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Optimization strategy for air traffic flow in multi-airport network

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To avoid the airport congestion and reduce flight delays, the paper studied the airport traffic balance from a system perspective at the strategic level. By considering the single-airport approach and departure as well as the correlation between the multi-airport connecting flights, the paper proposed network system on traffic flow which is open and has a direction in multi-airport; the paper set up multiairport open network assignment model which was based on constraint capacity and multiple connecting flights and which minimized the total delay of all flights in the network within the target. The model was simulated which combined on the three major airports' actual flight data in domestic. Simulation results show that: the proposed model can coordinate and optimize the matching of multiairport network traffic and capacity. It also can minimize the system's delayed flights based on using of system capacity; the paper can deploy optimization strategy in traffic flow for air traffic control sector and provide supporting decision-making basis for the civil aviation department of the flight plan.

Key words: Air traffic flow management, multi-airport, assignment, optimization.

INTRODUCTION

Recently, air transportation grows rapidly with the development of the economy in China. It is difficult to satisfy air transportation demand for airspace capacity. Due to airspace congestion and flight delays, the growth of air transportation has slowed. Airports, as the key point in air transportation, have been becoming the bottleneck of air transportation development. Airport flow management which is to allocate the airport capacity to optimize the airport capacity and minimize flight delays belongs to strategic management of air transportation field. In the past decade, various sophisticated techniques and algorithms have been developed to improve the efficiency of airport. Gilbo (1993) considered arrival and departure operations as interdependent processes and strategically allocated the airport capacity between arrivals and departures to optimize air traffic flow. Gilbo (1997) proposed a new model which took into account the interaction between runway capacity and capacities of arrival and departure fixes to optimize air traffic flow through the airport system. Gilbo and Howard

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(2001) introduced collaborative decision making (CDM) into the airport flow optimization problem, and proposed a collaborative optimization model which used flight priorities from airlines and other users in order to provide the optimal allocation of airport capacity as well as fixes capacity. Dell' Olmo and Lulli (2003) proposed a dynamic programming model with a corresponding backward solution algorithm for general airport capacity envelopes. Janic (2007) developed a heuristic algorithm for the allocation of airport runway capacity to minimize the cost of arrival and departure flight delays. In summary, many researches have been conducted on flow assignment for single airport while few studies have investigated the cases of multi-airport airport. For the multi-airport network, the interdependence among airports is significant and flight delays occurring at each airport could spread from one airport to others in the multi-airport network. Furthermore, the spread of flight delays could result in large-scale delays, even whole network delays.

The flow assignment strategy is a combinatorial optimization which is considered as NP-hard problem. In this paper, a multi-airport capacity allocation and flow assignment model are first proposed in which considers the effect of multi-airport network and aims to minimize



Figure 1. Open multi-airport network.

the total delays of the multi-airport system. In this paper, a new heuristic algorithm-shuffled complex evolution method (SCE-UA) (Duan et al., 1992, 1993) is also developed to optimize the flow assignment at multi-airport network. The remainder of the paper is organized as follows: subsequently, we briefly introduce the open multiairport network, after which we formulate the model. Then, we design an algorithm to solve the model. Thereafter, a numerical test is performed and we have a discussion. Finally, the conclusion is described.

OPEN MULTI-AIRPORT NETWORK

The airport network is composed of all airports in some area. The open multi-airport network is composed of some airports which are part of the airport network. In multi-airport network, traffic flow is among some airports. For example, in Figure 1, some flights from other airports arrive at airport A and some flights departure from airport A. For airport B, some flights' terminate, but other flights which are called connecting flights arrive at airport B, then departure from it. That is to say, the target airport is not airport B, but other airport which is in the open multi-airport network is the target airport. For the network, the flights from or to other target airports (for example, the airport B or C) as well as airports outside the network are considered. As the same time, the flights among the airports whose capacities are confined (orange in Figure

1) are also discussed. In this paper, airport flow is composed of arrival flow and departure flow, while airport capacity is consist of arrival capacity and departure capacity. In most of the literature on air traffic flow management, arrival and departure flights are considered independently and airport capacity is assumed constantly. In practice, arrival and departure flights are interdependent, while airport capacity is not constant. Airport capacity is significantly affected by weather and other factors, for example, the aircraft types and their ratio.

Figure 2 (axises represent the numbers of arrival and departure flights per 15 min respectively) shows the relation between arrival capacity and departure capacity. As the point p in Figure 2, when air traffic demand exceeds the airport capacity, flight delay can hardly be avoided. To minimize flight delay, searching Pareto optimal solutions on or inside the airport capacity curve is acceptable.

MODEL FORMULATION

The strategy of the model could be described as follows: assume that air traffic congestion occur in the multiairport network in some period. We divide the period into several time intervals. Based on the utilization of each airport capacity, we assign the optimal arrival or departure time span to each flight. We are devoted to



Figure 2. Airport capacity curve.

balance air traffic demand and airport capacity and make the flow and the airport capacity of each time interval coordinated. The assumptions in the model are as follows:

1) The capacity curves for each airport affected by different weather conditions in given period are known.

2) All flight delay cost is known and the flying time between airports of which the capacities are confined are constants.

3) The air traffic demand which can not be accommodated in given period could be achieved in an extra time span following the period. That is to say, the capacity in the extra time span is infinite and feasible solutions of the model can be gained.

4) Flights cannot arrive or depart in advance and be delayed infinitely.

5) There are only constraints on airport capacity and the capacities of en routes and sectors are not considered.

The parameters used in the model are defined as follows:

T: the given period composed by several time spans of which the duration is Δ (15 min), where $T = \{t_1, t_2, ..., t_M\}$ and $t \in T$. t_{N+1} is defined as the extra time span with infinite capacity mentioned earlier.

l: the set of airports of which the capacities are confined in multi-airport network.

F: the set of flights which depart from airport *i* and arrive

at airport *j*, where *i* and *j* \in *I*.

F_i: the set of flights which depart from and arrive at airport *i* of which the capacity is confined in set *F*, where *i* \in *I* and $F_i \subseteq F \cdot F_i$ is composed of the set Dep_i of arrival flights and the set Arr_i of departure flights, where $F_i = Dep_i \bigcup Arr_i \cdot Dep_i = Dep_i^{ext} \bigcup Dep_i^{int}$, where Dep_i^{ext} is the set of flights that depart from airport *i* and arrive at airports of which the capacities are not confined, and Dep_i^{int} is the set of flights that depart from airport *i* and arrive at the airport *j* of which the capacity is confined. $Arr_i^{int} = Arr_i^{ext} \bigcup Arr_i^{int}$, where Arr_i^{ext} is the set of flights that depart from airport *j* of which the capacity is confined.

X: the set of connecting flights in set *I*, $X = \{(f, f') | f, f' \in F\}$, where flight *f* is leading flight and flight f' is following flight.

 t_f : the real arrival or departure time of flight f, where t_f^d

represents the real departure time and t_f^{a} represents the real arrival time.

 $\Delta t_{f,f}$: the time separation between connecting flight couple *f* and *f'*, where $(f, f') \in X$.

e_f: the expected arrival or departure time of flight f, where

 $e_f^{\rm d}$ represents the expected departure time and $e_f^{\rm a}$ represents the expected arrival time.

 T_f : the set of possible arrival or departure time spans of flight f, $T_f = \{t \in T | t \ge e_f\} \cup \{t_{N+1}\}.$

 $\Psi_t^i(u, v)$: the capacity curve of airport *i* in time span *t*, where $t \in T$ and $i \in I$.

 U_t^i , V_t^i : the maximum arrival capacity and departure capacity of airport i in time span t.

 u_t^i : the arrival flow assigned to airport *i* in time span *t*, $u_t^i = \sum v_i(t) i \in I, t \in T$

$$u_t^{\cdot} = \sum_{f \in F_i^a \cup A_i} y_f(t), t \in I, t \in I^{\cdot}.$$

 v_t^{\prime} : the departure flow assigned to airport *i* in time span *t*,

$$v_t^i = \sum_{f \in F_i^d \cup D_i} x_f(t), i \in I, t \in T$$
$$x_f(t) = \begin{cases} 1, \text{ flight } f \text{ depart in time span } t \\ 0, \text{ otherwise} \end{cases},$$
$$y_f(t) = \begin{cases} 1, \text{ flight } f \text{ arrive in time span } t \\ 0, \text{ otherwise} \end{cases}$$

The aim of the model is to minimize the total delay time of all flights in the open multi-airport network including delays of the flights among airports of which the capacities are confined (represented by departure delays), delays of the flights just departing from airports of which the capacities are confined (represented by departure delays) and delays of the flights just arriving at airports of which the capacities are confined (represented by arrival delays). The objective function of the model is formulated as Equation 1.

$$\min \sum_{i \in I} \left\{ \sum_{f \in Dep_i^{\text{int}} t \in T_f} (t - e_f^d) \cdot x_f(t) + \sum_{f \in Dep_i^{\text{cut}} t \in T_f} (t - e_f^d) \cdot x_f(t) + \sum_{f \in Aer_f^{\text{cut}} t \in T_f} (t - e_f^d) \cdot y_f(t) \right\}$$
(1)

Equation 1 could be simplified as follows:

$$\min \sum_{i \in I} \left\{ \sum_{f \in Dep_i} \sum_{t \in T_f} (t - e_f^d) \cdot x_f(t) + \sum_{f \in Arr_f^{ext}} \sum_{t \in T_f} (t - e_f^a) \cdot y_f(t) \right\}$$
(2)

Subject to:

1) Constraints on time span assignment:

$$\sum_{t \in T_f} x_f(t) = 1, x_f(t) = \{0, 1\}, \forall f \in Dep_i, \forall i \in I$$
(3)

Equation 3 is the constraint on time span assignment for departure flights which ensures each flight only one departure time span. If time span t is assigned to flight f,

then, $x_f(t)$ is 1. Otherwise, $x_f(t)$ is 0.

$$\sum_{t \in T_f} y_f(t) = 1, y_f(t) = \{0, 1\}, \forall f \in Arr_i, \forall i \in I$$
(4)

Equation 4 is the constraint on time span assignment for arrival flights which ensures each flight only one arrival time span. If time span t is assigned to flight f, then $y_f(t)$

is 1. Otherwise, $y_f(t)$ is 0.

2) Constraints on airport capacities:

$$0 \le u_t^i \le U_t^i, 0 \le v_t^i \le V_t^i, \forall t \in T, \forall i \in I$$
(5)

$$\alpha_t^i \cdot u_t^i + \beta_t^i \cdot v_t^i \le \gamma_t^i, \forall t \in T, \forall i \in I$$
(6)

Equations 5 and 6 are constraints on airport capacities which compose of the feasible area of capacity allocation. The optimal solutions are on or inside the airport capacity curves, where α_t^i , β_t^i and γ_t^i are coefficients of capacity curve $\Psi_t^i(u, v)$. For example, in Figure 2, the feasible area of capacity allocation is composed of $0 \le u_t \le 7$, $0 \le v_t \le 8$, $u_t + v_t \le 12$ and $2u_t + v_t \le 18$.

3) Constraints on connecting flights:

$$t_{f}^{d} - t_{f}^{a} \ge \Delta t_{f,f}, \forall \quad (f, f') \in X$$

$$\tag{7}$$

Equation 7 is the constraint on connecting flights which ensures the time separation minima of connecting flights being accommodated. For example, (4) constraints on flying time.

$$0 \le \left(t_{f_i}^a - t_{f_j}^d\right) - \left(e_{f_i}^a - e_{f_j}^d\right) \le \delta; f_i \in Arr_i, f_j \in Dep_j; i, j \in I$$
(8)

Equation 9 is the constraint on airborne flight which ensures the airborne time of flights being constant. Equation 8 shows that the arrival time, departure time and expected airborne time accommodate inherent relations which ensure the feasible solutions being gained.

SCE-UA

Swarm intelligence is considered as an efficient way to solve combinatorial optimization problems (Yu et al., 2009, 2010, 2011). The SCE-UA technique has been successfully used in the area of surface and sub-surface hydrology for the calibration of rainfall-runoff models and identification of parameters of aquifer formation (Duan et al., 1994). The SCE-UA algorithm combines the strengths



Figure 3. An example of reflection point.

of the simplex procedure of Nelder and Mead (1965) with: 1) the concept of controlled random search after work of Price (1987); 2) competitive evolution after Holland (1975); and 3) the concept of complex shuffling (Duan et al., 1992, 1993). The synthesis of these three concepts makes the SCE-UA algorithm not only effective and robust but also flexible and efficient. A general description of the steps of the SCE-UA algorithm is given as follows (Duan et al., 1992, 1993; Nunoo et al., 2004):

Step 1: Select *s* points randomly from the feasible solution space;

Step 2: Sort the *s* points in increasing order such that the first point represents the smallest function value.

Step 3: Partition the *s* points into *p* complexes, each containing v points. The complexes are partitioned such that the first complex contains every p(j-1)+1 ranked point, the h^{th} complex contains every p(j-1)+h ranked point, and so on, where j = 1, 2, ..., h... v.

Step 4: Evolve the complexes with the competitive complex evolution (CCE) algorithm (which will be elaborated later).

Step 5: Combine the points in all evolved complex into a single sample population; sort the population in increasing order and shuffle (that is re-partition) them into p complexes according to procedure specified in Step 3.

Step 6: If convergence criteria are satisfied stop the calculation, otherwise, continue.

Step 7: If the minimum number of complexes required in the population p_{min} is less than p, remove the smallest complex and set p = p - 1, s = pv and return to Step 4. If $p_{min} = p$, return to Step 4 without reducing population size p.

One key component in the SCE-UA is the CCE algorithm. The CCE procedure employs the simplex downhill search method of Nelder and Mead (1965) in the generation of the offspring. The algorithm is summarized from the work (Duan et al., 1992, 1993; Nunoo et al., 2004) as follows:

Step 1: Construct a sub-complex by randomly selecting q points from the complex according to a trapezoidal probability distribution. The probability distribution is specified such that the better point has the higher chance of being chosen.

Step 2: Identify the worst point of the sub-complex and compute the centroid of the sub-complex by excluding the worst point.

Step 3: Attempt a reflection step by reflecting the worst point through the centroid. If the newly generated point is within the feasible space, go to Step 4, otherwise, randomly generate a point within the feasible space and go to Step 6 (Figure 3).

Step 4: If the newly generated point is better than the worst point, replace it with the new point and then go to Step 7, otherwise, go to Step 6.



Figure 4. An example of contracted point.

Step 5: Attempt a contraction step by computing a halfway point between the centroid and the worst point. If the contracted point (Figure 4) is better than the worst point, replace it with the contraction point and go to Step 7. Otherwise, go to Step 6.

Step 6: Randomly generate a point within the feasible space. Replace the worst point by the new point.

Step 7: Repeat Steps 2 to 6 α times, where $\alpha \ge 1$ is the number of consecutive offspring generated by the same sub-complex.

Step 8: Repeat Steps 1 to 7 β times, where $\beta \ge 1$ is the number of evolution steps taken by each complex before complexes are shuffled.

NUMERICAL TEST

To test how well the model may be applied in real word, a numerical test is performed. Set China Beijing Capital airport, Shanghai Pudong airport and Guangzhou Baiyun airport as an example, they compose of an open multi-airport network system. Select real flights data of the three airports during one of typical periods (8:00 to 12:00, N = 16, $\Delta = 15$ min). Initial air traffic demand of each airport is shown in Table 1. The capacity curves of each airport and the coordination between demand and

capacity are shown in Figure 5. It can be seen that, there is imbalance between demand and capacity for all three airports in some time intervals and demand exceeds capacity in some time intervals, flight delays will occur. The air traffic flow assignment after optimization is shown in Table 2 and Figure 5. In Figure 5, it is easy to see that flow assignment solutions are on or inside capacity curves of three airports, which shows the coordination between demand and capacity. The flow distribution before and after optimization is compared in Figure 6. It can be seen that peak traffic has been eliminated and the flow distribution in each time span tends to balance. The airport capacities are fully utilized and the arrival or departure time span for each flight is also assigned reasonably.

Performance of the proposed algorithm

To examine the efficacy of SCE-UA with same parameters, we solved the model twenty times and the results are shown in Figure 7. We found that fitness decreased fast before 100th generation and then it changed smoothly. The best fitness appeared at about 160th generation and then it hardly changed again. Furthermore, it can be understood that the results of the

Time	Beijng airport			Pudong airport			Baiyun airport		
	Dep	Arr	Total	Dep	Arr	Total	Dep	Arr	Total
8:00-8:14	15	0	15	12	0	12	16	1	17
8:15-8:29	16	1	17	17	0	17	12	3	15
8:30-8:44	16	3	19	14	6	20	9	1	10
8:45-8:59	15	10	25	14	4	18	8	5	13
9:00-9:14	7	3	10	26	3	29	11	3	14
9:15-9:29	5	2	7	12	5	17	4	5	9
9:30-9:44	10	13	23	4	5	9	2	7	9
9:45-9:59	8	19	27	12	39	51	11	8	19
10:00-10:14	5	2	7	14	9	23	8	4	12
10:15-10:29	1	5	6	6	28	34	3	4	7
10:30-10:44	17	11	28	6	13	19	10	3	13
10:45-10:59	16	22	38	7	18	25	12	13	25
11:00-11:14	9	7	16	32	9	41	7	3	10
11:15-11:29	3	11	14	5	16	21	3	6	9
11:30-11:44	10	7	17	13	9	22	9	5	14
11:45-12:00	21	20	41	12	11	23	12	18	30
Total	174	136	310	206	175	381	137	89	226

Table 1. The first traffic demand distribute of three airports.

Note: "dep" is the mean of the number of the aircrafts departure from the airport. "arr" is the mean of the number of the aircrafts arrival to the airport.

Time	Beijing airport			Pudong airport			Baiyun airport		
	Dep	Arr	Total	Dep	Arr	Total	Dep	Arr	Total
8:00-8:14	15	0	15	12	0	12	12	1	13
8:15-8:29	16	1	17	17	0	17	12	2	14
8:30-8:44	16	3	19	14	6	20	12	2	14
8:45-8:59	14	10	24	14	4	18	9	5	14
9:00-9:14	8	3	11	22	3	25	11	3	14
9:15-9:29	5	2	7	15	5	20	4	5	9
9:30-9:44	10	13	23	4	5	9	2	7	9
9:45-9:59	8	15	23	10	17	27	9	6	15
10:00-10:14	5	6	11	10	17	27	9	6	15
10:15-10:29	1	5	6	10	17	27	4	4	8
10:30-10:44	13	11	24	8	17	25	10	3	13
10:45-10:59	10	14	24	8	17	25	9	6	15
11:00-11:14	14	10	24	10	17	27	9	6	15
11:15-11:29	8	15	23	10	17	27	4	8	12
11:30-11:44	10	8	18	10	17	27	9	6	15
11:45-12:00	16	8	24	21	5	26	6	6	12
After 12:00	5	12	17	11	11	22	6	13	19
Total	174	136	310	206	175	381	137	89	226

Table 2. The first traffic demand distribution of three airports after optimization.

Note: "dep" is the mean of the number of the aircrafts departure from the airport. "arr" is the mean of the number of the aircrafts arrival to the airport.



Figure 5. The relation of traffic volume and capacity between the optimized and current conditions.

that the SCE-UA algorithm has a good convergence.

Conclusion

Considering the interdependence between arrival and departure and the relation among flights of multiple

airports, the paper studies air traffic flow assignment problem of multi-airport network. With the constraints on the capacities of multiple units, a capacity allocation and flow assignment model for open multi-airport network is presented. A case study based on China Beijing Capital airport, Shanghai Pudong airport and Guangzhou Baiyun airport is performed, and the correctness and effectiveness



Figure 6. Traffic volume in each period between the optimized and current conditions. a) Beijing airport, B) Shanghai airport and, c) Guangzhou airport.

of the model is verified. Further research will extend to

the whole air traffic network including airspace sectors



Figure 7. The result of each calculation.

and enroutes and the balance between demand and capacity for large scale area will be studied.

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