

Full Length Research Paper

Artificial neural network approach to buckling coefficients of laminated orthotropic rectangular plates with a centrally rectangular hole

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The objective of the present study is to describe the results of research that has been conducted on the buckling behavior of rectangular laminated orthotropic plates that have a rectangular hole. As a result of using some geometrical holes which cause stress concentrations on laminated orthotropic plates, the analysis about the buckling coefficient of the elements become very important. This work deals with buckling analysis of laminated orthotropic plates with a central rectangular hole under in-plane static loading (uniaxial compression). Initially, the critical buckling coefficients of the laminated plates are obtained by finite elements method. However, as a new study, an other numerical method is applied to this work which is called "artificial neural networks" in order to obtain those buckling coefficients. A PASCAL computer program is used for training and testing procedure of neural networks, and finally good results are achieved.

Key words: Artificial neural network, orthotropic plates, buckling analysis.

INTRODUCTION

The physical understanding and numerical simulation of the buckling of laminated plates have been the focus of intense efforts because of the extended use of fibrous composites in aerospace, automotive, ship building and other industries and the need to establish the practical limits of the load carrying capability of structures made from these materials (Jones, 1999). Summaries of the many buckling studies reported in the literature are given; a study of the buckling behavior of laminated plates with a central circular hole was presented by Lin and Kuo (1989). Finite element results were obtained for clamped and simply supported stress loaded plates loaded by uniaxial compression, biaxial compression or tension (compression biaxial loading). The effects of material parameter randomness on the initial buckling load of rectangular, specially orthotropic, composite laminates (Salim et al., 1998). The basic formulation for stability analysis is based on classical laminate theory. An

approximate analysis for buckling of a rectangular specially - orthotropic plate with a central hole is applied to symmetrically laminated angle - ply plates (Nemeth, 1988). buckling loads and modes of flat composite laminates were measured and compared with theory (Tuttle et al., 1999). Laminates were subjected to simply supported boundary conditions and biaxial loading. The objective of this study is to give an overview of the finite element buckling analyses of laminated plates and to explore an other method to calculate the buckling values. Therefore, at the first part, the finite element method (FEM) is applied to obtain the buckling responses of laminated plates subjected to various combinations of mechanical loadings. The sensitivity of the responses to variations in material and lamination parameters is searched.

At the second part, a new neural network program is performed to calculate the buckling coefficients of plates as an alternative method. The dimensions of the holes

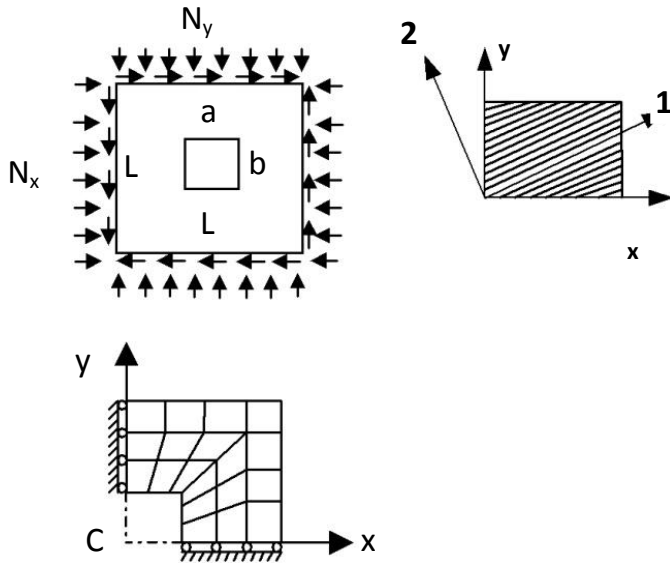


Figure 1. Geometry of the laminated rectangular plate.

and the number of lamina are used with the results of FEM for the training and testing phases of the neural network program.

BUCKLING OF LAMINATED PLATES

The laminated plates considered herein have rectangular platform and consist of a number of perfectly bonded laminates. Each lamina is assumed as a homogeneous anisotropic materials. Consider a rectangular laminated plate of length L central rectangular hole a and b in the x and y directions as shown in (Figure 1) and thickness t which consists of 2,4,6,8 orthotropic laminates. The plate is defined in the cartesian coordinates x, y and z with axes x and y lying on the middle surface of the plate, and is subjected to biaxial compressive forces N_x and N_y in the x and y directions, respectively, as shown in Figure 1. In the present study, a first-order shear deformable theory is employed to analyse the problem and the following displacement field is assumed

$$\begin{aligned} u(x, y, z) &= u_o(x, y) + z \psi_x(x, y) \\ v(x, y, z) &= v_o(x, y) + z \psi_y(x, y) \\ w(x, y, z) &= w(x, y) \end{aligned} \tag{1}$$

Where u_o , v_o and w are the displacements of the reference surface in the x, y and z direction, respectively, and ψ_x, ψ_y are the rotations of the transverse normal about the x and y -axes (Turvey and Marshall, 1995). In the finite element formulation, the deformation variables within a typical element (e) are approximated as:

$$\{u\} = [\Phi] \{a\} \tag{2}$$

Where $[\Phi]$ is the matrix of shape functions and $\{a^e\}$ generalized nodal displacements. For example, for the Reissner - Mindlin theory in which the deformation variable matrix $\{a^e\}$ includes u_o, v_o, w, ψ_x and ψ_y :

$$\{a^e\} = \{u_o, v_o, w, \psi_x, \psi_y\}^T \tag{3}$$

Using Equation 2, the strain energy can be expressed in the matrix form:

$$U = \sum_e (1/2 \{a^e\}^T [K^e] \{a^e\} - \{a^e\}^T \{F^e\}) = 1/2 \{a\}^T [K] \{a\} - \{a\}^T \{F\} \tag{4}$$

In which $\{a\}$, $[K]$ and $\{F\}$ represent the global displacement, stiffness and buckling load matrices, respectively. Likewise, the potential of the in-plane forces which develop during the prebuckling state, becomes:

$$V = 1/2 \sum_e \{a^e\} [K_G^e] \{a^e\} = 1/2 \{a\}^T [K_G] \{a\} \tag{5}$$

Where $[K_G]$ is the global geometrical stiffness matrix.

For buckling loading, the equations governing the prebuckling equilibrium state are found by minimizing Equation 4 with respect to the nodal displacement vector $\{a\}$, with the result:

$$[K] \{a\} = \{F\} \tag{6}$$

Equation 6 is solved for the nodal displacements, after which the corresponding stress resultant distributions and the geometric stiffness matrix $[K_G]$ can be found. The eigenvalue problem governing subsequent buckling is then established by requiring that the increase in total potential $\Pi = U + V$ be a minimum; this yields:

$$[K] + \lambda [K_G] = 0 \tag{7}$$

where the lowest eigenvalue λ corresponds to the amplitude of the critical buckling load (Bathe, 1982; Vasiliev and Jones, 1993; Vinson, 1975; Tung and Surdenas, 1987).

At the next part of this work, the critical buckling coefficients which are obtained by FEM are aimed to be calculated by using an other computing method, artificial neural network.

ARTIFICIAL NEURAL NETWORKS (ANN)

Nowadays, engineers and scientists are trying to develop intelligent machines. Artificial neural systems are present-day examples of such machines that have great potential to further improve the quality of our life. It is well known

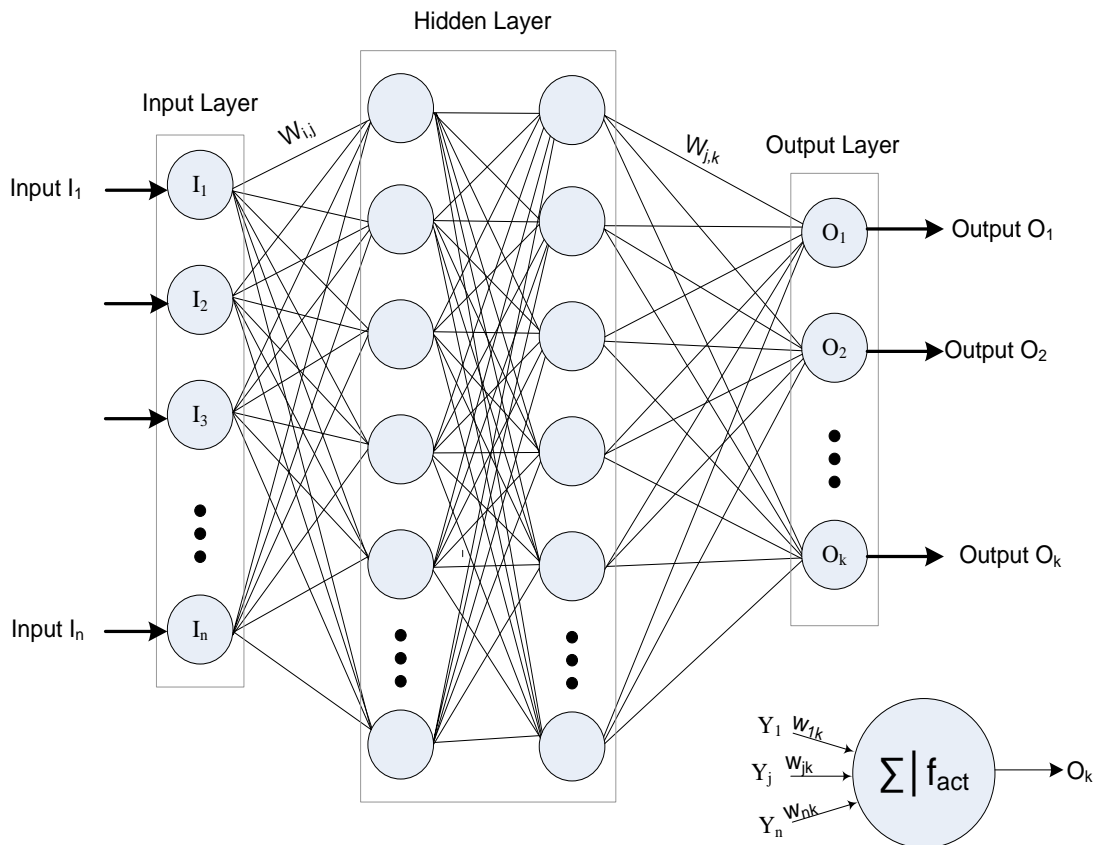


Figure 2. A multilayer perceptron architecture.

that people and animals are much better and faster at recognising images than most advanced computers. Although computers outperform both biological and artificial neural systems for tasks based on precise and fast arithmetic operations, artificial neural systems represent the promising new generation of information processing networks. Advances have been made in applying such systems for problems found intractable or difficult for traditional computation Karadag and Akgobek (2008). Neural networks can supplement the enormous processing power of digital computer with the ability to make sensible decisions and to learn by ordinary experience, as human do. Network computation is performed by a dense mesh of computing nodes and connections. They operate collectively and simultaneously on most or all data and inputs. The basic processing elements of neural networks are called artificial neurons, or simply neurons. Often they are simply called nodes. Neurons perform as summing and non-linear mapping junctions. In some cases they can be considered as threshold units that fire when their total input exceeds certain bias levels. Neurons usually operate in parallel and are configured in regular architectures. They are often organised in layers, and feedback connections both within the layer and toward

adjacent layers are allowed. Connection strength is expressed by a numerical value called a weight, which can be modified. Among the Artificial Neural Networks, the elementary multilayer perceptrons (MLP) with sigmoidal transfer function had been successfully applied to solve some difficult and diverse problems as non-linear discriminant function classifiers. The feedforward network learns from the input data by the supervision of the output data creating single linear discriminant functions by each sigmoid hidden unit and combines them. Thus, this piecewise linear discriminant function works as a non-linear discriminator (Zurada, 1993).

Training the network in a supervised manner with a highly popular algorithm known as the error back-propagation (BP) has become very popular. The back-propagation is an optimisation technique for implementing gradient descent in weight space for multilayer feedforward networks. The basic idea of the technique is to efficiently compute partial derivatives of an approximating function $F(w;x)$ realized by the network with respect to all the elements of the adjustable weight vector w for a given value of input vector x and output vector y . The weights are adjusted to fit linear piecewise discriminant functions to feature space for the best class separability. The difference between the network's output

Table 1. Optimum parameters and % errors of programs.

Program No.	Output variables	ϵ	A	Hidden layer node number	Average % error
1	Buckling coefficient (2 layers)	0.20	0.93	2	0.169
2	Buckling coefficient (4 layers)	0.70	0.90	2	0.302
3	Buckling coefficient (6 layers)	0.70	0.90	2	0.204
4	Buckling coefficient (8 layers)	0.70	0.90	2	0.112

and the supervisor output is minimized according to a predefined error function (performance criterion) such as mean square error (MSE). In this work, neural network system has been applied with multilayer perceptron (Figure 2), and backpropagation algorithm by supervised training (Çetinel et al., 2002). A computer program, which was written by the authors in PASCAL, has been used for this application. Obtained buckling coefficients produced as a function of orthotropic lamina numbers and the rectangular hole dimensions have been used for training operations.

Training procedure

The general aim of the training process is to teach the relations between input and output values to the program and get the results with the possible lowest errors. In this work, we have performed four separate programs due to the lamina numbers of plates which are 2, 4, 6 and 8 laminates. In all four programs, the input variables are the dimensions of rectangular hole at x and y directions while the output variables are buckling coefficients. Therefore, there are two input variables and one output variable at each program in this application. In neural network procedure, the training values are supposed to be reduced between 0 and 1 which is called the normalization process. That must be done before training phase by dividing the input and output values by some appropriate numbers.

NUMERICAL RESULTS AND DISCUSSION

The FEM results previously obtained in this paper are used now as input and output data that consist of 36 rows in each program (some of the input and output values were kept in order to be used for testing process after training). Many training iterations were made by changing the learning rates (α), momentum values (ϵ), and the node numbers of hidden layers. Training is completely a trial and error process and aims to get the appropriate network parameters to minimize the errors. These parameters are α , ϵ , and hidden layer node numbers. After many trials and performing numerous iterations, the optimum parameters and the final average errors of each program are obtained as shown in Table 1.

As seen, the average % errors are less than 0.3 s which are clearly acceptable for neural network applications. The final error values are obtained by 50000 training iterations. As it was aforementioned, the training process takes much more time because of many trials by changing the parameters. But this process is terminated when the optimum parameters are determined.

Testing

The final and most important step of neural network applications is testing the designed programs with real values. The programs used for training in this study are also available for testing procedure. Testing process is carried out by using different input values which were not used for training previously. In Table 2, the testing results are presented together with the results of FEM. As it is obviously seen, ANN test outputs (results) have enough agreement with the ones of the FEM method. Finally, it can be said that, this method has been an alternative way to calculate the buckling coefficients of laminated plates. It should also be emphasized that, the neural network program is able to work for different input values (that were not used by FEM), and gives good results. And an other important point is that the testing process takes only milliseconds unlike the training procedure. So the results are able to be obtained in very short time.

CONCLUSION

In this work, critical buckling coefficients of rectangular laminated plates which carry rectangular holes are obtained by two different methods. At the beginning, the stress of the laminated plates is analyzed by considering stress concentrations for critical buckling. This work shows that there is an important decrease in the critical buckling coefficients of laminated orthotropic plates. So a finite element solution is performed in order to obtain those buckling coefficients due to the number of lamina and the hole dimensions of plates. Secondly, an alternative method is performed to do the same work. That is artificial neural networks, and it is used to obtain the critical buckling coefficients which were obtained by FEM. The results of FEM and ANN are compared, and little differences are determined. The error rates about %

Table 2. The comparison between FEM and ANN results.

	1st Input (a/L)	2nd Input (b/L)	Buckling coefficients	
			FEM results	ANN results
2 laminate	0.125	0.1	5.40	5.26
	0.475	0.1	4.81	4.71
	0.275	0.2	4.05	4.06
	0.375	0.2	4.01	3.99
	0.225	0.3	3.67	3.64
	0.425	0.3	3.52	3.51
	0.175	0.4	2.75	2.77
	0.325	0.4	2.60	2.60
4 laminate	0.225	0.1	2.60	2.61
	0.475	0.1	2.42	2.42
	0.125	0.2	2.64	2.62
	0.375	0.2	2.31	2.30
	0.125	0.3	2.37	2.36
	0.425	0.3	1.78	1.84
	0.175	0.4	1.83	1.83
	0.425	0.4	1.60	1.57
6 laminate	0.125	0.1	2.39	2.39
	0.425	0.1	2.09	2.10
	0.175	0.2	2.19	2.17
	0.275	0.2	2.05	2.01
	0.125	0.3	2.00	1.98
	0.375	0.3	1.52	1.54
	0.225	0.4	1.45	1.44
	0.325	0.4	1.27	1.30
8 laminate	0.125	0.1	2.25	2.25
	0.475	0.1	1.93	1.92
	0.225	0.2	1.95	1.94
	0.375	0.2	1.76	1.76
	0.175	0.3	1.84	1.88
	0.325	0.3	1.65	1.66
	0.225	0.4	1.74	1.72
	0.475	0.4	1.49	1.48

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0.2 s are fairly granted by neural network applications.

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