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Full Length Research Paper

A novel extrapolation method for value prediction applications

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This paper presents a simple extrapolation method for value prediction applications. Classical statistical analysis methods are not used for the proposed method instead some basic mathematical and constitutional properties of the system responses are used, and relationships between the samples are modeled. Detailed mathematical description of the proposed method is given and obtained results are justified using illustrative examples. Limitations and possible applications of the proposed method are also discussed.

Key words: Extrapolation, predictive models, estimation, industrial control.

INTRODUCTION

Prediction concept is one of the most important topics of control (Mayne et al., 2000; Mayne and Rakovic, 2003; Guo and Billings, 2007), signal processing (Spriet et al., 2005), chemical engineering (Shi et al., 2003), statistics (Zayed Raqab, 2001), etc. Considering the previous samples and system-tendency, some foresights could be developed. In some problems, prediction of the systems' future response is important. For this reason some methods such as Kalman filtering (Ji and Sul, 1995), autoregressive (AR) modeling (Hashimoto et al., 2001), gray modeling (Lin and Bin-Da, 2005) and neural networks (Orlowska-Kowalska and Szabat, 2007; Erenturk et al., 2004) have been developed.

In most of the systems, dynamic behavior of the system is modeled using differential equations. Solutions of these equations are in exponential form. In general, electrical and most of the chemical processes are in first-order linear model. In control, controlled systems are closed-loop systems and they have second-order transfer functions. Solution of the closed-loop systems is combination of both exponential and sinusoidal functions.

In this study, a basic prediction and extrapolation method is proposed. The main aim of the proposed method is to provide a simple and a useful tool for both exponential and sinusoidal system behavior.

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Abbreviations: **y,** Time dependent function; **t,** time (s); **A, B,** constants; τ**,** sampling time (s); ***n*****,** sampling instant; ω**,** angular frequency (rad.).

MATERIALS AND METHODS

Mathematical basis of the proposed method

In control applications, predicting of the error values for the next sampling instant is very important. Using these predicted values provides more powerful control action compared to the classical or other well-known control techniques as one may estimate the error values before the error occurred.

In general, systems and processes are modeled using *n* differential equation, and *h* variables. This modeling is generally expressed as (*n, h*). In a real control environment, (1, 1) is the most commonly used model for studies and applications.

In the following exponential form part of the work, the proposed prediction technique and detailed mathematical descriptions will be discussed.

Exponential form

Here, (1, 1) denotes a single-variable sequence and first-order linear model. Most of these systems have an exponential solution as given in (1).

$$y(t) = A.e^{\pm Bt} \qquad (1)$$

The original series of data with *n* samples could be assumed as

$$y(t) = \{y^0, y^1, y^2, ..., y^n\} \qquad (2)$$

where superscription (0, 1, 2, …, n) of *y* represents the original data values for the different sampling instants. It is clear that each of the sampled values is obtained in the next sampling instant, and this sampling instant is determined by the sampling time, τ. If tendency of the systems is considered as descending, then sampled values could be expressed as

$$\begin{aligned} y(t = t_0) = y^0 = A.e^{-Bt_0} &\Rightarrow t_0 = -\frac{1}{B}\ln(\frac{y^0}{A}) \\ y(t = (t_0 + \tau)) = y^1 = A.e^{-B(t_0+\tau)} &\Rightarrow t_0 + \tau = -\frac{1}{B}\ln(\frac{y^1}{A}) \end{aligned} \qquad (3)$$

For $t = t_0 + 2\tau$ instant, y^2 could be predicted in terms of these two previous samples, such as

$$y(t = (t_0 + 2\tau)) = y^2 = A.e^{-B(t_0+2\tau)} = A.e^{-B(-\frac{1}{B}\ln(\frac{y^0}{A}) - \frac{2}{B}(\ln(\frac{y^1}{A}) - \ln(\frac{y^0}{A})))} \qquad (4)$$

Simplifying Equation (4), the following equations is obtained.

$$y(t = (t_0 + 2\tau)) = y^2 = \frac{(y^1)^2}{y^0} \qquad (5)$$

In order to generalize the above expression for n^{th} sampling instant, Equation (5) can be expressed in the following form

$$y(t = (t_0 + n\tau)) = y^n = A.e^{-B(-\frac{1}{B}\ln(\frac{y^0}{A}) - \frac{n}{B}(\ln(\frac{y^1}{A}) - \ln(\frac{y^0}{A})))} = \frac{(y^1)^n}{(y^0)^{n-1}} \qquad (6)$$

Regression analysis is not required to find the coefficients of the exponential form, *A* and *B*, since these coefficients are analytically obtained from the first two samples by using Equation (3).

Sinusoidal form

A sampled system with sinusoidal character can be expressed by the following equations.

$$\begin{aligned} y(t = t_0) = y^0 = A.\sin(\omega t_0) &\Rightarrow \omega t_0 = \arcsin(y^0 / A) \\ y(t = (t_0 + \tau)) = y^1 = A.\sin(\omega t_0 + \tau) &\Rightarrow \omega t_0 + \tau = \arcsin(y^1 / A) \end{aligned} \qquad (7)$$

y^2 for $t = t_0 + 2\tau$ instant can be predicted in terms of two previous samples, as

$$y(t = (t_0 + 2\tau)) = y^2 = A.\sin(2.\arcsin(y^1 / A) - \arcsin(y^0 / A)) \qquad (8)$$

In order to generalize the above expression for n^{th} sampling instant, equation (8) is expressed in the following form

$$y(t = (t_0 + n\tau)) = y^n = A.\sin(n.\arcsin(y^1 / A) - (n-1).\arcsin(y^0 / A)) \qquad (9)$$

In order improve the usefulness of the above expressions, following assumptions are made

1. All functions are continuous,
2 Functions are normalized to their maximum value,
3. Combination of these functions has homogeneity and superposition properties.

It is obvious that above assumptions do not result in intolerable restrictions and error for the most of the systems.

RESULTS

In order to make a comparison, gray modeling technique is considered and applied to all prediction applications. Results of three different types of applications are given in this part of the work. Firstly, a descending exponential form is considered and results are given in Figure 1. Characteristic discharge curve of a simple RC circuit of electronics is examined and illustrated in Figure 1. This curve also defines most of the chemical processes such as drying and reaction kinetic.

Secondly, a sinusoidal case is taken into consideration and results are shown in Figure 2.

Finally, a product of both exponential and sinusoidal form is considered. Here, an under damped second-order closed-loop system behavior for step response is predicted using both gray modeling technique and proposed model. Prediction results for this case are illustrated in Figure 3.

A drying curve of Echinacea angustifolia (Erenturk et al., 2004) is given in Figure 4, for air temperature of 30°C and air velocity of 0.3 m/s. In this figure, a real time experimental result is given.

DISCUSSION

Tabulated form of the results is also given in Table 1. Sum squared error (SSE) criterion, given in equation (10), is used to compare the performance of two different methods. As can be seen from the table, the proposed model provides more accurate predictions for the next response of the system than that of the gray model. One of the superiorities of the proposed system is that the proposed method does not require complex mathematical calculations hence it spends less CPU time.

$$SSE = \sum_{i=1}^{n}(x_i - \bar{x}_i) \qquad (10)$$

where n is the number of observations, x_i are observed values and $\bar{x}_i$ are their fitted values.

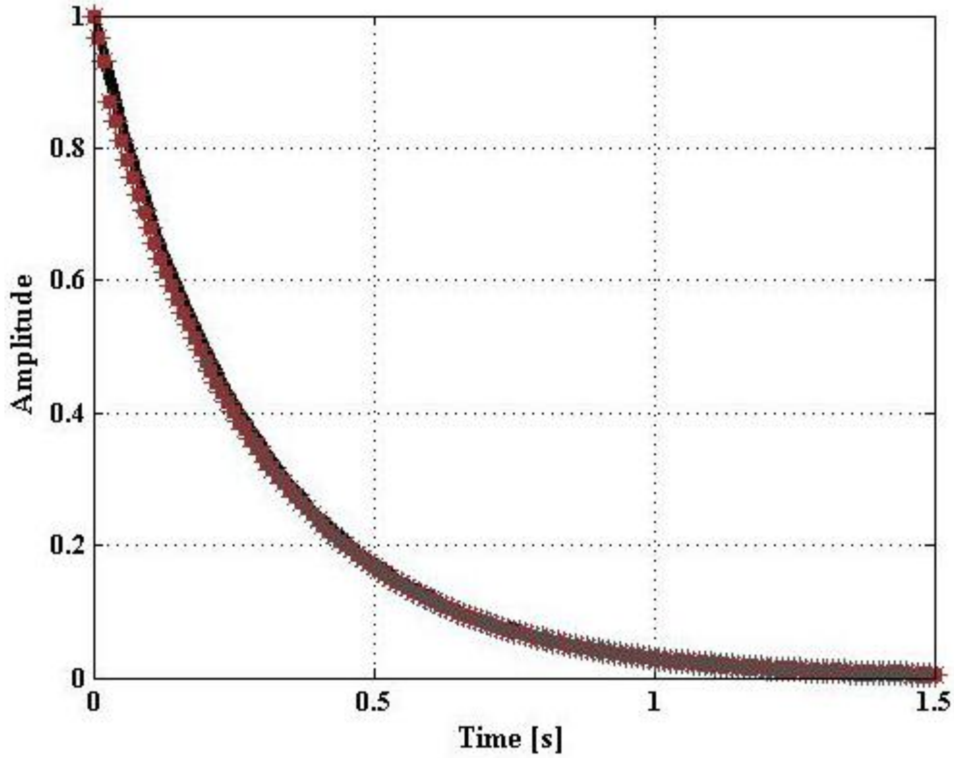

Figure 1. Results for exponential case. (solid line: real exponential curve, **.**: proposed method, *****: gray estimator).

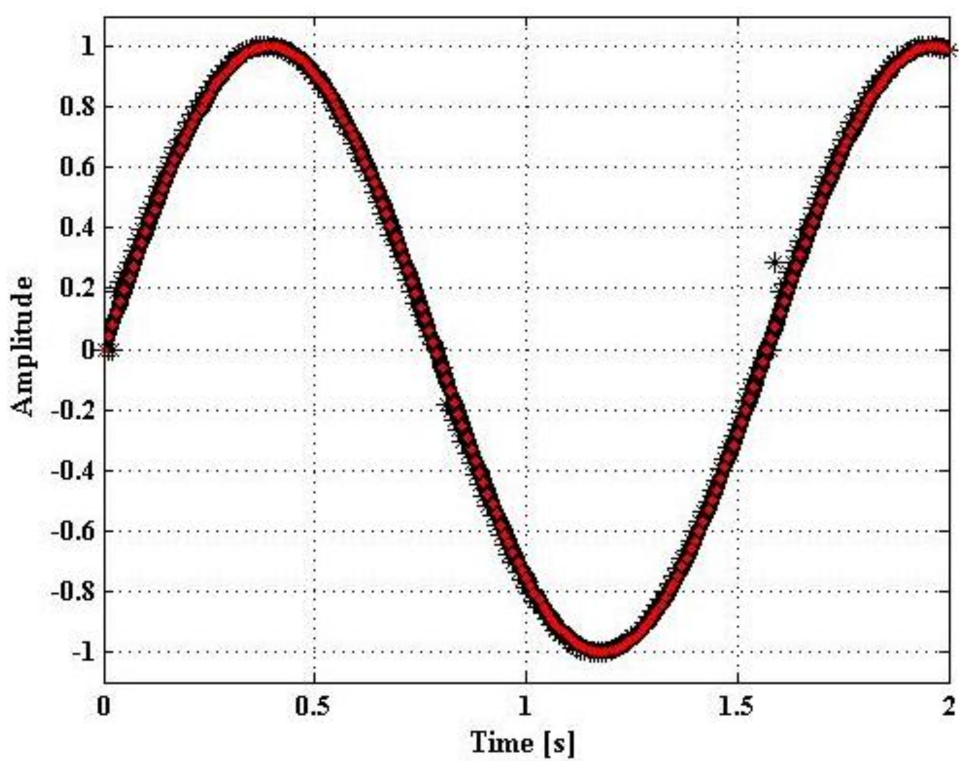

Figure 2. Results for sinusoidal case. (solid line: real exponential curve, **.**: proposed method, *****: gray estimator).

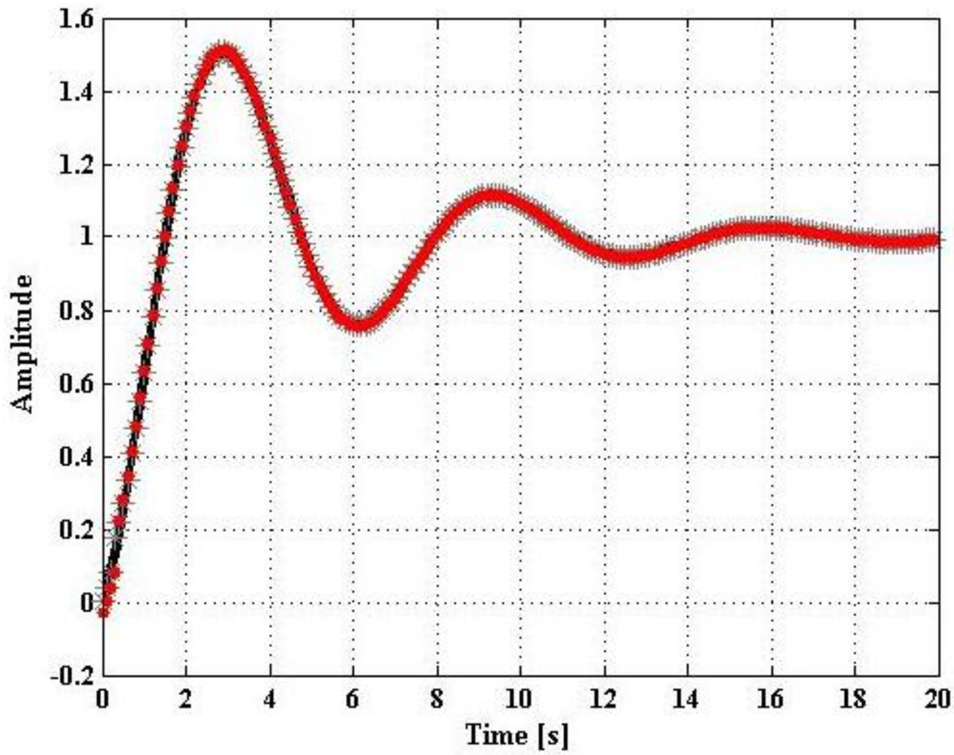

Figure 3. Results for combination of both exponential and sinusoidal cases. (solid line: real exponential curve, **.**: proposed method, *****: gray estimator)

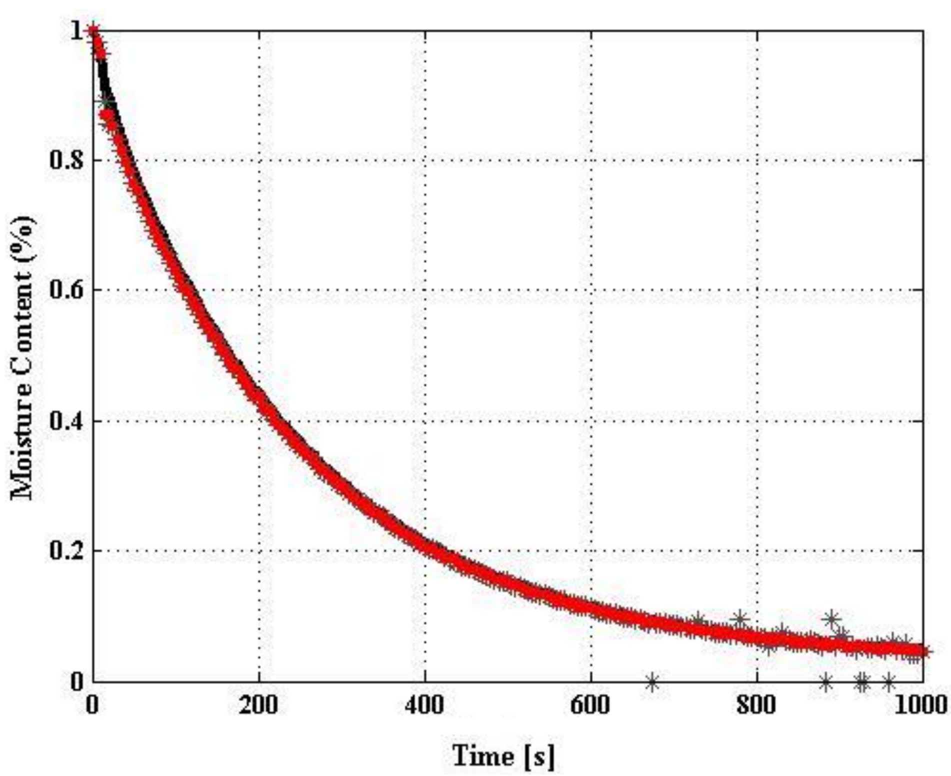

Figure 4. Results for *Echinacea angustifolia* drying case. (solid line: real experimental curve, **.**: proposed method, *****: gray estimator).

Table 1. Comparison results of the proposed method and gray modeling technique.

Sum Squared Error (SSE)	Proposed model	Gray Model
Exponential case	0.0142	0.143
Sinusoidal case	0.2952	0.3767
Combination of both sinusoidal and exponential cases	0.0053	0.1644
Echinacea angustifolia drying case	0.0102	0.0374

It can be seen from Figure 4, there are some deviations due to the fact that the regression coefficients of this type of process are modeled using Arrhenius-type approximation. These coefficients depend upon system parameters such as air velocity, air humidity, air inlet-outlet temperature and etc. For this reason, these coefficients should be re-calculated in each step when regression analysis is considered. However, as could be seen from the figure, the proposed prediction method gives more reliable result compared to regression analysis.

Conclusions

In this study, a basic prediction and extrapolation method are proposed. Detailed mathematical fundamentals of the proposed method and application results are explained in both graphical and tabular form. The results obtained from the proposed prediction method are not only superior in the prediction but also suitable for many practical application. The proposed method is applicable in the areas of control, signal and image processing, electrical power engineering such as fault detection in protective relaying, statistics, chemical process engineering, drying system modeling, and etc.

The main limitations of the proposed method are: 1) first two samples have to be obtained, 2) all amplitudes have to be normalized, 3) all considered functions have to be in continuous form and 4) all samples must be taken periodically, that is sampling periods should be equal.

It is obvious that above limitations do not bring out intolerable restrictions and error for most of the systems. It is also clear that the proposed method has many advantages for control applications such as error prediction and control action determination before the response of the system for the input.

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