Scientific Research and Essays Vol. 5(21), pp. 3298-3305, 4 November, 2010
Available online at http://www.academicjournals.org/SRE
ISSN 1992-2248 ©2010 Academic Journals

Full Length Research Paper

Control of Rabinovich chaotic system based on passive control

Selçuk Emiroğlu* and Yilmaz Uyaroğlu

Department of Electrical Electronics Engineering, Faculty of Engineering, Sakarya University, Turkey.

Accepted 7 October, 2010

This paper discusses the control of three-dimensional continuous Rabinovich attractor by using passive control technique. Based on the property of the passive system, the passive controller is designed and this controller is added to chaotic Rabinovich system for achieving the control of the system. The controller is ensured to the global asymptotical stability of the three dimensional Rabinovich chaotic system via Lyapunov theory. As a result, the control of Rabinovich chaotic system is realized. To confirm the validity of the proposed method, numerical simulations are presented graphically.

Key words: Passive control, Rabinovich chaotic system, chaos control.

INTRODUCTION

Chaos has been extensively interesting study area for many scientists, after Lorenz found the first attractor in 1963 (Lorenz, 1963). After Lorenz, many chaotic systems were introduced such as Liu system (Liu et al., 2004), Chen system (Chen and Ueta, 1999), Chua system (Chua et al., 1986), Rössler system (Rössler, 1976), Rabinovich system (Pikovski et al., 1978) and Rikitake system (Rikitake, 1958). Chaos control has received increasingly attentions from researchers, since OGY (Ott et al., 1990) method has been proposed. Many control methods have been proposed for the control of chaotic systems such as adaptive control (Wu et al., 1996; Zeng and Singh, 1997; Hong et al., 2001), sliding mode control (Konishi et al., 1998; Ablay, 2009), linear feedback control (Yassen, 2005), and passive control (Lin, 1995; Yu, 1999; Qi, et al., 2004; Kemih et al., 2006; Zhou, 2009; Kemih, 2009; Chen, 2010). Recently, the concept of passivity of nonlinear systems has intensively paid attentions and has been applied in chaos control (Byrnes et al., 1991). Wen Yu used the passive control technique to design the controller for Lorenz system (Yu, 1999). Qi applied this technique to Chen system (Qi et al., 2004) and also Chen et al. applied the passive control to unified chaotic system (Chen and Liu, 2010). Kemih designed a controller to control the Liu system (Kemih et al., 2006) and nuclear spin generator system (Kemih, 2009) based on passive control. Zhou et al. investigated the control of 4D chaotic system via passive control technique (Zhou, 2009). In this paper, the control of Rabinovich chaotic system is investigated based on the properties of a passive system. Feedback controller is designed to control chaotic system via passive control approach. Simulation results show that the controller designed based on passive control can regulate the chaotic system effectively.

*Corresponding author. E-mail: selcukemiroglu@sakarya.edu.tr.

SYSTEM DESCRIPTION

The Rabinovich chaotic system (Pikovski et al., 1978) is defined by

$$\begin{cases} \dot{x}_1 = -ax_1 + hx_2 + x_2x_3, \\ \dot{x}_2 = -bx_2 + hx_1 - x_1x_3, \\ \dot{x}_3 = -dx_3 + x_1x_2, \end{cases} \quad (1)$$

Where x_1, x_2 and x_3 are the state variables, and a, b, d, h are positive real constants. The Rabinovich system (1) exhibits a chaotic attractor for a=4, b=d=1, and h=6.75 as shown in Figures 2 - 3. Using a Matlab-Simulink model, as shown in Figure 1, the time series of x_1, x_2, x_3,

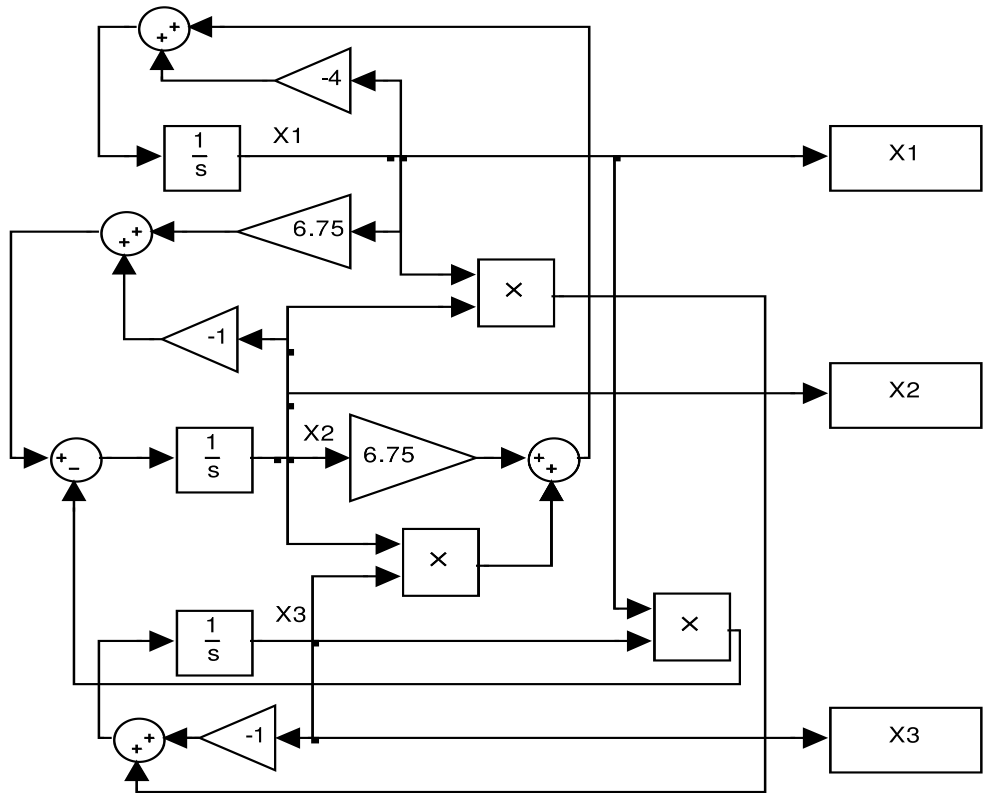


Figure 1. Matlab-Simulink model of Rabinovich chaotic system for a = 4, b = d= 1, h = 6.75.

and $x_1 - x_2$, $x_1 - x_3$, $x_2 - x_3$, $x_1 - x_2 - x_3$, phase portraits of the system are achieved as shown in Figure 2. Calculating the lyapunov exponents for parameter a=4, b=d=1, and h=6.75, lyapunov exponents are $\lambda_1 = 0.46 > 0$, $\lambda_2 = 0$ and $\lambda_3 = -6.46 < 0$. Figure 3 shows the Lyapunov spectrum of the Rabinovich system for parameter a=4, b=d=1, and h=6.75. As can be seen from the Lyapunov exponent spectrum, the system is chaotic.

THE THEORY OF PASSIVE CONTROL

Consider a nonlinear system (2) modelled by ordinary differential equation with input vector u(t) and output vector y(t) (Yu, 1999; Qi et al., 2004),

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{2}$$

Where the state variable $x \in \Re^n$, the input $u \in \Re^m$ and the output $y \in \Re^m$. f(x) and g(x) are smooth vector fields. h(x) is a smooth mapping. We suppose that the vector field f has at least one equilibrium point and without loss of the generality, we assume the equilibrium point x=0.

Definition 1. System (2) is a minimum phase system if Lgh(0) is nonsingular and x=0 is one of the asymptotically stabilized equilibrium points of f(x).

Definition 2. System (2) is passive if the following two conditions are satisfied:
(1) f(x) and g(x) exist and are smooth vector fields, h(x) is also a smooth mapping.
(2) For any $t \geq 0$, there is a real value β that satisfies the inequality

$$\int_0^t u^T(\tau) y(\tau) d\tau \geq \beta, \tag{3}$$

or there are real values β and $\rho \geq 0$ that satisfy the inequality

$$\int_0^t u^T(\tau) y(\tau) d\tau + \beta \geq \int_0^t \rho y^T(\tau) y(\tau) d\tau, \tag{4}$$

When we let $z = \Phi(x)$ system (2) can be changed into the following generalized form

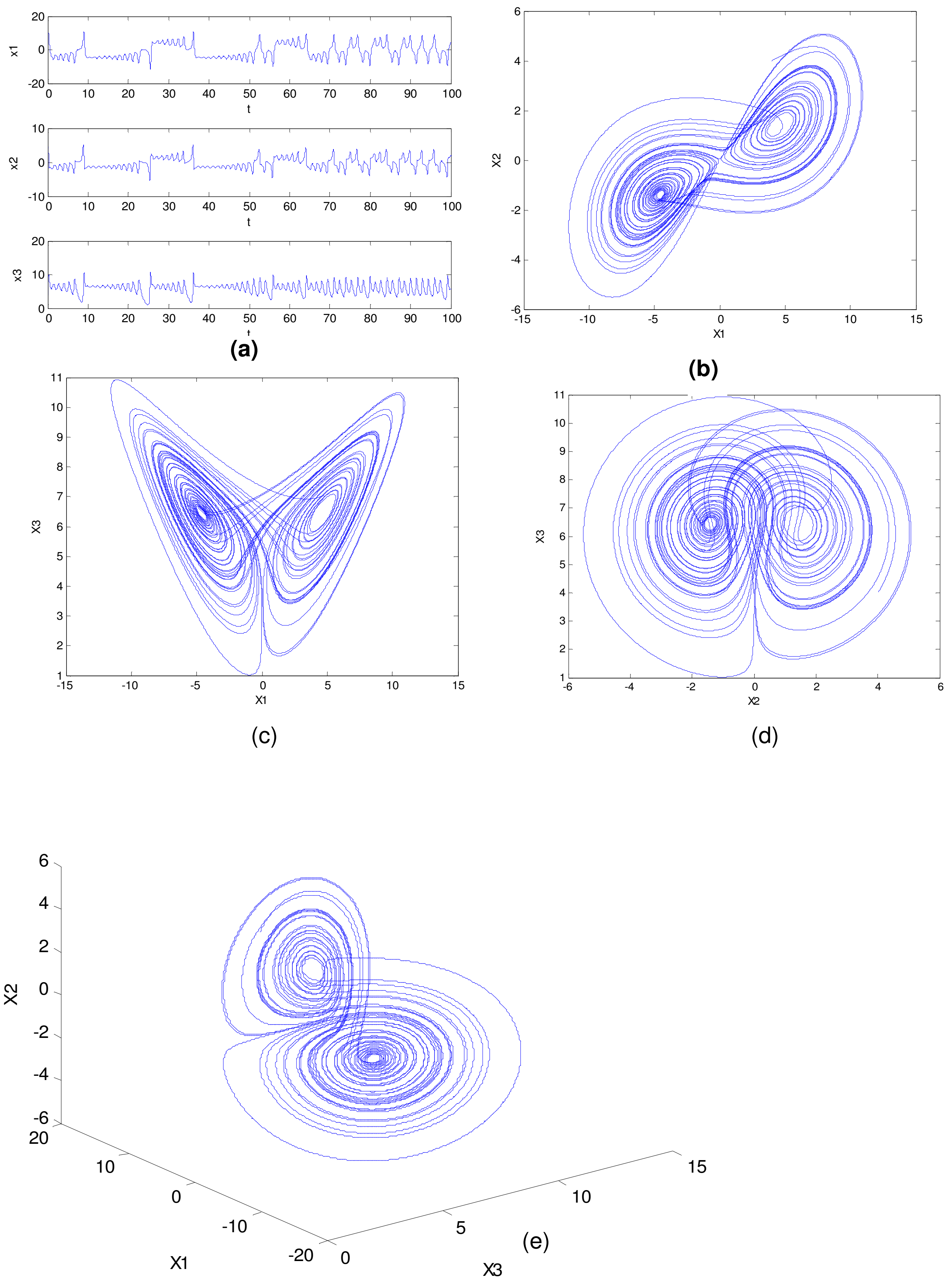


Figure 2. (a) X1, X2, X3 time series, (b) X1 – X2, (c) X1 – X3, (d) X2 – X3, (e) X1-X2-X3 phase portraits of Rabinovich attractor when a=4, b=d=1, and h=6.75.

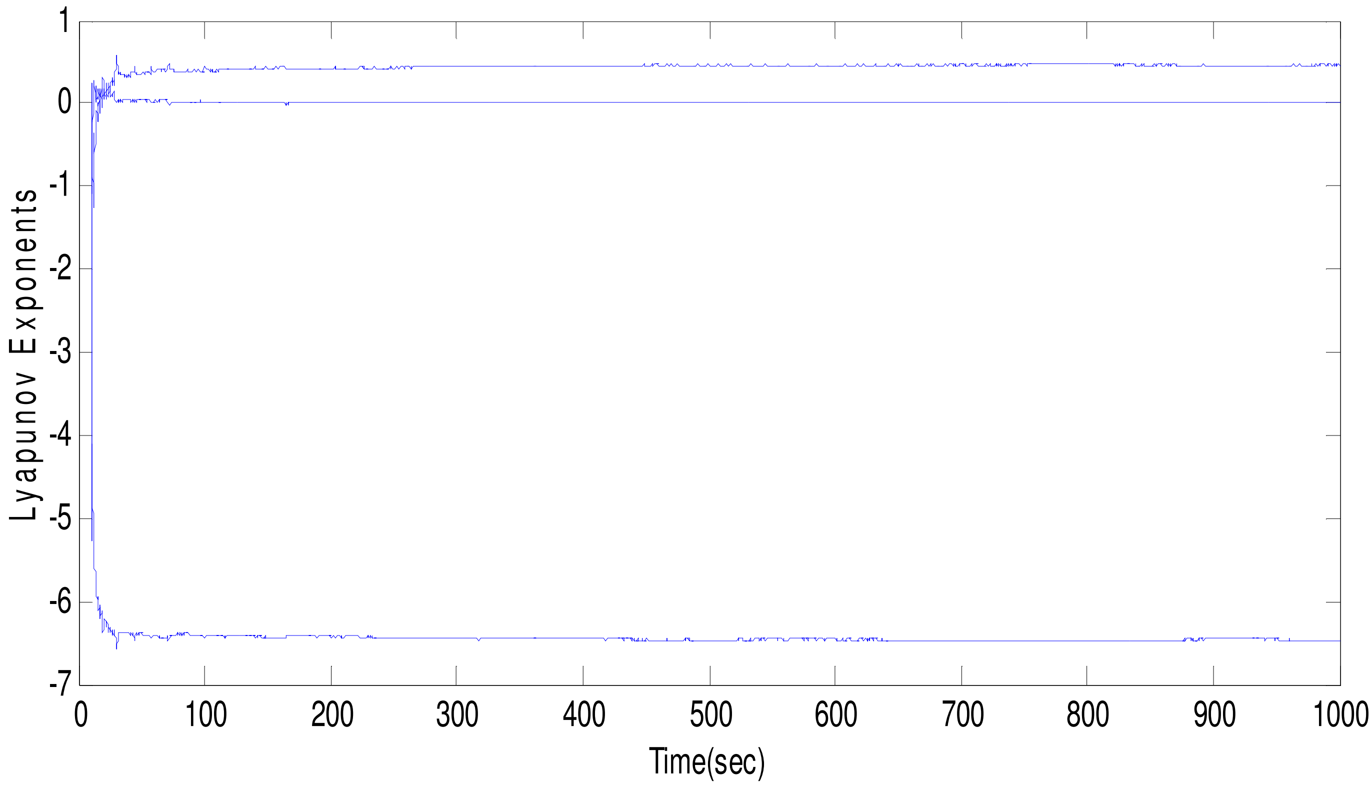


Figure 3. Lyapunov spectrum of system.

$$\begin{cases} \dot{z} = f_0(z) + p(z, y)y, \\ \dot{y} = b(z, y) + a(z, y)u, \end{cases} \tag{5}$$

Where $a(z, y)$ is nonsingular for any (z, y).

If system (2) has relative degree [1,1, ...] at x = 0 and system (2) is a minimum phase system, then system (5) will be equivalent to a passive system and will be asymptotically stable at equilibrium points through the local feedback control as follows:

$$u = a(z,y)^{-1}[-b^T(z,y) - \frac{\partial W(z)}{\partial z} p(z,y) - \alpha y + v] \tag{6}$$

Where $W(z)$ is the Lyapunov function of $f_0(z)$, α is a positive real value, and v is an external signal which is connected to the reference input.

CHAOS CONTROL OF RABINOVICH CHAOTIC SYSTEM

The control of chaotic system (7) is achieved using passive control theory. The controlled model given by

$$\begin{cases} \dot{x}_1 = -ax_1 + hx_2 + x_2 x_3, \\ \dot{x}_2 = -bx_2 + hx_1 - x_1 x_3 + u, \\ \dot{x}_3 = -dx_3 + x_1 x_2, \end{cases} \tag{7}$$

where u is controller to be designed. Suppose that state variable x_2 is the output of the system and suppose $z_1 = x_1$, $z_2 = x_3$, $y = x_2$, then the system can be expressed by normal form:

$$\begin{cases} \dot{z}_1 = -az_1 + hy + yz_2, \\ \dot{z}_2 = -dz_2 + z_1 y, \\ \dot{y} = -by + hz_1 - z_1 z_2 + u, \end{cases} \tag{8}$$

so

$$\begin{cases} \dot{z} = f_0(z) + p(z, y)y, \\ \dot{y} = b(z, y) + a(z, y)u, \end{cases} \tag{9}$$

where

$$f_0(z) = \begin{bmatrix} -az_1 \\ -dz_2 \end{bmatrix} \tag{10}$$

$$p(z,y)=\begin{bmatrix} h+z_2 \\ z_1 \end{bmatrix} \quad (11)$$

$$b(z,y)=-by+hz_1-z_1z_2, \quad (12)$$

$$a(z,y)=1. \quad (13)$$

Choose the following storage function

$$V(z,y)=W(z)+\frac{1}{2}y^2, \quad (14)$$

where

$$W(z)=\frac{1}{2}(z_1^2+z_2^2), \quad (15)$$

is the Lyapunov function of $f_0(z)$, and $W(0)=0$.

According to (15), taking the derivative of $W(z)$, we have

$$\dot{W}=\frac{d}{dt}W(z)=\frac{\partial W(z)}{\partial z}f_0(z)=\begin{bmatrix} z_1 & z_2 \end{bmatrix}\begin{bmatrix} -az_1 \\ -dz_2 \end{bmatrix}$$

$$=-(az_1^2+dz_2^2) \quad (16)$$

Since $W(z)\geq 0$ and $\dot{W}(z)\leq 0$, it can be concluded that $W(z)$ is the Lyapunov function of $f_0(z)$ and $f_0(z)$ is globally asymptotically stable which means that the zero dynamics of the controlled system (7) is Lyapunov stable. Meanwhile, $L_g h(0)=1\neq 0$, That is, $L_g h(0)$ is nonsingular, and the system has the relative degree $[1,...,1]$, so the system can be equivalent to a passive system using state feedback.

We have

$$\frac{d}{dt}V(z,y)=\frac{\partial W(z)}{\partial z}\dot{z}+y\dot{y}, \quad (17)$$

We substitute $\frac{\partial W(z)}{\partial z}=\begin{bmatrix} z_1 & z_2 \end{bmatrix}$ and $\dot{z}=f_0(z)+p(z,y)y$ into equation (17), then it yields

$$\frac{d}{dt}V(z,y)=\frac{\partial W(z)}{\partial z}f_0(z)+\frac{\partial W(z)}{\partial z}p(z,y)y$$

$$+\ yb(z,y)+\ ya(z,y)u.. \quad (18)$$

As the chaotic system is minimum phase, we obtain

$$\frac{\partial W(z)}{\partial z}f_0(z)\leq 0. \quad (19)$$

From equation (18) and inequality (19), we have

$$\frac{d}{dt}V(z,y)\leq\frac{\partial W(z)}{\partial z}p(z,y)y+\ yb(z,y)+\ ya(z,y)u. \quad (20)$$

Taking equation (u) into inequality (20), the above inequality can be written as

$$\frac{d}{dt}V(z,y)\leq-\alpha y^2+vy. \quad (21)$$

Taking integration of both sides (21)

$$V(z,y)-V(z_0,y_0)\leq\int_0^t v(\tau)y(\tau)d\tau-\int_0^t \alpha y(\tau)^2 d\tau, \quad (22)$$

as $V(z,y)\geq 0$, let $V(z_0,y_0)=\mu$, we obtain

$$\int_0^t v(\tau)y(\tau)d\tau+\mu\geq\int_0^t \alpha y(\tau)^2 d\tau+V(z,y)\geq,\int_0^t \alpha y(\tau)^2 d\tau. \quad (23)$$

According to definition 2, the system (7) is passive system.

The controlled system (7) can be equivalent to a passive system and globally asymptotically stabilized at its zero equilibrium by the following state feedback controller:

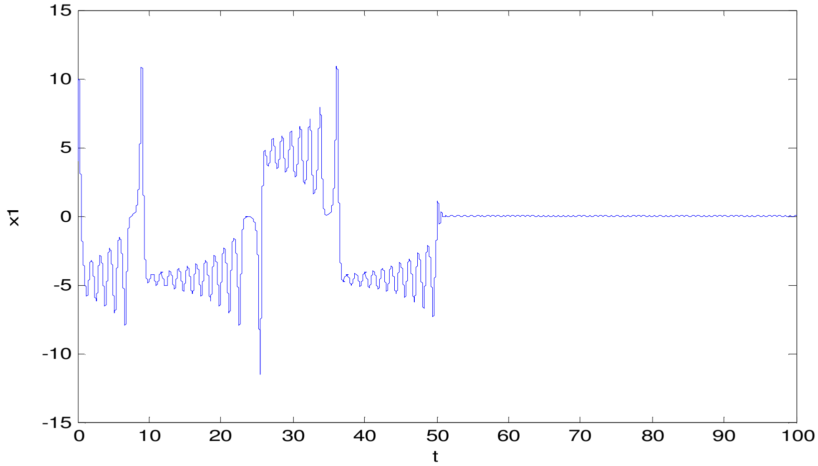
$$u=-2hz_1-z_1z_2+y(b-\alpha)+v, \quad (24)$$

Where α is a positive constant and v is an external input signal. When passive controller is activated at t=50 s, time trajectories of Rabinovich chaotic system are shown in Figure 4.

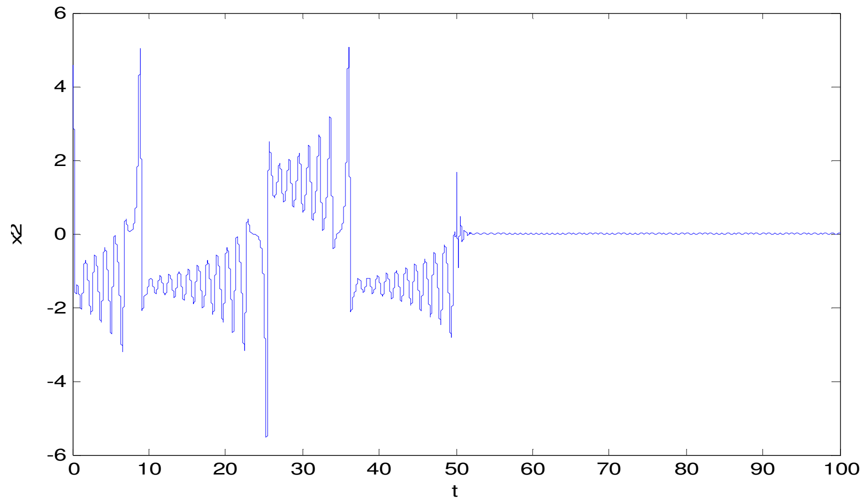
The controlled Rabinovich chaotic system with the larger α (alpha) quickly converges to equilibrium point as shown in Figure 5.

NUMERICAL RESULTS

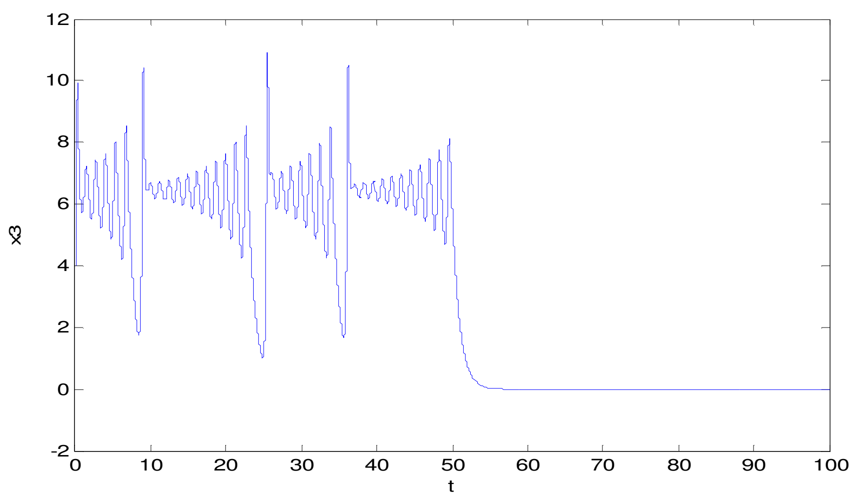
The fourth order Runge-Kutta method is used to solve the system with step size 0.001. Using passive theory, the feedback controller is designed to control the system. The control signal with v=0 is added to Rabinovich chaotic system at t=50s, when parameter a=4, b=d=1,



(a)



(b)



(c)

Figure 4. (a) X1, (b) X2, (c) X3 time series when controller is activated at t=50 s.

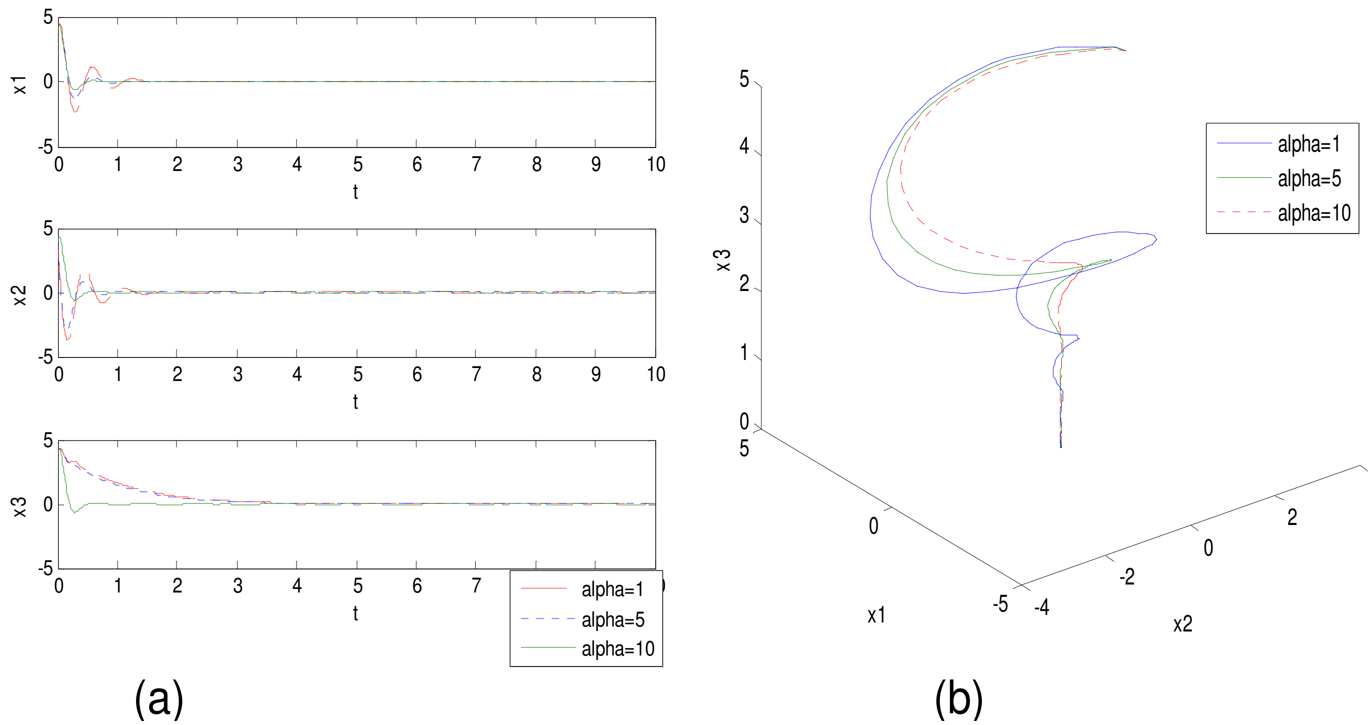


Figure 5. (a) X1, X2, X3 time series, (b) Phase-space trajectory.

and h=6.75 in Figure 4. The trajectories and time series of the controlled Rabinovich chaotic system are shown in Figures 4 and 5. As can be seen in Figures 4 and 5, the system converge to origin (0, 0, 0), after the controller is activated. So, the controller (24) can regulate the Rabinovich chaotic system effectively to zero equilibrium point and also the larger α (alpha) provides the better performance as shown in Figure 5.

CONCLUSION

This work addresses controlling chaos of Rabinovich chaotic system by using passive control technique. Based on the passive system theory, passive controller is proposed to realize the global asymptotical stability of the controlled system. In the proposed method, stability of the system is guaranteed by applying appropriate control signal based on Lyapunov stability theory. All the theoretical analyses are verified by numerical simulations to show the effectiveness of the proposed control method.

REFERENCES

Byrnes CI, Isidori A, Willem JC (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems, IEEE Trans. Automat. Contr., 36: 1228-1240.

Chen X, Liu C (2010). Passive control on a unified chaotic system, Nonlinear Analysis: Real World Applications, 11: 683-687.

Chua LO, Komuro M, Matsumoto T (1986). The double scroll family, IEEE Trans. Circ. Syst., 33(11): 1073-1118.

Chen G, Ueta T (1999). Yet another chaotic attractor, Int. J. Bifurcation and Chaos, 9(7): 1465-1466.

Ablay G (2009). Sliding mode control of uncertain unified chaotic systems, Nonlinear Analysis: Hybrid Systems, 3: 531-535.

Hong Y, Qin H, Chen GR (2001). Adaptive synchronization of chaotic systems via state or output feedback control, Int. J. Bifur. Chaos, 11(3): 1149-1158.

Kemih K (2009). Control of nuclear spin generator system based on passive control. Chaos, Solitons and Fractal,. 41: 1897-1901.

Kemih K, Filali S, Benslama M, Kimouche M (2006). Passivity-based control of chaotic Lu system. Int. J. Innovat. Comput. Inf. Control, 2(2): 331-337.

Konishi K, Hirai M, Kokame H (1998). Sliding mode control for a class of chaotic systems, Phys. Lett. A, 1(245): 511-517.

Lin W (1995). Feedback stabilization of general nonlinear control system: A passive system approach, Syst. Contr. Lett., 25: 41-52.

Liu C, Liu T, Liu L, Liu K (2004). A new chaotic attractor, Chaos, Solitons and Fractals, 22(5): 1031-1038.

Lorenz EN (1963). Deterministic non- periodic flows, J. Atmos. Sci., 20(1): 130-141.

Ott E, Grebogi C, YorkJA (1990). Controlling chaos, Phys.Rev.Lett., 64: 1196-1199.

Pikovski AS, Rabinovich MI, Trakhtengerts VY (1978). Onset of stochasticity in decay confinement of parametric instability, Sov. Phys. JETP, 47: 715-719.

Qi, D, Zhao G, Song Y (2004). Passive control of Chen chaotic system, Proc. of the 5th World Congress on Intelligent Control and Automation, Hangzhou, China, pp.1284-1286.

Rikitake T (1958). Oscillations of a System of Disk Dynamos,Proc. Cambridge Philos. Soc., 54: 89.

Rössler OE (1976). An equation for continuous chaos, Phys. Lett. A, 57: 397-398.

Wu CW, Yang T, Chua LO (1996). On adaptive synchronization and control of nonlinear dynamical systems, Int. J. Bifurcat Chaos, 6: 455-471.

Yassen MT (2005). Controlling chaos and synchronization for new chaotic system using linear feedback control, Chaos, Solitons and Fractals, 26: 913-920.

Yu W (1999). Passive equivalence of chaos in Lorenz system, IEEE Trans.Circuits Syst. I, 1.46(7): 876-878.

Zeng Y, Singh SN (1997). Adaptive control of chaos in Lorenz system, Dynamic Contr., 7: 143-154.

Zhou X, Jiang M, Zhang H, Zhou J (2009). Passivity-based Chaos Control and Synchronization of a Four dimensional Chaotic System, IEEE, 978-1-4244-4888-3.