# Verification of the analytical equation for power penalty measurement in OXADM device 

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#### Abstract

This paper presents a computational investigation of OXADM device for measuring power penalty. The OXADM are located in the nodes, which have more than two switching directions in ring networks. The function of OXADM is to flexibly switch the wavelengths among the different input and output ports. Because of the OXADM's imperfect performance, the insertion loss and crosstalk are induced in the system. This paper highlights the verification of analytical modeling method which is used to analyze the OXADM structure in crosstalk or leakage power that leads to the power penalty. The power penalty depends on few parameters but our investigation here focused on a number of operating wavelengths input and output ports, ratio of optical power as well as, the $Q$ factor. The variation of this parameters will affect the amount of the desired power penalty.


Key words: OXADM, analytical, verification, power penalty, crosstalk.

## INTRODUCTION

The rapid growth in optical communication technology has boosted the development and realization of novel optical switching device. Optical Cross Add and Drop Multiplexer (OXADM) is one of the new optical hybrid device that apply combined concept of both Optical Cross Connect (OXC) and Optical Add and Drop Multiplexing (OADM) between two main transmission lines for the wavelength routing operations. It is designed to improve network efficiency, reliability and survivability and can be used from point to point, ring or mesh network topology (Ab-Rahman et al., 2006; Ab-Rahman and Shaari, 2006; Ab-Rahman et al., 2007). In general, an OXADM consists of three main subsystems including a wavelength selective demultiplexer, a switching subsystem and a wavelength multiplexer (Ab-Rahman et al., 2008a). It allows the operating wavelength on two different optical trunks to be switched to each other while implementing

[^0]add and drop function at the same time. Each OXADM can handle at least two distinct wavelength channels each with granularities of $2.5 \mathrm{Gbit} / \mathrm{s}$ or higher (Eldada and Nunen, 2000). The signals can be re-routed to any output port or accumulated/multiplexed onto one path before exit from any desired output port. These features also differ from OXADM with the other existing devices (Ab-Rahman et al., 2008b).

The imperfection in OXADM device has led to induction of crosstalk in the system where the extent of crosstalk depends on the number of input/output ports connected to the fiber lines and the number of wavelengths per line (Figure 1). The presence of crosstalk will impair the signal and additional signal power is required to maintain a desired bit error rate (Ab-Rahman et al., 2010). This power is known as power penalty and is defined as:

$$
\begin{equation*}
P_{\text {penalty }}(\mathrm{dB})=10 \log _{10} \frac{P_{1}}{p_{2}} \tag{1}
\end{equation*}
$$

Where $\mathrm{P}_{1}=$ power required with impairment, $\mathrm{P}_{2}=$ power required without impairment.


Figure 1. Induction of incoherent crosstalk due to the leakage signal of each channel operates in OXADM.

This paper presents a computational investigation of OXADM device for measuring power penalty. Four parameters have been chosen which are number of operating wavelengths, number of ports, ratio of optical power, and $Q$ factor. These four variables are individually treated as manipulated variable. Meanwhile, the power penalty is treated as the responding variable. The responses between the maximum power penalty and MN , $\varepsilon, L$, and $Q$ are investigated by plotting MATLAB graphs. The power penalty is significant parameter and the value must be considered during calculating power budget of optical network.

## THEORY AND DERIVATION

The structure of OXADM for the analytical modeling is shown in Figure 1. The OXADM consists of a total N optical demultiplexers and N optical multiplexer. Each of the input fiber to optical demultiplexer contains M
different wavelengths. Demultiplexer is used to isolate the entire wavelength into M pathway.
When an optical signal passes through an OXADM, many crosstalk contributions are combined with the signal due to non ideal cross talk specification. Assuming OXADM is fully loaded; each signal passing through OXADM will be interfered by MN-1 homodyne crosstalk which is caused by leaked signal from demultiplexer/multiplexer pair.

For facilitating the description, lets $\lambda_{11}$ act as a main signal which has wavelength 1 in input fiber 1. At optical multiplexer, the main signal $\lambda_{11}$ will be interfered by M ( N 1) crosstalk contributions leaked from the signals with wavelength 1 in the other $\mathrm{N}-1$ input fiber: $\lambda_{21}, \lambda_{31}, \ldots$, $\lambda_{N 1}$ and ( $\mathrm{M}-1$ ) crosstalk from own $\lambda_{11}$ signal leakage. Thus, the main signal is degraded by a total of $\mathrm{M}(\mathrm{N}-1)+$ $(\mathrm{M}-1)=\mathrm{MN}-1$ crosstalk and causes signal impairment. The main signal is multiplexed with these MN-1 crosstalk signals at optical multiplexer. This situation is illustrated in Figure 2.

Homodyne crosstalk can be divided into coherent crosstalk and incoherent crosstalk. The phase of coherent crosstalk is correlated with the signal considered. On the other hand, the phase of incoherent crosstalk is not correlated with the signal considered. Coherent crosstalk is believed not to cause noise but cause fluctuation of signal power if the optical propagation delay differences are much less than the time duration of one bit of the signal. However, when this condition is not satisfied, the coherent crosstalk will cause noise to the overall system (Shen et al., 1995).
The $\mathrm{M}(\mathrm{N}-1)$ crosstalk contributions from leaked signals with wavelength 1 in $\mathrm{N}-1$ input fibers can be treated as generated by different lasers and therefore their phases are uncorrelated with the main signal, $\lambda_{11}$ and with each other. On the other hand, the ( $\mathrm{M}-1$ ) crosstalk contributed by $\lambda_{11}$ itself are also uncorrelated with $\lambda_{11}$ and other crosstalk signals if the optical propagation delay differences in an OXADM exceed the coherent time of the laser, that is, $\mathrm{T}>\mathrm{T}_{\text {coherent }}$ (Shen et al., 1995). Therefore, the field of the main signal $\lambda_{11}$ and all the $\mathrm{MN}-1$ incoherent crosstalk can be generally expressed as (Shen et al., 1995):

$$
\begin{aligned}
& \vec{E}(t)=E b_{s}(t) \cos \left[\omega_{s} t+\phi_{s}(t)\right] \vec{P}_{s} \\
& +\ldots \sum_{l=1} \sqrt{\varepsilon} E b_{l}(t) \cos \left[\omega_{l} t+\phi_{l}(t)\right] \vec{P}_{l}
\end{aligned}
$$

The first term of the right part of (2) is the field of main signal and the second term accounts for the incoherent crosstalk contributions. From the probability density function (PDF) for the accumulation of interferences is the Gaussian assumption with further derivation of power penalty (Takahashi et al., 1996). Power Penalty is given as:
$P_{\text {penalty }}=-5 \log _{10}\left[1-4 \cdot \sigma_{\text {RIN }}^{2} \cdot Q^{2}\right]$
With:
$\mathrm{Q} \quad=\mathrm{Q}$ factor for specified BER
$\sigma_{\text {RIN }}^{2}=$ Auto variance for noise from interferometric noise.
$=\varepsilon \sum_{l=1}^{M N-1} \cos ^{2} \theta_{l}$
$\cos ^{2} \theta_{1}=$
Where, $\theta$, is the polarization angle difference between the main signal and the th crosstalk contribution.

The maximum power penalty is equal to the sum of the optical power of the crosstalk contributions. From (4), it
can be seen that the maximum value of $\sigma^{2}$ RIN is equal to $\varepsilon(M N-1)$ when $\cos ^{2} \theta_{l}=1$. Therefore:
$P_{\text {penalty ( } O X A D M, ~ m a x)}$
$=-5 \log _{10}\left[1-4 \cdot \max \left(\sigma_{\text {RIN }}^{2}\right) \cdot Q^{2}\right]$
$=-5 \log _{10}\left[1-4 \cdot \varepsilon(M N-1) \cdot Q^{2}\right]$

The maximum power penalty after passing through L number of OXADM are given as:
$P_{\text {penalty (L_OXADM, max) }}$
$=-5 \log _{10}\left[1-4 \cdot \max \left(\sigma_{\text {RIN }}^{2}\right) \cdot Q^{2}\right]$
$=-5 \log _{10}\left[1-4 \cdot \varepsilon\left[\left(N_{1} M_{1}-1\right)+\left(N_{2} M_{2}-1\right)+\ldots+\left(N_{L} M_{L}-1\right)\right.\right.$.
$\left.\left.Q^{2}\right]\right]$
$=-5 \vec{P}_{s} \cdot \vec{P}_{l} \log _{10}\left[1-4 \cdot \varepsilon\left[\left(\Sigma \mathrm{M}_{\mathrm{i}} \mathrm{N}_{\mathrm{i}}-1 \cdot \mathrm{~L}\right) \cdot \mathrm{Q}^{2}\right]\right]$
$=-5 \log _{10}\left[1-4 \cdot \varepsilon\left[\left(\Sigma M_{i} N_{i}-L\right) \cdot Q^{2}\right]\right.$
If we assume all $M_{i} N_{i}$ in each OXADM of length $L$ is the same (that is, $\sum \mathrm{M}_{\mathrm{i}} \mathrm{N}_{\mathrm{i}}=\mathrm{MN}{ }^{*} \mathrm{~L}$ )

Then, maximum power penalty:
$=-5 \log _{10}\left(1-4 \cdot \varepsilon\left(M N^{*} L-L\right) \cdot Q^{2}\right)$

Where, Bit Error Rate, BER is calculated as:
$B E R=1 / 2 \operatorname{erfc}\left(Q / 2^{1 / 2}\right)$
Max $Q$ Factor, $Q$ is calculated thus:
$Q=(2)^{1 / 2} \mathrm{erfc}^{-1}(2 \cdot \mathrm{BER})$
From equation (7), it can be seen that the maximum power penalty achieves infinity as:
$1-4 \cdot \varepsilon \cdot\left[\left(\Sigma M_{i} N_{i}-L\right) \cdot Q^{2}\right]=0$.
Therefore, saturated $\mathrm{P}_{\text {penalty ( L_OXADM, max) occur }}$ when:
$1-4 \cdot \varepsilon \cdot\left[\left(\Sigma M_{i} N_{i}-L\right) \cdot Q^{2}\right]<0$
Lets:
$\varepsilon=-44 \mathrm{~dB}=3.98107 \times 10^{-5}$
Q = 6
$\mathrm{P}=\left(\Sigma \mathrm{M}_{\mathrm{i}} \mathrm{N}_{\mathrm{i}}-\mathrm{L}\right)=(\mathrm{MN}-1) \mathrm{L}$
Thus:
$1-4 \cdot \varepsilon \cdot P \cdot Q^{2}=0$
$\mathrm{P}=174.4$
Therefore, if (NM-1) L is greater than 173, then the power


Figure 2. Crosstalk due to the leakage signals in $2 \times 2$ OXADM

## penalty will achieve infinity.

In our previous research (Ab-Rahman et al., 2008a) showed the maximum of nodes that can be cascaded both OXADM and OXC in comparison. In point to point configuration, the maximum ports at two operating wavelengths in OXADM is 43 ports but at three operating wavelengths the figure decreases to 29 , as compared to OXC device with 86 ports and 85 ports respectively.
The increments of number of channel also give much effect to the number of OXADM nodes that can be cascaded. With 2 ports, the maximum channels that can be operated are 43 channels but in OXC the figure is double. The maximum channels that can be handled by OXADM at 4 ports are 26 channels compare to that of OXC which is 84 channels. Although, the accumulation features decrease the scalability of OXADM but its application is wide.

## MATERIALS AND METHODS

## Relationship among the variables of interested

We can observe that there are four main variables, MN (which depends on the value of the individual M and N ), $\varepsilon, \mathrm{L}$ and Q . These four variables are individually treated as manipulated variable. Meanwhile, the power penalty is treated as the responding variable. The responses between the maximum power penalty and $\mathrm{MN}, \varepsilon, \mathrm{L}$, and $Q$ are investigated by plotting MATLAB graphs.

## Power penalty versus $\mathbf{Q}$ factor

## Case 1

7vary: MN = $4(2 \times 2), 6(2 \times 3$ or $3 \times 2), 8(2 \times 4$ or $4 \times 2), 10(5 \times 2$ or $2 \times 5)$, $12(2 \times 6,3 \times 4,4 \times 3$ or $6 \times 2)$.
constant: $\varepsilon=4 \times 10^{-5}, \mathrm{~L}=1$
Figure 3 show the power penalties at one OXADM node with the varying MN product. The power penalties will increase exponentially as the MN product increase. The gap of power penalties between each others at $\mathrm{Q}=10$ are almost identical ( 0.08 dB ). When MN product increases, the signal will be interfered by more crosstalk since they passed through more input/output port or more optical switch.

## Case 2

vary: $L=1,2,3,4$
constant: $\varepsilon=4 \times 10^{-5}, \mathrm{MN}=2 \times 2$
Figure 4 show that the power penalties obtain at a cascaded $2 \times 2$ OXADM with the increment of the number of OXADM used. Power penalties increase exponentially as the $Q$ factor increase. The increase in power penalty is greater for $Q>5$ which suggests that $Q$ factor more than $5(Q=6,7,8,9,10)$ are not suitable in OXADM as there will be more crosstalk induced. An increase in the number of OXADM used will lead to a higher power penalty too.

## Case 3

vary: $\varepsilon=3 \times 10^{-5}, 3.5 \times 10^{-5}, 4 \times 10^{-5}, 4.5 \times 10^{-5}$, and $5 \times 10^{-5}$ constant: $L=1, M N=2 \times 2$

From Figure 5, power penalties at cascaded $2 \times 2$ OXADM increase exponentially with $Q$ factor and it is higher at greater ratio of optical power from crosstalk contribution. The difference in the power penalties between consecutive optical power ratios at lower Q factor is not significant.

## Power penalty versus number of cascaded OXADM, L

## Case 1

vary: $\mathrm{MN}=4(2 \times 2), 6(2 \times 3$ or $3 \times 2), 8(2 \times 4$ or $4 \times 2), 10(5 \times 2$


Figure 3. Power penalties at one OXADM with the changing MN product ( $\mathrm{L}=1$ ).


Figure 4. Power penalties at cascaded $2 \times 2$ OXADM with the increment of the number of OXADM used.
or $2 \times 5), 12(2 \times 6,3 \times 4,4 \times 3$ or $6 \times 2)$
constant: $\varepsilon=4 \times 10^{-5}, Q=1$
Figure 6 depicted the power penalties at one OXADM with the changing MN product. The increment in the number of operating wavelength or the number of input/output ports will increase the power penalty linearly. At the same time, a greater number of the cascaded OXADM being used will induce more crosstalk and increase the power penalty.

## Case 2

vary: $Q=0,1,2,3,4$.
constant: $\varepsilon=4 \times 10^{-5}, \mathrm{MN}=2 \times 2$
From Figure 7, the power penalties at cascaded $2 \times 2$ OXADM increase linearly with the $Q$ factor except for $Q=1$, where it has a fairly constant low level of power penalty. The gap of power penalties increase significantly for larger $Q$ factor as well (at $L=10$ ).


Figure 5. Power penalties at cascaded $2 \times 2$ OXADM with the increment of ratio of optical power $(L=1)$.


Figure 6. Power penalties with the changing in $M N$ product $(Q=1)$.

## Case 3

vary: $\varepsilon=3 \times 10^{-5}, 3.5 \times 10^{-5}, 4 \times 10^{-5}, 4.5 \times 10^{-5}, 5 \times 10^{-5}$, constant: $\mathrm{Q}=1, \mathrm{MN}=2 \times 2$

From Figure 8, the power penalties at cascaded $2 \times 2$ OXADM increase linearly with the increment in the ratio of optical power for crosstalk contribution and the number of OXADM used. The gap of the power penalties between consecutive ratios of optical power at $L \leq 10$ is small.

Power penalty versus ratio of optical power, $\varepsilon$

## Case 1

vary: $\mathrm{MN}=4(2 \times 2), 6(2 \times 3$ or $3 \times 2), 8(2 \times 4$ or $4 \times 2)$, $10(5 \times 2$ or $2 \times 5)$, $12(2 \times 6,3 \times 4,4 \times 3$ or $6 \times 2)$
constant: $L=1, Q=1$
Figure 9 shows the graph of the power penalties versus ratio of optical power. At $L=1$ and $Q=1$, the power penalties increase


Figure 7. Power penalties at cascaded $2 \times 2$ OXADM with the increment in $Q$ factor.


Figure 8. Power penalties at cascaded $2 \times 2$ OXADM with the increment in the ratio of optical power, $\mathcal{E}(\mathrm{Q}=1)$.
linearly with the ratio of optical power for crosstalk contribution. The power penalty becomes greater when the MN product increase as well. Higher $\varepsilon$ indicates more induction of crosstalk to the
transmitting signal while higher MN product cause the signal that pass through the OXADM interfered by MN-1 crosstalk distribution which is leaked by the demultiplexer/multiplexer pairs.


Figure 9. Power penalties at one OXADM with changing MN product ( $L=1, Q=1$ ).


Figure 10. Power penalties at cascaded $2 \times 2$ OXADM with the increment in $Q$ factor ( $L=1$ ).

## Case 2

vary: $Q=0,1,2,3,4$.

Constant: $\mathrm{L}=1, \mathrm{MN}=2 \times 2$
From Figure 10, the power penalties increase linearly for every $Q$


Figure 11. Power penalties at cascaded $2 \times 2$ OXADM with the increment in the number of OXADM used $(Q=1)$.
factor except at $Q=1$ where the graph appears to be fairly constant at low level of power penalty.

## Case 3

vary: $L=1,2,3,4,5$
constant: $\mathrm{Q}=1, \mathrm{MN}=2 \times 2$
From Figure 11, power penalties at cascaded $2 \times 2$ OXADM increase linearly if the number of OXADM used increased as well. The higher power penalty at higher ratio of optical power for crosstalk contribution ( $\varepsilon>$ $5 \times 10^{-5}$ ) suggests that higher $\varepsilon$ will degrade the signal handling.

## RESULTS AND DISCUSSION

As the number of operating wavelength ( $M$ ) or the number of input/output port (N) increase, the MN product will increase accordingly. The signals that pass through more N demultiplexer/multiplexer and M optical switch will experience more crosstalk and it will become weaker. Hence, more power are required to account for this degradation and this will cause an increase in the power penalty as shown in Figures 3, 6 and 9.

From Figures 4 and 11, an increase in the number of cascaded OXADM used (L) will increase the power penalty as well. We can deduce that as the number of OXADM increase, the imperfection of each device will lead to the power leakage where this leaked power will
sum up to produce a larger power penalty required to maintain the output signal at specified BER. By referring to Figures 5 and 8, an increment in the ratio of optical power $(\varepsilon)$ for crosstalk contributions will increase the power penalties exponentially and linearly with respect to the $Q$ factor and $L$ respectively. As the ratio $\mathcal{E}$ increases, more optical power will contribute to the crosstalk in the signal lead to the increase in power penalty. From Figures 7 and 10, the power penalties are fairly constant at $Q=1$ and increase linearly with respect to other $Q$ factor. The higher the $Q$ factor, the greater will be the $B E R$. Therefore, more power is needed to counteract this effect and maintain the BER at acceptable level for desired signal quality.

## Conclusion

The power penalty of OXADM is depending on parameters such as the $M, N, L$ and $Q$. Therefore, emphasis needs to be given to these parameters in order to minimize the power penalty and form an optimized system as well as to avoid occurrence of infinity power penalty in cascaded OXADM system. The obtained results show that OXADM has higher power penalty as compared to other devices such as OXC. This is because OXADM is able to multiplex all the input signals into a single path and causes the maximum auto variance, max $\left(\sigma_{\text {RIN }}^{2}\right)=\varepsilon \cdot(\mathrm{MN}-1)$ which is larger than maximum auto
variance of OXC, max $\left(\sigma_{\text {RIN }}^{2}\right)=\varepsilon \cdot(\mathrm{M}+\mathrm{N}-2)$ (Shen et al. 1995). Therefore, trade off exists between a device capability and power penalty.

Notation: E, Signal field amplitude; $\mathbf{b}_{\mathbf{s} \text {, }}$ Binary data sequences with values of 0 or 1 in bit period T of main signal; $\mathbf{w}_{\mathbf{s}}=$ Center frequency of main signal; $\vec{P}_{s}=$ Unit magnitude polarization vector of main signal; $\mathbf{b}_{\mathbf{1}}=$ Binary data sequences with values of 0 or 1 in bit period T of th crosstalk contribution; $\mathbf{w}_{\mathbf{1}}=$ Center frequency of th crosstalk contribution; $\vec{P}_{l}=$ Unit magnitude polarization vector of th crosstalk contribution; $\boldsymbol{\Phi}=$ Phase noise of the lasers; $\boldsymbol{\varepsilon}=$ Ratio of optical power from each contribution of cross talk to signal; $\mathbf{M}=$ Number of wavelength for each fiber; $\mathbf{M}_{\mathbf{i}}=$ Number of wavelength for each fiber for specified i-OXADM; $\mathbf{N}=$ Number of input/output terminal for OXADM; $\mathbf{N}_{\mathbf{i}}=$ Number of input/output terminal for i-OXADM; L = Number of OXADM that the signal passing through; $\mathbf{Q}=\mathbf{Q}$ factor for specified BER; $\varepsilon=$ Ratio of optical power from each contribution of crosstalk to signal.

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