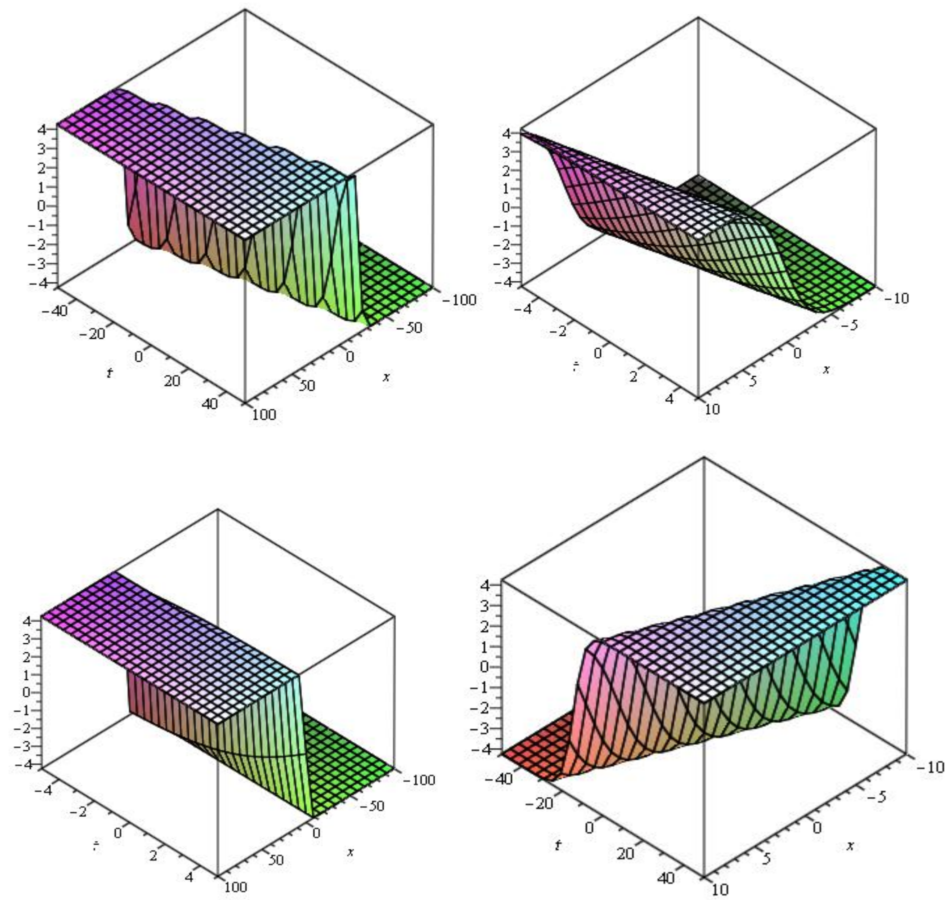

Figure 3. 1st solution set (Case 1 - When $\lambda > 0$ for Sin Gordan Equation.

$$a_1 = \sqrt{3\omega^2 - 3k^2},\; b_1 = \sqrt{6}\sqrt{\sigma},\; \lambda = \frac{2}{-\omega^2 + k^2},\; \mu = 0.$$

Which yields

$$u(x,t) = \frac{2\sqrt{3\omega^2 - 3k^2}\sqrt{2}\left(\frac{1}{-\omega^2+k^2}\right)^{3/2}\left(A_1 \cos\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right) - A_2 \sin\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right)\right)}{\frac{2A_1 \sin\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2} + \frac{2A_2 \cos\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}}$$

$$+\frac{2\sqrt{6}\sqrt{A_1^2 + A_2^2}}{(-\omega^2+k^2)\left(\frac{2A_1 \sin\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2} + \frac{2A_2 \cos\left(\sqrt{2}\sqrt{\frac{1}{-\omega^2+k^2}}\,(kx+\omega t)\right)}{-\omega^2+k^2}\right)}.$$

Conclusion

(*G*′/*G*, 1/*G*)-expansion method is applied to obtain generalized solitary solutions of nonlinear (2+1) dimensional generalized KdV, Sine-Gordan and Landau-Ginzburg-Higgs Equations. The main advantage of this scheme over others is that it possesses all types of exact solutions. Moreover, reliability of the algorithm and the reduction in the size of computational domain give this proposed technique a wider applicability.

REFERENCES

Abdou MA (2007). The extended tanh-method and its applications for solving nonlinear physical models. Appl. Math. Comput. 190:988-996.

Ablowitz MJ, Clarkson PA (1991). Nonlinear Evolution Equations and Inverse Scattering Transform. Cambridge University Press, Cambridge.

Akbar MA, Ali NHM (2011a). Exp-function method for Duffing Equation and New Solutions of (2+1) Dimensional Dispersive Long Wave Equations. Prog. Appl. Math. 1(2):30-42.

Akbar MA, Ali NHM, Zayed EME (2012a). A generalized and improved (G'/G)-expansion method for nonlinear evolution equations. Math. Prob. Eng. 22 pp. doi: 10.1155/(2012)/459879.

Akbar MA, Ali NHM (2012). New Solitary and Periodic Solutions of Nonlinear Evolution Equation by Exp-function Method. World Appl. Sci. J. 17(12):1603-1610.

Akbar MA, Ali NHM (2011b). The alternative (G'/G)-expansion method and its applications to nonlinear partial differential equations. Int. J. Phys. Sci. 6(35):7910-7920.

Akbar MA, Ali NHM, Mohyud-Din ST (2012b). The alternative (G'/G)-expansion method with generalized Riccati equation: Application to fifth order (1+1)-dimensional Caudrey-Dodd-Gibbon equation. Int. J. Phys. Sci. 7(5):743-752.

Akbar MA, Ali NHM, Zayed EME (2012c). Abundant exact traveling wave solutions of the generalized Bretherton equation via the improved (G'/G)-expansion method. Commun. Theor. Phys. 57:173-178.

Chen Y, Wang Q (2005). Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic functions solutions to (1+1)-dimensional dispersive long wave equation. Chaos Solit. Fract. 24:745-757.

El-Wakil SA, Abdou MA, El-Shewy EK, Hendi A (2010). (G'/G)-expansion method equivalent to the extended tanh-function method. Phys. Script. 81:35011-35014.

Fan EG (2000). Extended tanh-function method and its applications to nonlinear equations. Phys. Lett. A. 277:212-218.

He JH, Wu XH (2006). Exp-function method for nonlinear wave equations. Chaos Solit. Fract. 30:700-708.

Hirot R (1971). Exact solution of the KdV equation for multiple collisions of solutions. Phys. Rev. Lett. 27:1192-1194.

Inan IE (2010). Generalized Jacobi elliptic function method for traveling wave solutions of (2+1)-dimensional breaking soliton equation. Cankaya Univ. J. Sci. Eng. 7(1):39-50.

Kawahara T (1972). Oscillatory solitary waves in dispersive media. J. Phys. Soc. Jpn. 33:260-264.

Kudryashov NA (2009) .Seven common errors in finding exact solutions on nonlinear differential equations. Commun. Nonlin. Sci. Numer. Simul. 14:3507-3523.

Liu S, Fu Z, Liu SD, Zhao Q (2001). Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. Phys. Lett. A.289:69-74.

Lu D (2005). Jacobi elliptic function solutions for two variant Boussinesq equations. Chaos Solit. Fract. 24:1373-1385.

Miura MR (1978). Backlund Transformation. Springer-Verlag, Berlin.

Naher H, Abdullah FA, Akbar MA (2011). The Exp-function method for new exact solutions of the nonlinear partial differential equations. Int. J. Phys. Sci. 6(29): 6706-6716.

Naher H, Abdullah FA, Akbar MA (2012). New traveling wave solutions of the higher dimensional nonlinear partial differential equation by the Exp-function method J. Appl. Math. 14:doi: 10.1155/2012/575387.

Parkes EJ (2010). Observations on the basis of (G'/G)-expansion method for finding solutions to nonlinear evolution equations. Appl. Math. Comput. 217:1759-1763. doi:10.1016/j.amc.2010.03,073.

Rogers C, Shadwick WF (1982). Backlund Transformations. Academic Press, New York.

Wang DS, Ren YJ, Zhang HQ (2005). Further extended sinh-cosh and sin-cos methods and new non traveling wave solutions of the (2+1)-dimensional dispersive long wave equations. Appl. Math. E-Notes 5:157-163.

Wang M, Li X, Zhang J (2008). The (G'/G)-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A 372:417-423.

Wang Z, Zhang HQ (2007). A new generalized Riccati equation rational expansion method to a class of nonlinear evolution equation with nonlinear terms of any order. Appl. Math. Comput. 186:693-704.

Wazwaz AM (2008a). New solutions of distinct physical structures to high-dimensional nonlinear evolution equations. Appl. Math. Comput. 196:363-368.

Wazwaz AM (2008b). The extended tanh-method for new compact and non-compact solutions for the KP-BBM and the ZK-BBM equations. Chaos Solit. Fract. 38:1505-1516.

Wazzan L (2009). A modified tanh-coth method for solving the KdV and KdV-Burgers equation. Commun. Nonlin. Sci. Numer. Simul. 14: 443-450.

Yang X-J (2012a). Advanced Local Fractional Calculus and Its Applications, World Science Publisher, New York, USA.

Yang X-J (2012b). Cantor-type cylindrical-coordinate method for differential equations with local fractional derivatives. Phys. Lett. A 377(38-30):1696-1700.

Yang X-J (2013). Fractional heat conduction problem solved by local fractional variation iteration method. Therm. Sci. 17(2):625-628.

Yomba E (2008). A generalized auxiliary equation method and its application to nonlinear Klein-Gordon and generalized nonlinear Camassa-Holm equations. Phys. Lett. A 372:1048-1060.

Yusufoglu E, Bekir A (2008). Exact solution of coupled nonlinear evolution equations. Chaos Solit. Fract. 37:842-848.

Zayed EME, Abourabia AM, Gepreel KA, Horbaty MM (2007). Traveling solitary wave solutions for the nonlinear coupled KdV system. Chaos Solit. Fract. 34:292-306.

Zayed EME, Zedan HA, Gepreel KA (2004a). Group analysis and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations. Int. J. Nonlin. Sci. Num. Simul. 5:221-234.

Zayed EME, Zedan HA, Gepreel KA (2004b). On the solitary wave solutions for nonlinear Euler equations. Appl. Anal. 83:1101-1132.

Zhang S, Xia TC (2007). A generalized new auxiliary equation method and its applications to nonlinear partial differential equations. Phys. Lett. A 363:356-360.

Zhang S, Xia TC (2008). A further improved tanh-function method exactly solving the (2+1)-dimensional dispersive long wave equations. Appl. Math. E-Notes 8:58-66.

Zhang SL, Wu B, Lou SY (2002). Painleve analysis and special solutions of generalized Broer-Kaup equations. Phys. Lett. A. 300:40-48.