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An automated height transformation using precise geoid models

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In most of the countries, national height systems are referenced to the mean sea level (geoid) whereas using GPS technique in positioning provides ellipsoidal heights. However, these GPS ellipsoidal heights h can be transformed into the orthometric heights from the geoid H depending on a simple relation among the heights, and this transformation requires to know the geoid undulations N ($H = h - N$). The methods for deriving the geoid undulation at a point are various and practicality of the method is one of the major concerns as well as its precision. In this study, an experiment on calculating local geoids using an algorithm based on surface polynomials with weighted corrections was performed and a program, which computes the geoid undulation at a point in the local area using the model was developed, and an executable file of the program was inserted into the commercial GPS data processing software as "height transformation module". With this *ad hoc* module, the software provides the orthometric heights to the user at the same time with the three dimensional geodetic coordinates of the points. Hence, an automated height transformation, without user intervention, simultaneously with GPS data post-processes is provided. The experiment data includes dense and well distributed 301 GPS/levelling benchmarks in the west of Turkey. All the decisions regarding the data preparation and modelling process using polynomials (such as detecting the blunders in the data, determining the optimal degree of the polynomial and testing significance of the polynomial coefficients etc.) are critical in practice and were dealt in the study as well. The orthometric heights of the test benchmarks were provided with 3.5 cm accuracy by the automated height transformation module.

Key words: Height transformation, geoid modelling, orthometric height, GPS/levelling, surface polynomials, programming, Turkey.

INTRODUCTION

The use of heights, such as orthometric heights that are connected to the Earth's gravity field is important in many fields, not only in all Earth sciences but also in other applications such as cartography, oceanography, civil engineering, hydraulics, high-precision surveys and last but not least geographical information systems. Traditionally, these heights are determined by combining geometric levelling and gravity observations with millimetre precision in smaller regions. This technique, however, is very time consuming, expensive and makes providing vertical control difficult, especially in areas which are hard to access. Another disadvantage is the loss of precision over longer distances since each height system usually refers to a benchmark point close to the sea level which is connected to a tide gauge station representing the mean sea level.

In order to counteract these drawbacks and because of the wide and increasing use of global navigation satellite systems (GNSS) in all kind of geodetic and surveying applications, the modern technique called GNSS levelling can be considered as an alternative for practical height determination. In GNSS levelling, the orthometric heights H based on the geoid which is approximated by mean sea level are determined by converting the ellipsoidal heights h with respect to a reference ellipsoid by applying the fundamental equation $H = h - N$, instead of levelling and gravity observations (Heiskanen and Moritz, 1967; Hofmann-Wellenhof and Moritz, 2006). In the formulation, N is the deviation between the two reference surfaces, geoid and reference ellipsoid, along the ellipsoidal normal and called geoid undulation (or height). Hence, determining the geoid height with an accurate geoid

model make GNSS levelling possible, that is a simple and cost-effective way of obtaining the orthometric heights (Gilliland, 1986; Schwarz et al., 1987; Zilkoski and Hothem, 1989; Gilliland and Jaksa, 1994; Ananga and Sakurai, 1996; Collier and Croft, 1997; Featherstone et al., 1998; Lee and Mezera, 2000; Erol et al., 2008; Gucek and Bašić, 2009). Since the ellipsoidal heights are nowadays mostly obtained from GPS technique and the ellipsoidal heights of the benchmarks used in this study are provided by GPS, GNSS term is replaced by GPS in the sequel.

The methods for geoid modelling are various and each method has its merits and limitations. The studies on determining a precise geoid model for Turkey has been started in 1970's and various regional geoid models based on different methods have been computed since then (Ayan, 1976; Ayhan, 1993; Ayhan and Kilicoglu, 1993; TNFGN, 2002; TNGC, 2003). In 2003, the General Command of Mapping released the most recent regional gravimetric geoid model of Turkey, TG03. This is a hybrid geoid model which was gravimetrically determined and fitted to the regional vertical datum at 197 homogeneously distributed GPS/levelling benchmarks throughout the country, by the adjustable tension continuous curvature surface gridding algorithm (TNGC, 2003). However, the absolute accuracy of TG03 is reported as 10 cm in the central territories and 20 cm along the coastlines and boundaries of the country (TNGC, 2003) that the decimetre level accuracy of transformed orthometric heights stay rough in many applications (Erol and Çelik, 2004; Erol et al., 2005a, b).

Therefore, because of the accuracy concerns in GPS levelling, in Large Scale Maps and Spatial Data Production Regulation of Turkey (legalized in 2005) (LSMSDPR, 2005; Deniz and Çelik, 2008), the determination and use of GPS/levelling surface type geoid models for height transformation is encouraged in respectively small areas where the dense and precise data is available (LSMSDPR, 2005). Local GPS/levelling geoid models are often used in Turkey and provide a practical and fast solution to the height transformation since the absence of precise regional geoid yet in the country (Ayan et al., 1996a; Ayan et al., 1996b; Ayan et al., 1999; Çelik et al., 2002; Ayan et al., 2001; Ayan et al., 2006).

The accuracy of local GPS/levelling geoid models as height transformation tool is restricted by many factors, such as the data accuracies, the density and distribution of reference benchmarks, modelling methods etc. There are many researches, done on the effects of data quality, the density and distribution of reference benchmarks in local geoid modelling, so far and can be found in the literature (Featherstone et al., 2001; Fotopoulos et al., 2001; Fotopoulos et al., 2003; Fotopoulos, 2003; Yilmaz, 2005; Erol et al., 2006; Erol, 2008). However, this study concentrated mainly on the role of practicality and applicability of a local geoid model in the fast solution of height transformation problem. In this manner surface

polynomials are obviously easy models to determine and use, comparing to the other more complicated ones such as method of finite elements, geostatistical kriging, least squares collocations, artificial neural networks etc. (Çepni and Deniz, 2005; Kavzaoğlu and Saka, 2005; Kutoğlu, 2007).

Therefore this investigation on developing an automated height transformation utility integrated on GPS processing software employed surface polynomials with additive corrections approach. The data of numerical evaluations consist of dense and well distributed 301 benchmarks of geodetic GPS/levelling network in the west of Turkey. In the content, detailed discussions on the critical decisions of both data preparation and modelling phases of the investigation, which directly affect the quality of final model and thus the accuracy of the transformed heights, are also included. At the end, a program, which applies the local geoid model to derive geoid undulations at the points, was developed in Visual C environment, and added to a commercial GPS data processing software to automate the height transformation without user intervention and to derive the point heights in regional vertical datum simultaneously with the computation of their three dimensional GPS coordinates. Hence the possible personal mistakes, which may cause gross errors, are cancelled out from the transformation results, and also the speed of transformation process is considerably increased. In the results of numerical experiment, the comparisons at the test benchmarks provided a 3.5 cm accuracy of the transformed orthometric heights using the automated height transformation module.

DATA AND METHODOLOGY

In relatively small areas, a geometrical method for deriving GPS/levelling surface (so called local geoid) based on bivariate polynomial equations in various orders is often used (Yilmaz, 2005; Kavzaoğlu and Saka, 2005; Stopar et al., 2006; Kutoğlu, 2007; Deniz and Çelik, 2008; Erol et al., 2008). The solution can also be performed using either weighted average interpolators or multivariate regression equations in numerous forms, which are not going to be mentioned here. The geometrical method provides a practical and fast transformation of GPS heights where a regional precise geoid model is not available. The data used in numerical tests on height transformation using geometrically derived local geoid are 329 (and 28 of these benchmarks were removed as blunder in data screening) GPS/levelling benchmarks of İzmir geodetic reference system 2001 (IzGRS01) network (Ayan et al., 2001).

These are the common points of C1, C2 and C3 orders GPS benchmarks of Turkish National Fundamental GPS Network (TUTGA99A) and high order levelling benchmarks of Turkish National Vertical Control Network (TUDKA99). Thus, the Helmert orthometric heights of the benchmarks are in TUDKA99 Datum and their absolute accuracy is 2.5 cm. GPS coordinates of the benchmarks refer to the ITRF96 datum. The accuracies of the GPS-derived coordinates are 1.5 cm and 2.3 cm in the horizontal and vertical directions (Ayan et al., 2001). The network area covers 50 × 45 km² and the density of the benchmarks is around 1 benchmark per 8 km² with relatively homogeneous distribution
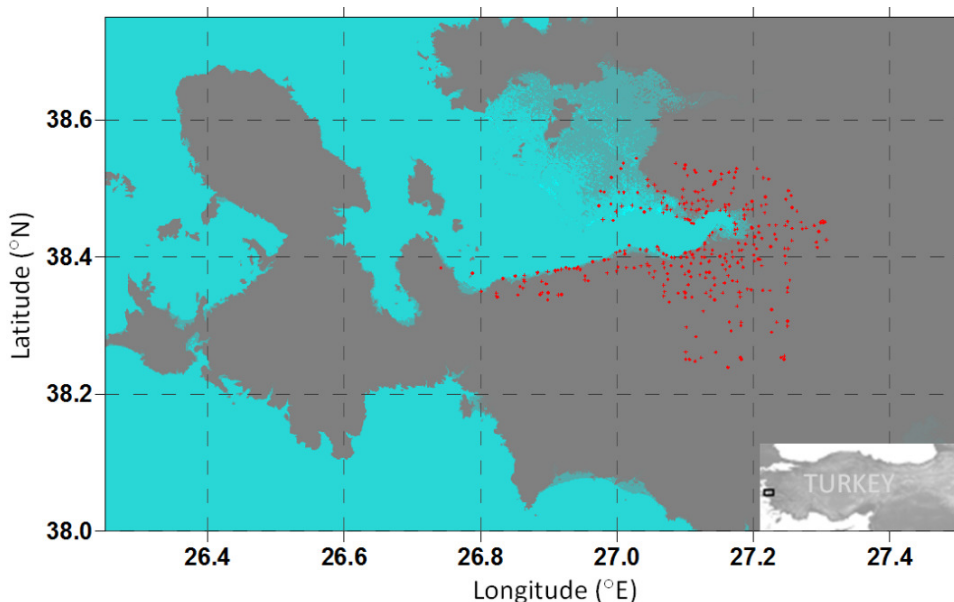
Figure 1. GPS/levelling benchmarks, İzmir, Turkey.

(Figure 1). The topographic heights in the region range from the sea level to 1500 m.

Data screening

In preparing the reference data for geoid modelling, first of all screening the data and detecting the blunders possibly contained by the data is very essential step and affects the accuracy of the final model very much. In many of the blunder detection approaches, cross validating the reference data is a very practical method which localizes the blundered data quickly. In the cross validation procedure, the relative quality of the observations at let's say given *n* observation locations (the reference data set) are assessed by comparing the observed value with the interpolated value from the surrounding observations (residual = interpolated value – observed value).

In the algorithm, the errors are calculated by removing the first observation from the data set, and using the remaining data and the specified algorithm to interpolate a value at the first observation location. Then, the first observation is put back into the data set and the second observation is removed from the data set. Using the remaining data (including the first observation), and the specified algorithm, a value is interpolated at the second observation location. Using the known observation value at this location, the interpolation error is computed as before.

The second observation is put back into the data set and the process is continued in this fashion for the third, fourth, fifth observations, etc., all the way through up to and including observation *n*. This process generates *n* residuals, which may provide a measure to assess whether the observation at a location has blunder or not. Thus, cross validation process can be summarized in four steps: i) selecting an interpolation method (such as inverse distance weighting, triangulation with linear interpolation, Kriging, polynomial etc.), along with all of the defining parameters, ii) for each observation location, interpolating the value using the neighbouring data, but not the observation itself, iii) computing the resulting errors (residuals), iv) assessing the quality of the observation using a pre-defined statistical criteria for the residual at each data point, e.g. deciding the observation, having the residual

bigger than three times of standard deviations of the residuals (3σ test, with 99.7% confidence interval), as having blunder (Sen and Srivastava, 1990; Draper and Smith, 1998; Fotopoulos, 2003; Surfer, 2009).

Modelling local GPS/levelling geoid with surface polynomials and additive corrections

In modelling local GPS/levelling geoid with geometric approach, a geoid reference benchmarks network having coverage of entire area is constituted. The geoid reference benchmarks are generally selected from the common points of C1, C2, C3 order GPS benchmarks and the 1st and the 2nd order levelling network points, and must have homogeneous distribution at the characteristic locations of topography. The density of the reference benchmarks is suggested to be 6 benchmarks per 20 km^2, and an additional bench-mark for each 15 km^2 enlargement of the model area, by LSMSDPR (2005). With existing *n* reference benchmarks having GPS ellipsoidal and levelling heights (and hence with known geoid heights: $N_{GPS/lev.} = h_{GPS} - H_{levelling}$) in a local area, the general equation of polynomial interpolation to estimate GPS/levelling geoid heights at unknown points in the area can be given as:

$$N\left(u,v\right) = \sum_{m=0}^{L}\ \sum_{n=0}^{M} a_{mn} u^{m} v^{n} \tag{1}$$

where u and v represent the position coordinates, a_{mn} symbolize the polynomial coefficients, and L is the degree of the polynomial. The position coordinates can be constituted in various ways, and in this study they are obtained from the ellipsoidal geographical coordinates as

follows:

$$u = k\left(\varphi - \varphi_o\right) \ , \quad v = k\left(\lambda - \lambda_o\right) \tag{2}$$

where φ_o and λ_o are the arithmetic averages of the latitudes and longitudes of the data set, $k = 100/\rho^o$ is scaling and unit adjustment factor. The polynomial coefficients are determined according to Least Squares Adjustment (LSA) method. The summation of observation (N_i) and its correction (residual, V_i) at each reference benchmark is described with the function of unknowns as follows:

$$N_i + V_i = a_{00} + a_{10}u + a_{11}v$$
$$+ a_{20}u^2 + a_{21}uv + a_{22}v^2$$
$$+ a_{30}u^3 + a_{31}u^2v + a_{32}uv^2 + a_{33}v^3 \tag{3a}$$
$$+ a_{40}u^4 + a_{41}u^3v + a_{42}u^2v^2 + a_{43}uv^3 + a_{44}v^4$$
$$\ldots$$

$$N + V = A\ X \tag{3b}$$

$$\begin{bmatrix} N_1 \\ N_2 \\ . \\ . \\ N_i \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_i \end{bmatrix} = \begin{bmatrix} 1 & u_1 & v_1 & \ldots \\ 1 & u_2 & v_2 & \ldots \\ . & . & . & \ldots \\ . & . & . & \ldots \\ 1 & u_i & v_i & \ldots \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{10} \\ . \\ . \\ a_{mn} \end{bmatrix} \tag{3c}$$

and hence the unknown polynomial coefficients (a_{mn} as the elements of X vector in Equation 4a) with their covariance information Q_{XX} (in Equation 4b) are simply determined according to LSA principles:

$$X = \left(A^T A\right)^{-1} A^T 1 \tag{4a}$$

$$Q_{XX} = \left(A^T A\right)^{-1} \tag{4b}$$

where A is called coefficients matrix and ℓ is the observations vector of which elements are the geoid heights (N_i). The degree of the polynomial is one of the critical parameters which should be decided in local geoid modelling using surface polynomials. Figure 2 shows samples of surface plots with varying polynomial degrees from the first to the fourth degree (Fotopoulos, 2003).

Decision procedure of an optimum degree of the surface polynomial will be discussed with details subsequently.

After calculating geoid undulation at a new point using the derived surface polynomial (in Equation 3a), the

additive corrections $\left(dN_P\right)$ calculated with weighted averages of geoid undulation residuals at the neighbouring geoid reference benchmarks can improve the precision of the geoid undulation at the calculation point:

$$dN_P = \frac{\dfrac{V_1}{S_{P-1}^2} + \dfrac{V_2}{S_{P-2}^2}\ldots}{\dfrac{1}{S_{P-1}^2} + \dfrac{1}{S_{P-2}^2}\ldots} = \frac{\sum\left(\dfrac{V}{S^2}\right)}{\sum\left(\dfrac{1}{S^2}\right)} \tag{5}$$

where V_i is the geoid undulation residual at the corresponding reference benchmark, S_{P-i} is the distance between the calculation point P and the ith reference benchmark in km, and hence dN_P is the additive correction to the geoid undulation at point P. At the case study, the S distance was limited with 3 km, and the only reference benchmarks, staying in the circle having the computation point in the center and the radius of 3 km, were considered in calculation of the additive corrections. The distance limit can be decided considering the topographic character of the area. With practicality concerns of height transformation using the local geoid model, the residuals at the geoid reference benchmarks were derived and included in an input file of the developed program codes.

Testing the polynomials for an optimal geoid model

One of the main difficulties of using polynomial interpolation is determining the optimal form of the model having an appropriate degree and significant coefficients, as it defines the accuracy of the approximation. Whilst the use of a low-degree polynomial usually results in an insufficient or rough approximation of the surface, the use of a higher-degree function may produce an over fitted surface. After determining the model with its coefficients using LSA (Equation 3), it is tested with statistical tests to assess its performance and to select the best model for the data, and these tests are quite arbitrary. In this study, the test procedure as suggested by Fotopoulos (2003) was adopted. Figure 3 summarizes the evaluation of the polynomials in various orders, determined in the result of LSA, for determining an optimal model of geoid data.

In the evaluation of the polynomials, at first, testing the statistical significance of the polynomial coefficients is critical, since the insignificant parameters may bias others in the model. With the purpose of significance test of the model parameters, F-test with the null hypothesis $H_0 : X = 0$ and the alternative hypothesis $H_1 : X \neq 0$ was applied (Draper and Smith, 1998; Koch, 1999; Fotopoulos, 2003). The F-statistic is used to verify the null hypothesis and computed as a function of observations (Dermanis and Rossikopoulos, 1991):
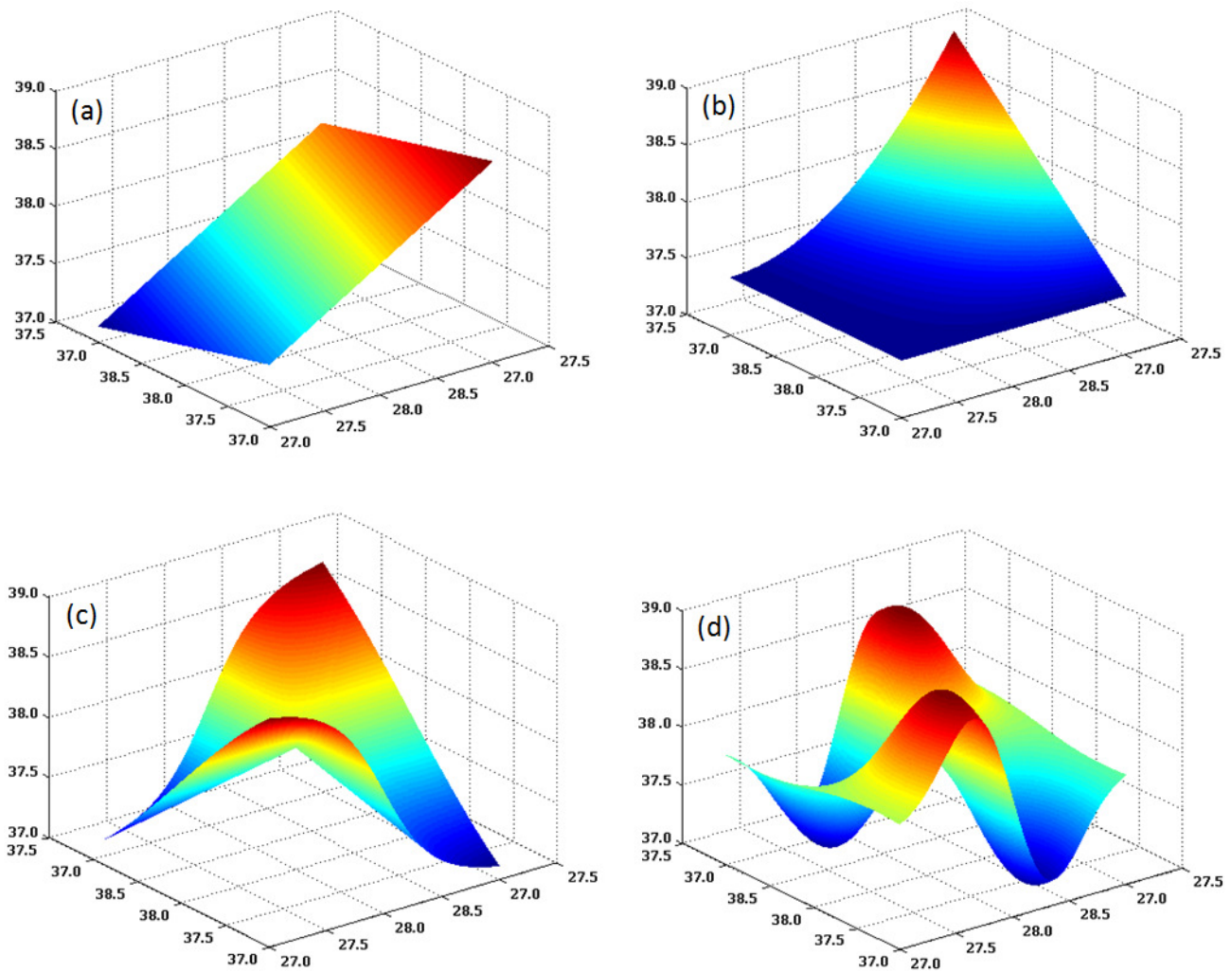
Figure 2. (a) First, (b) second, (c) third and (d) fourth degree surface polynomial fit samples.
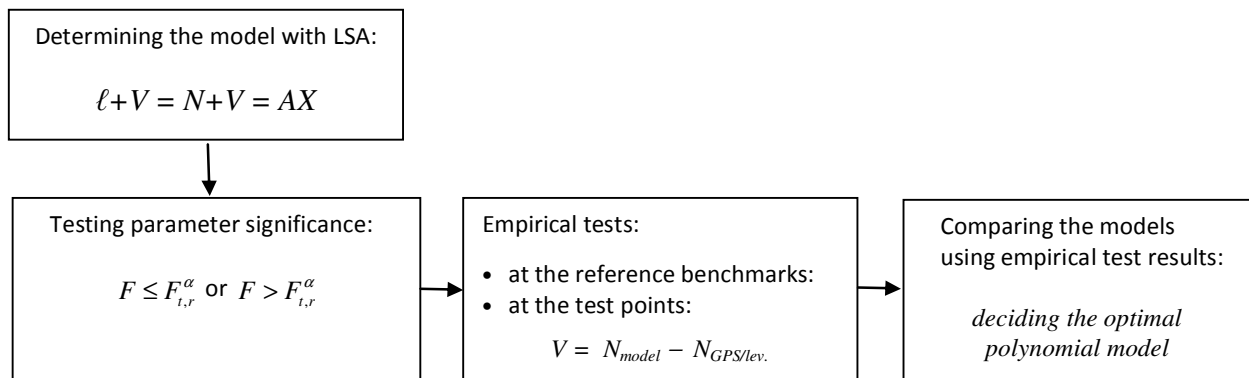

Figure 3. Performance assessment of the polynomial model.

$$F = \frac{X_i^T Q_{X_i X_i}^{-1} X_i}{t \, \hat{\sigma}^2} \tag{6}$$

where X_i is the estimate of the parameter vector, $Q_{X_i X_i}$ is the cofactor matrix of X_i, $\hat{\sigma}^2$ is a-posteriori variance

Table 1. The statistics of geoid heights and residuals before and after removing blunders in the data.

	Reference data (meter)			
	Min	**Max**	**Mean**	**σ**
Before (329 BM*)	37.10	38.81	38.07	0.18
After (301 BM)	37.59	38.72	38.06	0.16
	Geoid height residuals (centimeter)			
Before (329 BM)	-69.5	109.1	0.1	11.6
After (301 BM)	-8.9	10.4	0.0	3.0

*BM: Benchmark

Table 2. The statistics of geoid height residuals at the cross-validated benchmarks for the polynomials in varying degrees (centimeter).

	Min	**Max**	**Mean**	**σ**
First degree	-43.3	33.2	0.0	13.5
Second degree	-28.4	18.9	0.0	8.4
Third degree	-24.2	14.1	0.0	5.3
Fourth degree	-15.2	18.5	0.0	4.6
Fifth degree	-10.6	9.1	0.0	3.5
Sixth degree	-19.5	12.1	0.0	3.3

factor, t is the number of parameters tested. The null hypothesis is accepted if $F \leq F_{t,r}^{\alpha}$, where $F_{t,r}^{\alpha}$ is obtained from the standard statistical tables for a confidence level α and degrees of freedom r, which means the corresponding parameters of the test are insignificant and deleted from the model. If the contrary is true and $F > F_{t,r}^{\alpha}$ is fulfilled, then the tested parameters remain in the model. F-test can be applied to the parameters of the model with stepwise algorithm based on backward elimination and forward selection procedures. The test starts with the lowest degree form of the model (forward selection), and the parameters to be tested are selected one by one or a few at a time and examined for significance. The backward elimination is embedded in the stepwise algorithm. After removing insignificant parameters in the result of the statistical test and determining the final form of the polynomial model, its performance is tested empirically, considering the residuals at the benchmarks of the network $(V = N_{\text{model}} - N_{GPS/lev.})$. Among the determined polynomials in various degrees of expansions, the model having the smallest residuals can be used as the most appropriate model for the geoid data.

However, it should be noticed that the empirical tests of the models using the residuals at the reference benchmarks introduce optimistic measures on the precision of the model. Therefore, the tests with the residuals at the independent test benchmarks result more objective and realistic measures on the accuracy or prediction capability of the model. Only the handicap with selecting homogeneously distributed test points among the data may distort the densification and homogeneity of geoid reference benchmarks, used for determination of the model. In this case, cross validation can be used to derive a realistic criterion for the performance of the polynomials, and the estimated geoid height values in the results of iterative procedure of cross validation at each benchmark can be compared with the known geoid heights. In the empirical tests of the polynomials with cross-validation, considering the point sets, consist of few benchmarks, instead of single point at each iteration step is recommended to reduce the correlation of the results.

CASE STUDY: HEIGHT TRANSFORMATION WITH GPS/LEVELLING GEOID MODEL

The determination and use of a GPS/levelling geoid model for height transformation in the local area in the west of Turkey is explained using the reference GPS/levelling network of the case study. Data preparation for the determination of the geoid model constitutes the blunder detection and format arrangements of the input data files, including the matrices for LSA calculation of the polynomial coefficients. In the data screening and blunder detection stage, 28 of the 329 reference benchmarks were detected as blunders and removed from the data.

Figure 4 visualizes the GPS/levelling surface before and after removing the blunders from the reference data with cross-validation, and absurd changes like erections and collapses stem from the blunders in the reference data were disappeared after removing the blunders and hence the geoid surface became smoother. Table 1 shows the data statistics before and after blunder detection, where the third order polynomial was used and the geoid height residuals of the original data and the polynomial model at the cross validated benchmarks are also included in the table, for the data sets, consisting 329 and 301 benchmarks, respectively.

After preparing the reference geoid data, the regression equations in the form of bivariate orthogonal polynomials in varying degrees up to six were calculated using LSA method and tested with the procedure summarized in Figure 3. The basic statistics of the geoid height residuals at the cross-validated benchmarks for each polynomial are shown in Table 2, that provide a comparison among the polynomial models for geoid data. Another comparison among the polynomials was done considering the coefficients of determinations, R^2, which provides a measure on the goodness of the parametric model to fit the data ($0 \leq R^2 \leq 1$). The equation to derive the coefficient of determination is as follows:
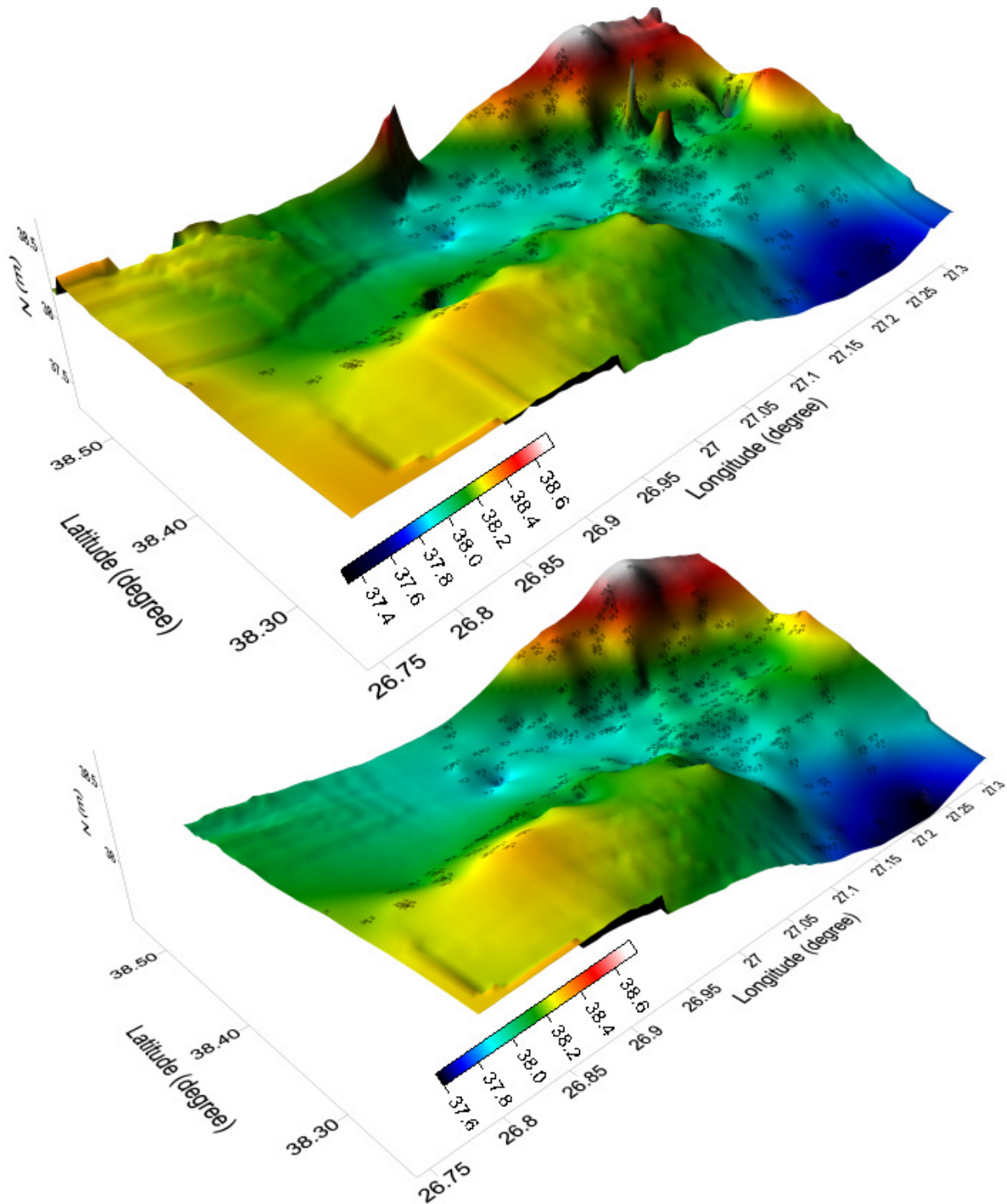
Figure 4. GPS/levelling surface before and after removing the blunders in the data.

$$R^2 = 1 - \frac{\sum\limits_{i=1}^{j}\left(1_i - \hat{1}_i\right)^2}{\sum\limits_{i=1}^{j}\left(1_i - \overline{1}\right)^2}$$

(7)

and in the equation, j is the number of observations, $\hat{1}_i$ is the computed value of the geoid height using the model $\left(\hat{1}_i = N_{model}\right)$, and $\overline{1}$ is the mean value of the observations, $1 = N_{GPS/lev.}$ (Sen and Srivastava, 1990).
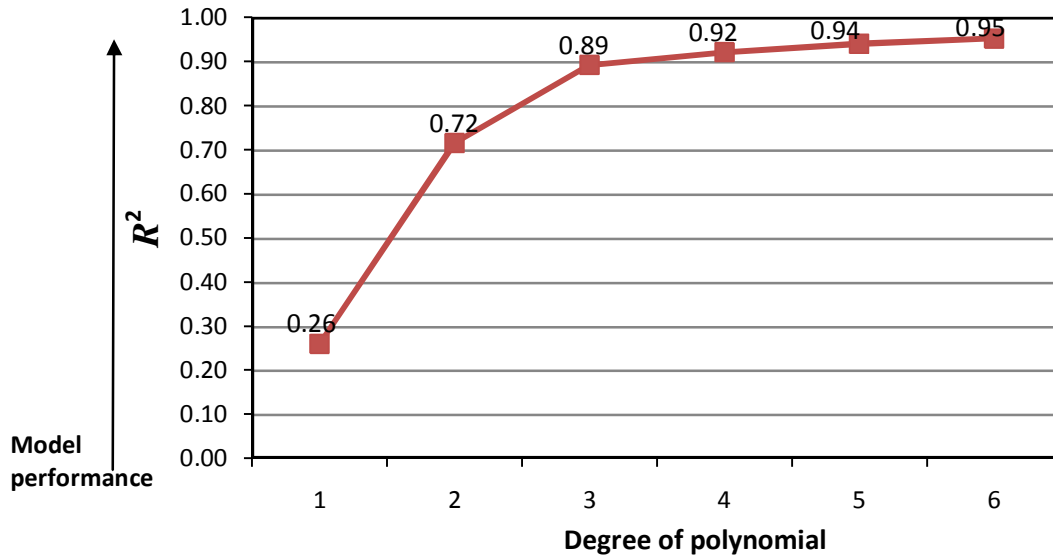
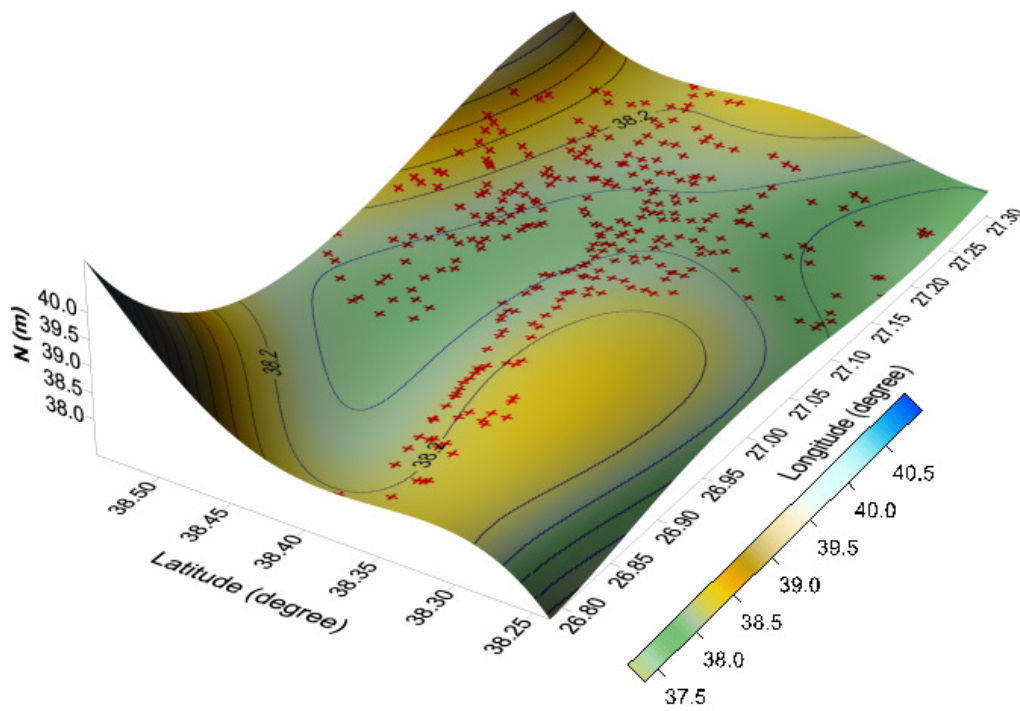**Figure 5.** The coefficients of determinations (R^2) versus polynomial degrees.

Figure 6. Local GPS/levelling geoid (5th degree polynomial).

Figure 5 shows the computed coefficients of determinations using each polynomial.

In the comparisons among the polynomial models considering the validation results against the observed data, and coefficients for determinations, it is seen that the fifth and the sixth order polynomials provided a better fit at the validation benchmarks, with higher coefficient of determination. However, the differences between the fifth and the sixth order polynomial models are minor in terms of standard deviation of geoid height residuals and coefficient of determination. Hence, the fifth order polynomial having 18 significant parameters is decided as the optimal model for the local geoid in the territory. The coefficients of the determined polynomial model are provided in Table 3. The local geoid surface using the calculated polynomial is shown in Figure 6.

Table 3. Coefficients of the fifth degree polynomial.

The 5th degree polynomial coefficients										
a_{00}	37.971	a_{20}	3.824	a_{30}	42.292	a_{40}	-0.482	a_{50}	-175.544
a_{10}	-0.600*	a_{21}	9.063	a_{31}	-7.854	a_{41}	41.343	a_{51}	416.621
a_{11}	-0.175*	a_{22}	2.059*	a_{32}	-6.265	a_{42}	4.689	a_{52}	150.027
				a_{33}	8.102	a_{43}	-67.811	a_{53}	-151.087
						a_{44}	-4.096	a_{54}	-9.877
								a_{55}	-25.992

* The coefficients are assigned insignificant and ignored in the model.

The 5th degree polynomial model

$$N_i = a_{00} + a_{10}u + a_{11}v + a_{20}u^2 + a_{21}uv + a_{22}v^2$$
$$+ a_{30}u^3 + a_{31}u^2v + a_{32}uv^2 + a_{33}v^3$$
$$+ a_{40}u^4 + a_{41}u^3v + a_{42}u^2v^2 + a_{43}uv^3 + a_{44}v^4$$
$$+ a_{50}u^5 + a_{51}u^4v + a_{52}u^3v^2 + a_{53}u^2v^3 + a_{54}uv^4 + a_{55}v^5$$

$$u = k(\varphi - \varphi_o) , \ v = k(\lambda - \lambda_o) , \ k = 100/\rho^o , \ \varphi_o = 38^o.4143 , \ \lambda_o = 27^o.1063$$

Program development for practical implementation of height transformation using local GPS/levelling geoid

With the purpose of practical implementation of height transformation using local GPS/leveling geoid model, a program was developed in Visual C by Geodesy division at Istanbul Technical University (Ayan et al., 2001). Hence, in addition to accelerate transformation of the heights, it was also aimed to reduce the calculation mistakes by the users of the model and the differences in the results stem from the numerical ignoring. The first version of the program accepts manual input of the geodetic latitude, longitude and ellipsoidal height of the computation point in ITRF96 datum via keyboard. When the program runs, an initial screen provides the program related information (Figure 7a), and gives the instructions to input the required data. In the process, at first, it is detected whether the computation point is in the cover area of the local geoid model. If it is not in the model area, the process is stopped and a warning message appears on the screen, because extrapolating the geoid heights is not allowed by the program. When the computation is done, the geoid height of the computation point derived from the polynomial model and refined geoid height with additive corrections, separately, are shown in addition to residuals at contributed reference benchmarks in computation of the additive correction, on the screen (Figure 7b).

A modified version of the program accepts the data of a list of computation points in a specified input file and provides their orthometric heights in an output file, and hence deriving the orthometric heights of a point group in a project is possible. Recently, many commercial GPS data processing software in the market provide an option,

using a geoid model for transforming the ellipsoidal heights from the GPS data processing to the orthometric heights in regional datum, and in order that the user can either use one of the valid geoid models in the data base of the software, which are generally global geoid models having accuracy in metre (Erol et al., 2009)), or a regional geoid model, developed as external program and added to the software. Hence, the orthometric heights of the benchmarks are derived at the same time with their geodetic coordinates from the GPS data.

In this study, the developed program based on the polynomial type local GPS/levelling geoid model was added to the employed GPS data processing software, Leica SKI Pro. Figure 8a describes the procedure, which is followed in introducing a new geoid model to the software as an executable program file. In the procedure, the path of the executable file of the geoid program should be introduced following the "Coordinate System Management" option of the Tools pull-down menu, and the geoid model related information, such as the name and reference ellipsoid of the new geoid model, must be entered via the "New Geoid Model" window. In this study, we use GRS 1980 ellipsoid for the geoid model, because this model will then be attached to Turkish national datum, which is ITRF96 with its reference ellipsoid GRS 1980. After adding the new geoid model file to the GPS processing software, it is ready to be attached and run in a project to compute the geoid heights and the orthometric heights of the benchmarks having three dimensional geodetic coordinates, calculated. Figure 8b shows running the geoid model program for deriving the orthometric heights, following the "Compute Geoid Separations" option in the tools menu. After a message appeared on the screen, saying that the computation is
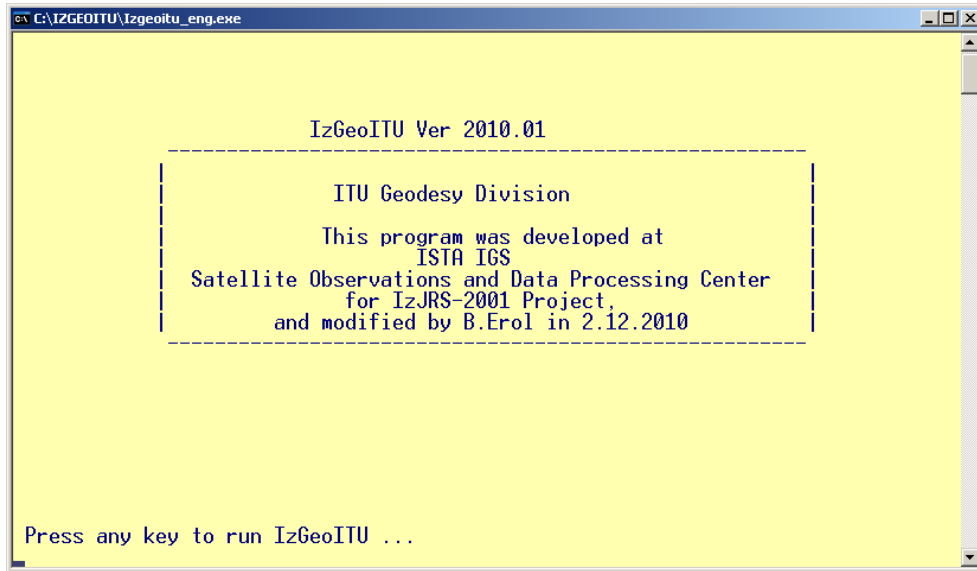
(a)
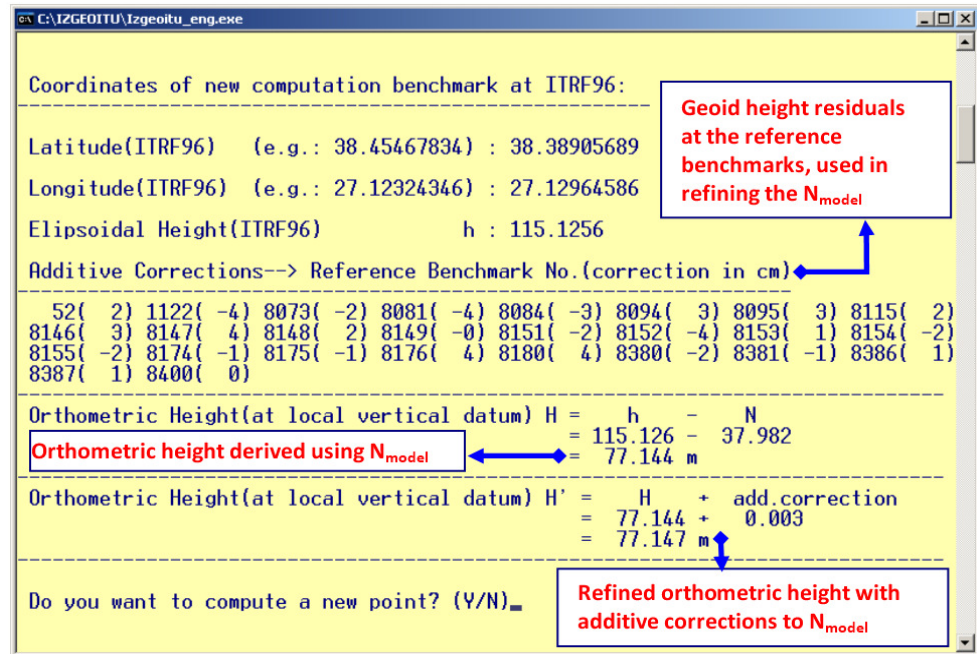
(b)

Figure 7. User interface of the height transformation program based on Local Geoid model during the computations. (a) Introductory screen of the program for height transformation with local geoid model. (b) Input data and the results of the computation on the screen.

successfully completed, the geoid undulations and the orthometric heights are included in the "Point Lists" view in addition to the geodetic coordinates of the benchmarks.

Conclusions

In this article, determining local geoid model with precise GPS/levelling data and it's use for automated height transformation via developed program working in a GPS processing software was reviewed. Then, the article is followed by a numerical case study, which was done using 301 high-order reference GPS/levelling benchmarks in the west of Turkey, and consists of data preparation, decision of an optimal polynomial model for the geoid data and model performance tests, as well. The determined polynomial model with additive corrections and the software program can be applied for deriving the
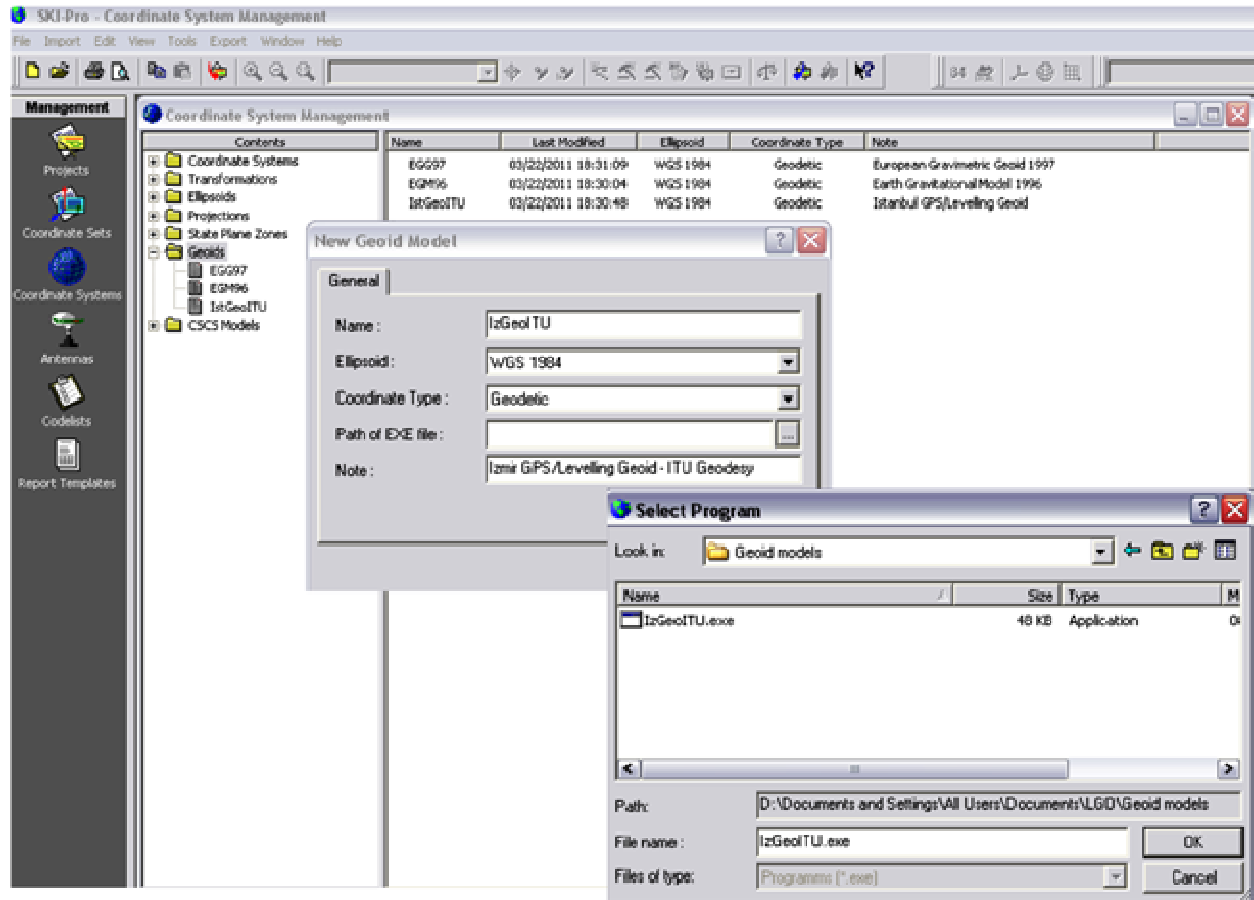
(a)
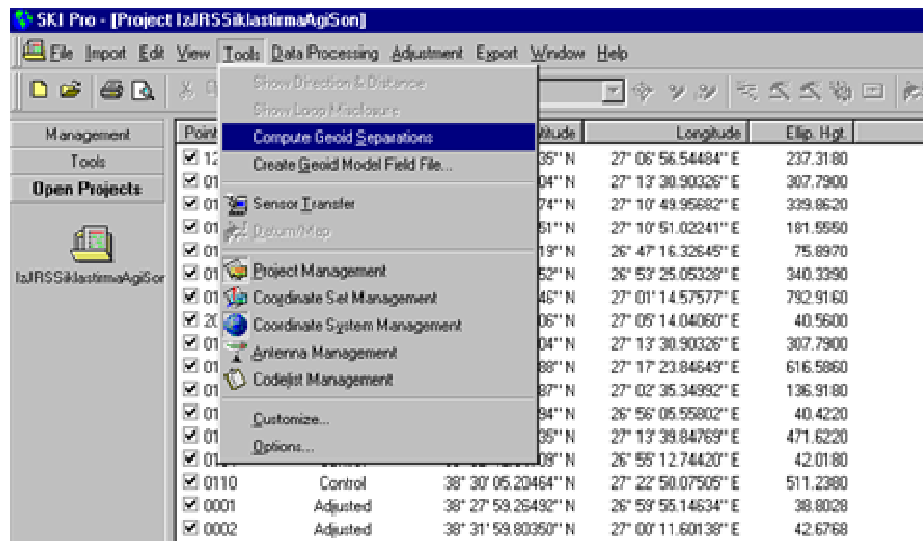
(b)

Figure 8. Automated height transformation using Leica SKI Pro GPS data processing software and added local geoid model program: (a) Adding a new geoid model to the Leica SKI Pro GPS data processing software. (b) Computing the point orthometric heights in the active project, using the geoid model program.

orthometric heights with 3.5 cm accuracy in the described local area, and presented methodology is suggested for

fast and precise determination of the heights in regional vertical datum. In addition, the following points on precise

modelling and use of geoid are also emphasized in the results of the study:

1. Multivariate polynomial equations provides practical solution in modelling the local geoid with precise, homogeneously distributed and dense GPS/levelling data, in the areas where a precise regional geoid model is not available, and hence precise transformation of GPS ellipsoidal heights to regional vertical datum.

2. However modelling procedure consists of critical decisions, such as detecting and removing the blunders in the reference data properly, determining optimal form of the polynomial and clarifying the performance of the model, which must be considered carefully, because each decision affects the accuracy of the model.

3. Automated height transformation with integration of the developed software program, which applies the local geoid polynomial model with additive corrections, on GPS processing software benefits to the accuracy of height transformation, because in this way the geoid undulations and the orthometric heights of the benchmarks are derived simultaneously with their geodetic coordinates from the GPS data processes, without user intervention. Hence the transformation process is accelerated, the personal mistakes, possible during the calculations using the parametric model, are reduced and uniform results are provided.

4. Beside the precision and practicality advantages of local GPS/levelling geoids, the handicap in their use is that, these models provide reliable results only for the benchmarks staying in the local area that the model covers, and the accuracy of the model decreases in the territories close to the area boundaries. Extrapolating the geoid undulation for a benchmark out of the area is not recommended at all, which limits the use of these local geoid models in practical studies. Therefore, although the high accuracy of the local models, a precise regional geoid model for the entire country, is always required as a part of national geodetic infrastructure and for providing uniform height systems.

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