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Full Length Research Paper

Modification of the dynamic scale of marks in analytic hierarchy process (ahp) and analytic network approach (anp) through application of fuzzy approach

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There are two mainstreams in the use of the analytic network process (ANP) and analytic hierarchy process (AHP). One is the standard applications of crisp distributive and ideal mode versions. The other is characterised by fuzzification of the AHP/ANP methodology and by attempts to tackle better inherently uncertain and imprecise decision processes with quantitative and qualitative data. This paper presents modification of the AHP/ANP method, in which fuzzy numbers have been used for determining weight values of criteria and alternatives.Unlike the papers describing the procedure of fuzzification of the AHP/ANP method, the method described here takes into account the level of uncertainty of the decision maker. After application of the AHP/ANP method in this way, the values of the functions criteria for each considered alternative are obtained. Certain values of the level of certainty are corresponding to the obtained values of the functions criteria. It is possible to generate various sets of the values of criterion functions. Since large number of experts often participate in decision making, the model deals with possibility of synthesis of the optimality of criterion values in case of group decision making. The proposed methodology has been used for the assessment of management plans in West Serbia.

Key words: Fuzzy logic, fuzzy multicriteria decision making, fuzzy ANP, fuzzy AHP.

INTRODUCTION

The basic problem in decision making is how to determine weights, or priorities of the considered alternatives. The importance of the alternative is usually estimated in relation to several criteria, and the activities concerning setting of the weights of the alternatives are the central task of all multi-criteria methods and techniques. Analytic Hierarchy Process (AHP) falls into the category of multi-criteria methods, defining, in its essence, the priorities of all the elements within the given hierarchy. The concept of the method is to determine simultaneously in rational and intuitive ways the best one among several alternatives within the consistent procedure of evaluation of all alternatives in relation to the given set of criteria, sub-criteria etc. While the process is in progress, the decision maker does only the basic comparison in pairs, and AHP integrates them up to the level of determining of the weights and ranking of all the alternatives.

AHP allows inconsistency in evaluation and enables correction of consistency during the very process. It is especially suitable for practical decision making when the criteria and decisions are complex, when there are many criteria and dilemmas, risks, and conflict of interests as well as, quantitative and qualitative data on the subject of the decision making. Establishing of the decision making ambience in AHP way means that the decision maker can successfully structure the problem and analyse its elements in order to find the best possible solution.

AHP models are decision making problem using a

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framework that assumes a unidirectional hierarchical relationship among decision levels. The top element of the hierarchy is the goal for the decision model, the subsequent level model, criteria, sub-criteria and alternatives. The hierarchy is basically a system where one group of entities influences another set of entities in another level of the hierarchy. The hierarchy decomposes from the general to the more specific attributes until a manageable level of decision criteria is reached.

AHP allows a set of complex issues to be compared with the importance of each issue relative to its impact on the solution to the problem. Since the introduction of AHP, numerous applications have been published in the literature (Zahedi, 1986; Shim, 1989; Kleindorfer and Partovi, 1990; Corner and Corner, 1991, 1995; Ghodsypour and O_Brien, 1998). Analytic Network Process (ANP) is a more general form of AHP, incorporating feedback and interdependent relationships among decision attributes and alternatives (Saaty, 1996). This provides a more accurate approach for modelling complex decision environment (Meade and Sarkis, 1999; Lee and Kim, 2000; Agarwal and Shankar, 2002b, 2003; Yurdakul, 2003).

Many decision problems cannot be structured hierarchically because they involve the interaction and dependence of higher level elements on lower level elements. Structuring a problem involving functional dependence provides feedback among clusters. This is a network system. Saaty accomplished a comprehensive study of this problem. He suggested the analytic network process used to solve the problem of dependence among alternatives or criteria.

Analytic Hierarchy Process (AHP) is similar to ANP, but cannot capture interdependencies (Meade et al., 1997; Meade and Sarkis, 1999). Hierarchical representation is an important component of ANP, however, strict hierarchical structure is not recommended, as it is the case of AHP. The ANP technique allows for more complex relationships among the decision levels and attributes. The ANP consists of coupling of two phases. The first phase consists of a control hierarchy of network of criteria and sub-criteria that control interactions. The second phase is a network of influences among elements and clusters. The network varies from criteria to criteria and thus different super-matrices of limiting influence are computed for each control criteria. Finally, each one of these super-matrices is weighted by the priority of its control criteria and results are synthesized through addition for the entire control criterion (Saaty, 1996).

ANP is a relatively new methodology that is still not well-known to the decision-making community (Meade and Preseley, 2002; Sarkis and Sanadarraj, 2002; Shang et al., 2004). Its application has been limited to academic settings (Garuti and Escudey, 2005) and large scope projects (Shang et al., 2004). Furthermore, there are some concerns whether ANP is too complex to be used outside the academic decision-making community and by average practitioners.

In assessing the various interests of participating groups in a decision process, it is necessary to clearly identify the overall goal and the hierarchically structured sets of criteria and sub-criteria that should be used in evaluating scenarios as the decision alternatives. The problem that arises is that traditional multi-criteria methods (such as various mathematical programming methods) are not robust when dealing with limited experimental data, human judgments and the various metrics of decision variables. The main difficulties appear when quantitative measures should be combined with linguistic expressions and the decision makers attitudes toward risk need to be modelled appropriately.

FUZZY AHP / ANP IN LITERATURE

The original crisp AHP method (Saaty, 1980), and its extension fuzzy AHP (Laarhoven and Pedrycz, 1983; Boender et al., 1989), proved to be an efficient tool. The core of both versions of the method is the hierarchical structuring of the decision problem, followed by the systematic process of the synthesis of various judgments in order to derive priorities amongst criteria and subsequently the performance of alternatives. AHP uses pairwise comparisons of criteria and alternatives to form a reciprocal decision matrix, thus transforming qualitative data to crisp or fuzzy ratios. The eigenvector method is used to solve the reciprocal matrix and to determine the importance of criteria and the performance of alternatives with respect to criteria. The additive weighting method is used to calculate the utility of each alternative across criteria. In the case of fuzzy AHP and ANP, defuzzification is necessary at the end to obtain crisp weights and finally rank the alternatives.

The ANP and its variations have become a landmark in modern decision making due to several factors: (a) its ability to handle uncertain, imprecise and subjective data; (b) its robustness when solving practical ranking problems; (c) its methodological clearness and mathematical simplicity; and (d) its transparency to fuzzy logic and fuzzy sets theory (Tong and Bonissone, 1984; Zimmermann, 1987; Chen and Hwang, 1992; Deng, 1999). To solve multi-criteria problems involving qualitative data, Laarhoven and Pedrycz (1983) and Buckley (1985) extended Saaty's crisp AHP to deal with the decision maker's subjectivity in judgments by imbedding it into a fuzzy environment. A fuzzy version of the method was based on the use of triangular fuzzy numbers in pairwise comparisons in order to compute criteria weights and the overall utilities of alternatives, known as fuzzy utilities. In order to arrive at the final stage where alternatives are prioritised, the fuzzy utilities are required to be defuzzificated and ranked.

Using the fuzzy Delphi, ANP and zero-one goal programming methods, Chang et al. (2008) deve-loped

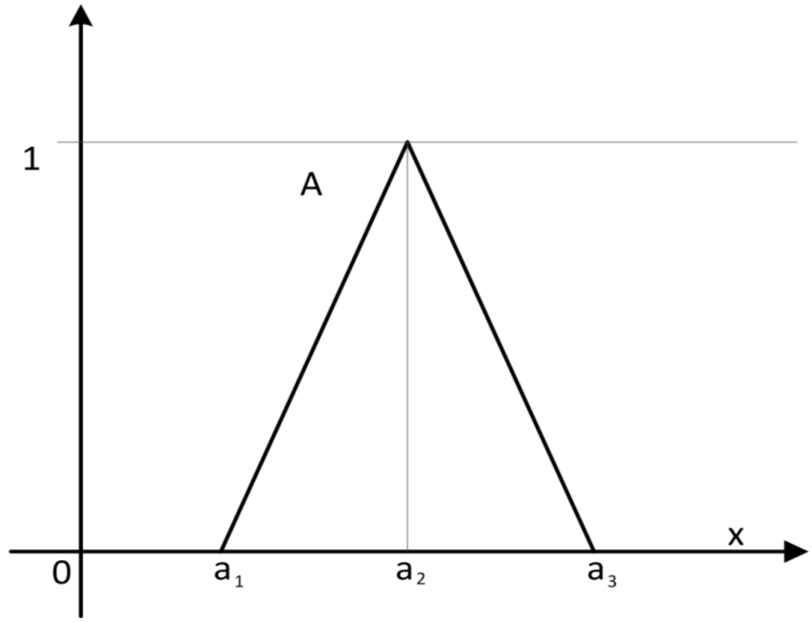

Figure 1. Triangular fuzzy number.

the model for the strategic project selection for the Alishan forest railway in Taiwan. Dagdeviren and Yueksel (2007) took into consideration the interdependencies of the factors relevant for the personal selection, applying the ANP method. Gencer and Guerpinar (2007) proposed using the ANP in supplier selection. Jharkharia and Shankar (2007) presented the use of the ANP in the process of selection of the logistics service provider. Cheng and Li (2005) applied the ANP method for the selection of the projects. Meade and Presley (2002) proposed the ANP for the project selection in a research and development environment. ANP is improved by AHP model, because it can consider relations and feedback among elements at the higher level and at the lower level in the hierarchy structure of a system. Bojovic et al. (2010) applied the ANP method for the selection among alternatives of organizational structure. Chun-Chu (2011) proposed two-dimensional decision model that integrates fuzzy data envelopment analysis and AHP to perform this essential task.

Some other fuzzy methods for prioritization in ANP which are worth mentioning, for example, those based on poly optimisation as proposed by Wagenknecht and Hartmann (1983); fuzzy least squares by Xu (2000); or pseudo-inverse generalisation by Kwiesielewicz (1998). Although, the latter three methods have gained a certain level of attention and are considered to be theoretically better, the fuzzy extent analysis method, as proposed in (Laarhoven and Pedrycz, 1983) and (Buckley, 1985), is more widely accepted in practice. This is probably due to its transparency and simplicity in handling uncertainties imbedded into decision making which includes quantitative, qualitative and 'grey' decision variables.

As highlighted by Deng (1999), the application of fuzzy AHP may produce unreliable results if: (a) an unbalanced 9-point scale is used; (b) the scale of fuzzification is not fully justified; and (c) an inappropriate defuzzification method is applied. Consequently, any application of fuzzy ANP requires considerable computations, careful handling of fuzzy operations and consistent interpretation of any results obtained (Wasil and Golden, 2003).

Fuzzy sets, norms and extensions

Fuzzy sets theory (Zadeh, 1965) defines fuzzy set A as a set of ordered pairs:

$$A=\left\{\left(x,\mu_A\left(x\right)\right)\middle|\, x\in X, 0\le\mu_A\left(x\right)\le 1\right\} \quad (1)$$

Where $\mu_A\left(x\right)$ is a membership function which shows to what extent $x\in X$ meets the criteria for membership in a set A. For the membership function $0\le\mu_A\left(x\right)\le 1$, for every $x\in A$, that is, $\mu_A : X\rightarrow\left[0,1\right]$.

According to the fuzzy theory the choice of membership functions that is, the form of the function and confidence intervals width are usually made based on subjective estimates or experience. Trapezoidal and triangular fuzzy numbers are most commonly used. Triangular fuzzy numbers with membership functions shown in Figure 1 are used in this paper. Triangular fuzzy numbers are usually given in the form $A=(a_1,a_2,a_3)$, where a_2 is the value with the membership function of the fuzzy number being 1.0, a_1 is the left distribution of the confidence interval, and a_3 the right distribution of the confidence interval of the fuzzy number A. Fuzzy number A membership function is defined as:

$$\mu_A(x)=\begin{cases} 0, & x<a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2\le x\le a_3 \\ 0, & x>a_3 \end{cases} \quad (2)$$

METHODS OF DEFUZZIFICATION

For defuzzification and mapping of the fuzzy number $A=(a_1,a_2,a_3)$ value into a real numbers, numerous methods are used. Two methods have been used in this paper namely (Seiford, 1996):

1. The centre of gravity: $defuzzy\ \mathrm{A}=\left[(a_3-a_1)+(a_2-a_1)\right]\cdot 3^{-1}+a_1$
2. The total integral value: $defuzzy\ \mathrm{A}=\left[\lambda a_3+a_2+(1-\lambda)a_1\right]\cdot 2^{-1}$ (with $\lambda,\ \lambda\in[0,1]$ being an optimism index).

The FDM approach

The fuzzy decision making (FDM) approach is proposed here. It is inspired by crisp AHP/ANP and it comprises several principles and procedures that pave the way for consequent analysis and the solving of a network structured decision problem. FDM involves qualitative assessments in fuzzy framework and it is based on the following premises:

1. Crisp AHP/ANP is fuzzified preserving its crisp logic and method of manipulating the priority vectors.
2. The full range of Saaty's evaluation scale is fuzzified, not only odd positive integer entries; triangular fuzzy numbers are also used.
3. An aggregation principle is implemented when manipulating criteria that split into sub-criteria. The criteria and sub-criteria levels aggregate into a unique level.
4. Fuzzy extent analysis is applied in all instances.
5. The total integral value method is used for defuzzification and the final ranking of alternatives.

Fuzzy ANP/AHP approach

Saaty (1996) suggested the use of AHP to solve the problem of independence on alternatives or criteria, and the use of ANP to solve the problem of dependence among alternatives.

The ANP, also introduced by Saaty, is a generalization of the AHP (Saaty, 1996). Whereas AHP represents a framework with a unidirectional hierarchical relationship, ANP is designed for the subjective evaluation of a set of alternatives based on multiple criteria organized in a hierarchical structure (Figure 2). While the AHP method is a decision-making framework using a unidirectional hierarchical relationship among decision levels, ANP allows for more complex interrelationships among the decision levels and attributes. In AHP, the top element of the hierarchy is typically the overall goal for the decision model. The hierarchy decomposes the general to more specific attributes until a level of manageable decision criteria is achieved. The ANP does not require this strict hierarchical structure; it allows factors to `control' and be `controlled' by the varying levels or `clusters' of attributes. Some controlling factors are also present at the same level.

This interdependency among factors and their levels is defined as a "system with feedback" approach. The AHP does not contain feedback loops among the factors that can adjust weightings and lessen the possibility of the reverse ranking phenomenon. The relative importance or the strength of the impacts on a given element is measured on a ratio scale similar to AHP. ANP allows for complex interrelationships among decision levels and attributes. The ANP feedback approach replaces hierarchies with networks in which the relationships between levels are not easily represented as higher or lower, dominated or being dominated, directly or indirectly (Meade and Sarkis, 1999). For instance, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives may have an impact on the importance of the criteria. Therefore, a hierarchical structure with a linear top-to-down form is not applicable for a complex system.

Since fuzzification of AHP/ANP method is primarily based on fuzzification of the grading scale, the remaining part of this paper will deal with different approaches to modification of Saaty's scale. After that, new approach to optimization of the dynamic grading scale realized by fuzzy approach will be explained.

Fuzzifying judgment scale

When making decision, qualitative characteristics of the element of importance for the decision are the most often evaluated facts. Spontaneous statements of qualitative evaluation, such as "very important", are verbally expressed in AHP/ANP in such a way that a language

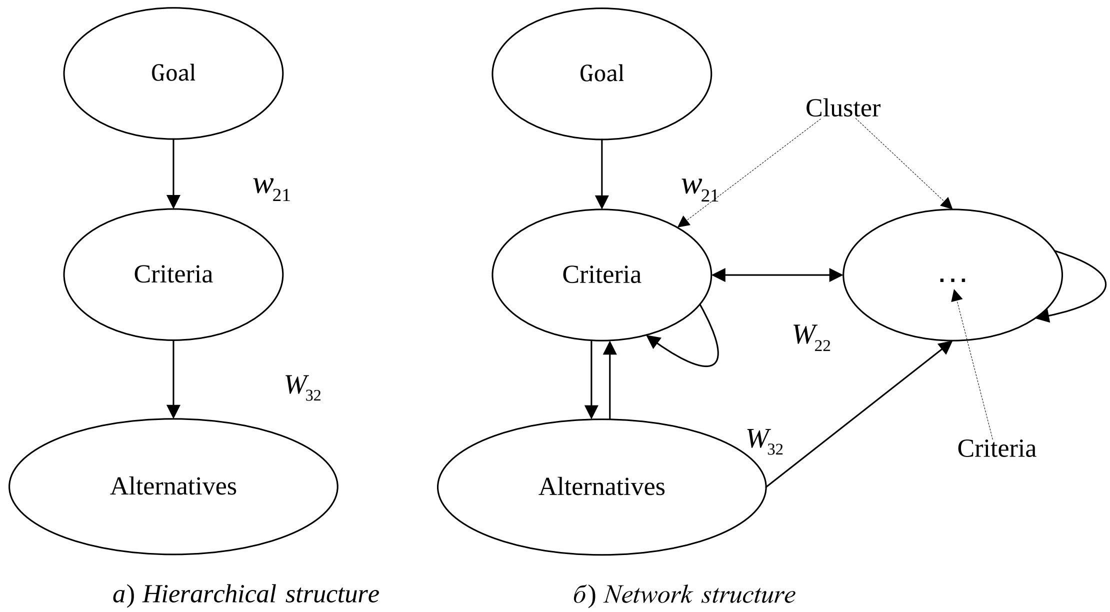

Figure 2. Structural difference between hierarchy and network.

determinant is added to the mark intensity. An appropriate scale of numeric values which transforms the language expressed marks into figures is connected to the determinants, while the associative procedure should lead towards logical outcome and results which can be confidently compared to already known measurements and marks. In other words, association maps the language expressed marks into appropriate discrete sets of numerical values. It is possible to take out the information which can be measured or compared with the information from the analogue set. In the process of comparing of the element pairs at given hierarchic level, the scale of relative priorities is derived for all the elements sharing the same characteristic and the minor changes in the marks value (language or numerical) result in small changes in the derived priorities.

One of the basic questions in multicriteria optimization is how to reliably evaluate the data which significantly influence the solution. The problem increases if it is necessary to get qualitative information from the decision maker, since it is difficult to express qualitative data as absolute, numerical values. There are two approaches to the development of the scale evaluation: (1) linear (Saaty, 1980) and (2) exponential (Lootsma, 1988, 1990; Lootsma et al., 1990). Both approaches are based on certain theories from the field of psychology and in this case, the first approach is of interest since it is convincingly dominant in application.

It is considered that people generally are not able to make their choices if the set of possibilities is endless; for example, it is difficult for them to notice the difference between values such as 5.00 and 5.09. Since psychological experiments have shown that an individual cannot compare more than seven objects (plus or minus 2) (Miller, 1956), Saaty defined the scale with its highest value 9, the lowest value 1 and the increment difference 1. Saaty's scale is generally considered to be standard for ANP. According to this scale, which will due to identification be called Scale 1, the available values for comparison in pairs are the elements of the discrete set of 17 values:

Scale 1 = Saaty's = {9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9}

Values from the Scale 1 can be grouped into two intervals: (1, 9) and (1/9, 1). As shown, the values from the interval (1, 9) are uniformly distributed, while the values from the interval (1/9, 1) are grouped on the right side of the interval. There is not a reason good enough for the values in the defined scale from the interval (1/9, 1) to be fairly distributed and to have other values as their reciprocal values. Ma and Zheng (1991) suggested usage of this kind of scale (Scale 2):

Scale 2 = Ma and Zheng's scale = {9, 9/2, 9/3, 9/4, 9/5, 9/6, 9/7, 9/8,1, 8/9, 7/9, 6/9, 5/9, 4/9, 3/9, 2/9, 1/9}.

In the interval (1/9, 1) the distance between successive values is (1–1/9)/8=1/9, so that the values are fairly distributed. The values in the interval (1, 9) are reciprocal values from the interval (1/9, 1).

Other scales can be defined in a similar way as for scale 2, for example by weighting the values from the previous scales. For the interval [1/9, 1] the values can be calculated using the formula:

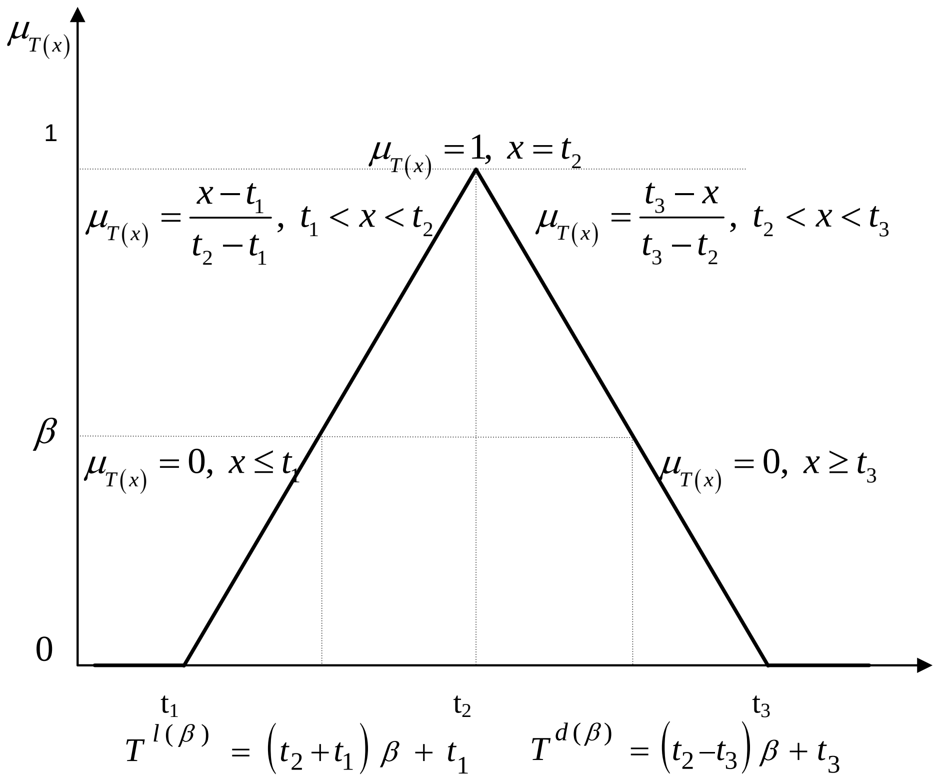

Figure 3. Defining of the left and right distribution of the confidence interval of fuzzy number T .

NV = V (Scale1) + [V (Scale 2) – V (Scale 1)]*(α/100)

In which symbols NV and V stand for the new value and the value and the parameter α can vary from 0 to 100. The values in the interval (1, 9) are reciprocal values for the values calculated by means of the aforementioned formula. Scale 1 is obtained for α = 0, and Scale 2 for α = 100. Recent analysis has shown that there is neither an all-purpose scale which is best for all the cases nor the worst one (Triantaphyllou et al., 1998). Saaty's Scale 1 is predominantly applied.

The differences among fuzzy versions of AHP/ANP are mostly those mirroring the way of the scale fuzzification and the method of the result defuzzification. The methods of defuzzification that are most often applied are centroid method, various kinds of geometrical comparison of triangle fuzzy numbers, or the methods of integration combined with so-called α – scaling and usage of λ – optimism index of the decision maker. Fuzzification of the analytic network process method is described in many works (for instance Triantaphyllou and Lin, 1996; Raju and Pillai, 1999; Arslan and Khisty, 2006).

What is common for all the aforementioned approaches is "sharp" fuzzification of linguistic expressions in Saaty's scale which are represented by triangle fuzzy numbers. "Sharp" fuzzification represents the case when for certain fuzzy number $T = (t_1, t_2, t_3)$, certain interval of confidence is determined in advance, that is, it is defined in advance that value of fuzzy number will not be bigger than t_3 nor smaller than t_1 (Figure 3). In other words, we are sure that value of the linguistic expressions belongs to the closed interval $[t_1, t_3]$.

Fuzzification of the basic AHP/ANP method has been done in such a way that triangle fuzzy numbers have been used for determining of the essential criteria values and fuzzy arithmetic has been used for the whole procedure. Fuzzy numbers are intuitively easy to use when expressing the decision maker's qualitative assessments.

This way of defining the confidence interval does not take into account the level of uncertainty used for evaluation of linguistic expressions. The level of uncertainty is represented by the length of the fuzzy number base. In other words, the greater the uncertainty in assessment of the linguistic expression, the bigger the length of the base (certainty interval) of the fuzzy number. Unlike the aforementioned works, the model represented in this work takes into account the level of uncertainty which is marked with parameter β . In this case, the greatest possible uncertainty is described by the

Table 1. Fuzzified Saaty's scale.

Importance intensity	Definition	Fuzzified values	
		Fuzzy number	**Inversive fuzzy number**
1	Same importance	(1, 1, 1) compared with oneself, $(\beta, 1, 2-\beta)$ in other cases	$(1/\beta, 1, 1/(2-\beta))$ in other cases
3	Weak dominance	$(3\beta, 3, (2-\beta)3)$	$(1/(2-\beta)3, 1/x, 1/3\beta)$
5	Strong dominance	$(5\beta, 5, (2-\beta)5)$	$(1/(2-\beta)5, 1/5, 1/5\beta)$
7	Very strong dominance	$(7\beta, 7, (2-\beta)7)$	$(1/(2-\beta)7, 1/7, 1/7\beta)$
9	Absolute dominance	$(9\beta, 9, (2-\beta)9)$	$(1/(2-\beta)9, 1/9, 1/9\beta)$
$x = 2,\ 4,\ 6,\ 8$	Inter-values	$(x\beta, x, (2-\beta)x)$	$(1/(2-\beta)x, 1/x, 1/\beta x)$

value $\beta = 0$, while the value $\beta = 1$ corresponds to the situation in which we are totally sure that the linguistic expression corresponds to given comparisons of the optimality criteria. Value of the parameter β can be any number which is within the interval $[0,1]$. In this way, upper and lower limits of the confidence interval of the fuzzy number are chosen randomly for the given value of the parameter β, so that they are within the limits defined by the expression:

$$T = (t_1, t_2, t_3) = \left\{ \begin{array}{lll} t_1 = \beta t_2, & t_1 \le t_2, & t_1, t_2 \in [1/9, 9] \\ t_2 = t_2, & & t_2 \in [1/9, 9] \\ t_3 = (2-\beta) t_2, & t_3 \le t_2, & t_2, t_3 \in [1/9, 9] \end{array} \right\} \quad (3)$$

Applying of the described procedure means realization of Saaty's scale fuzzification (Table 1).

Where fuzzy number $T = (t_1, t_2, t_3) = (x\beta, x, (2-\beta)x)$, $x \in [1,9]$ is:

$$t_1 = x\beta = \begin{cases} x\beta, & \forall\ 1 \le x\beta \le x \\ 1, & \forall\ x\beta < 1 \end{cases} \quad (4)$$

$$t_2 = x,\ \forall\ x \in [1,9] \quad (5)$$

$$t_3 = (2-\beta)x = \begin{cases} (2-\beta)x, & \forall\ x \le (2-\beta)x \le 9 \\ 9, & \forall\ (2-\beta)x > 9 \end{cases} \quad (6)$$

Where inversive fuzzy number

$$T^{-1} = (1/t_1, 1/t_2, 1/t_3) = (1/(2-\beta)x, 1/x, 1/\beta x),$$

$x \in [1/9, 1)$ is:

$$1/t_1 = 1/(2-\beta)x = \begin{cases} 1/(2-\beta)x, & \forall\ x < 1/(2-\beta)x < 1 \\ 1, & \forall\ 1/(2-\beta)x > 1 \end{cases} \quad (7)$$

$$1/t_2 = 1/x,\ \forall\ 1/x \in [1/9, 1] \quad (8)$$

$$1/t_3 = 1/\beta x = \begin{cases} 1/\beta x, & \forall\ 1/9 \le 1/\beta x \le 1/x \\ 1/9, & \forall\ 1/\beta x < 1/9 \end{cases} \quad (9)$$

Figure 4 shows fuzzificated Saaty's scale with the decision maker's certainty degree $\beta = 0.5$. It means that the shape of the linguistic scale differs depending on the decision maker's certainty degree.

In this way, the values of criteria functions for every considered alternative are obtained after application of AHP/ANP method. Certain value of parameter β corresponds to obtained values of criteria functions. It is possible to generate various sets of values of the criteria functions for various values of parameter β. This work will deal with four sets of values of criteria functions for four different values of parameter β as follows: $\beta = 0.3$, $\beta = 0.5$, $\beta = 0.7$ и $\beta = 1$.

In fuzzy versions of AHP/ANP, everything is carried out in the same way as the standard version, but the whole procedure is fuzzificated, starting from Saaty's scale and assessment in pairs up to all matrix operations. Numerous authors stress in their scientific works that decision making based on only personal thinking or intuition is almost impossible. In modern times and whenever possible, decisions are made within groups.

Since the described model is applied even when group

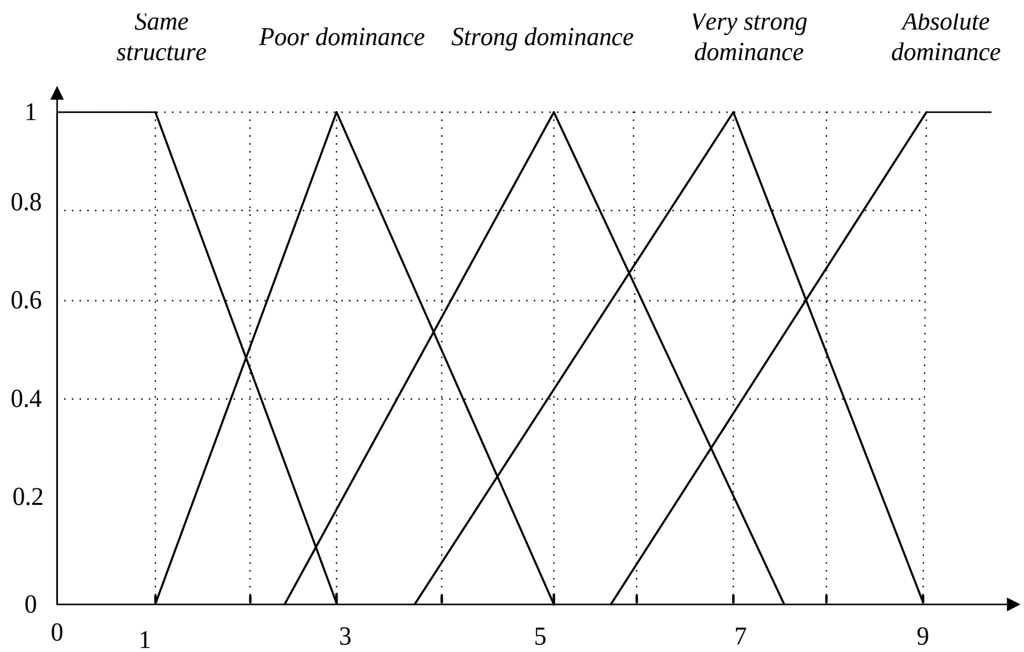

Figure 4. Graphic picture of fuzzificated Saaty's scale with the certainty degree of the decision maker $\beta = 0.5$.

decision making is concerned, it is necessary to define way of application of fuzzy AHP/ANP method in group decision making.

Application of fuzzy AHP/ANP method in group decision making

Unlike some other methods, ANP and AHP are used not only for individual decision making but it is more and more used for group decision making. The way it is applied in this area mostly depends on the attitude of the group members because some of them can act as isolated individuals or they can be prepared for cooperation and consensus having in mind the common goal. Group decision making by means of fuzzy AHP/ANP means the following:

1. Number of the group members is $K \geq 2$.
2. The group members give their opinion on their preferences of individual elements of hierarchy according to ANP rules and using fuzzificated Saaty's scale and
3. It is not obligatory for the group members to talk in public about their preferences.

The first characteristic separates group from individual application of fuzzy AHP/ANP method. Synthesis of the mark or priority is done for group application and it does not exist in individual decision making.

The second characteristic means that all hierarchy comparisons are done in pairs during application of fuzzy AHP/ANP. While comparing the elements E_i and E_j at a given hierarchy level in relation to a given element at a higher level, the decision maker semantically expresses intensity of his preference of the first element in relation to the second one. The verbal mark is automatically accompanied by value a_{ij} from linear part of Saaty's scale of fuzzy numbers. It means that fuzzificated Saaty's scale will be used in other considerations, since the shown group synthesis are related to it and make it different from the synthesis at other scales.

The third characteristic separates the cases of group decision making when all group members give complete information from the cases when a part of the information does not exist or is not available.

Group synthesis with complete information

Supposing that all the members $(k = 1, 2, ..., K)$ of group G for given hierarchy have done all necessary assessments of the elements in pairs, there are two ways of determining priority alternatives in relation to the goal in fuzzy AHP/ANP methodology:

1. The first way is to apply fuzzy AHP/ANP for every decision maker separately and to aggregate the obtained priority vectors. This procedure is known as procedure of

the priorities aggregation (Aggregating Individual Priorities - AIP) and it has character of additional synthesis of individual decisions and
2. The second way is to immediately aggregate the individual marks of preferences at all hierarchy levels and then to perform synthesis for a fictive decision maker in the same way as with the individual fuzzy AHP/ANP application. (Aggregating Individual Judgments - AIJ).

Supposing that all the members $(k=1,2,...,K)$ of group G are considered to be equal decision makers and that all decision makers for the same hierarchy have performed all assessments in pairs, the situation is treated as group decision making with complete information. In that case, synthesis of the priorities of alternatives can be performed in two ways; the first way is to perform AHP/ANP synthesis for every individual separately and to aggregate the obtained vectors of priorities of alternatives. There are two characteristic aggregations:

1. Weighted Arithmetic Mean Method - WAMM. The alternative A_i is given as well as its weight value (priority) $z_i(k)$ for the person k. If all G group members are given appropriate weight values α_k, weight mean is:

$$z_i^G = \sum_{k=1}^{K} \alpha_k z_i(k) \tag{10}$$

In which $\tilde{z}_i^G$ is final (composite) priority of alternative A_i. Single weights α_k of the group members were previously additively normalized.
2. Geometric Mean Method - GMM. Aggregation is performed as follows:

$$\tilde{z}_i^G = \prod_{k=1}^{K} \left[z_i(k) \right]^{\alpha_k} \tag{11}$$

In which α_k weights are also additively normalized.

It is necessary for both the methods to perform final normalization of priorities of all alternatives. The second way is to aggregate information at the local level, to get a synthetic set of matrix for a fictive decision maker and then to perform AHP/ANP synthesis as described in the previous part of the work. Aggregation consists of many micro aggregations. Let us suppose that all the group members in pairs have compared elements at the given hierarchical level in relation to the same element on the higher level. The formed matrixes: $A(1)=\{a_{ij}(1)\},...,A(K)=\{a_{ij}(K)\}$ are aggregated into unique matrix for the group $A^G=\{a_{ij}^G\}$ in such a way that geometrical mean processing is done at each position (i,j):

$$a_{ij}^G = \left[\prod_{k=1}^{K} a_{ij}(k) \right]^{1/K} \tag{12}$$

Further synthesis is performed in the same way as with the individual fuzzy AHP/ANP application.

Group synthesis with incomplete information

When some decision makers have not performed all necessary assessments, there are some symmetric gaps in relation to the main diagonal and then we have a case of group decision making by means of fuzzy AHP/ANP with incomplete information. In that case, the most applicable methods are WAMM and GMM, because there are not composite vectors of priorities of alternatives for some group members. The solution is to use the available information to perform micro aggregation of assessment in pairs for every element of every matrix in hierarchy and at the position (i,j) in given matrix A geometric mean of assessment in pairs is performed only for the group members who have stated their preferences of element E_i in relation to element E_j in previously described way.

It is necessary for at least one decision maker to state his attitude about the given element (or its reciprocal element a_{ij}). If not, the procedure is not applicable and there is possibility for the missing values to be assessed by means of consensus or voting of all the group members. Unlike in the case of complete information, micro aggregation is performed by means of the expression:

$$a_{ij}^G = \left[\prod_{l \in L} a_{ij}(l) \right]^{1/M} \tag{13}$$

In which L stands for set of the group members who have assessed a few elements (E_i, E_j), and M stands for number of the members.

APPLICATION OF THE PROPOSED METHODOLOGY: AN ILLUSTRATIVE PROBLEM

A three-level hierarchy was created to test the proposed

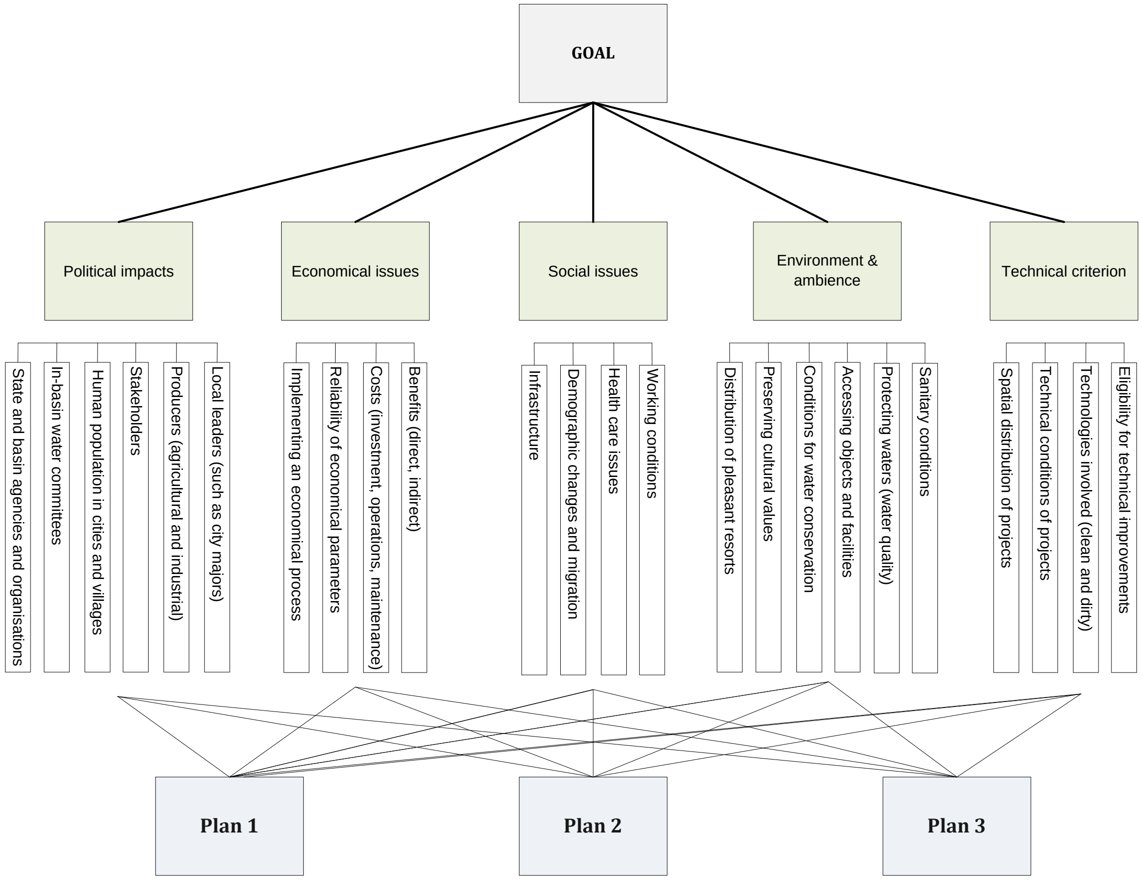

Figure 5. Hierarchy of the decision problem.

fuzzy decision making (FDM) approach and to verify its applicability in further developments. The overall goal has been stated as selecting the best long-term water management scenario in West Serbia. Three management scenarios were used as decision alternatives to be evaluated by means of 5 criteria which were split into a total of 24 sub-criteria. The decision hierarchy is defined as follows, Figure 5:

Plan 1: Demands related to human supply and animal supply should be fully satisfied in the future at present level needs. The remaining waters should be used giving priority to irrigation according to future needs. In the case of any surplus waters, ecological demands should be satisfied.
Plan 2: Priority should be given to attending to the demands of both human and animal supply, followed by irrigation demands, all according to future needs. Once more, in case of available water surplus, ecological demands should be satisfied.
Plan 3: This alternative considers fulfilling the necessities of human and animal supplies as the major priority; firstly according to future necessity values and then followed by ecological demands. Only in the case of available water surplus, should irrigation demands be satisfied.

To determine the relative importance of the evaluation criteria C_1–C_5, they were pairwise compared with respect to the goal by using the fuzzified scale given in Table 1. Linguistically expressed preferences among criteria have been used to create a judgment matrix A. The ranking procedure starts with the determination of the importance of criteria with respect to the goal. By using a fuzzified scale, a fuzzy reciprocal judgment matrix for criteria is determined as:

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1M} \\ a_{21} & a_{22} & \dots & a_{2M} \\ \dots & \dots & \dots & \dots \\ a_{M1} & a_{M2} & \dots & a_{MM} \end{bmatrix} \quad (14)$$

Where $a_{ij}=1$ for all $i=j\ (i,j=1,2,...,M)$, and $a_{ij}=1/a_{ji}$.

$$A = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & \tilde{1} & \tilde{3}^{-1} & 2 & \tilde{3} & 4 \\ C_2 & \tilde{3} & \tilde{1} & 2 & 3 & \tilde{5} \\ C_3 & 2^{-1} & \tilde{3}^{-1} & \tilde{1} & \tilde{3} & \tilde{3} \\ C_4 & \tilde{3}^{-1} & \tilde{3}^{-1} & \tilde{1} & \tilde{1} & 4 \\ C_5 & 4^{-1} & \tilde{5}^{-1} & \tilde{3}^{-1} & 4^{-1} & \tilde{1} \end{matrix} \tag{15}$$

Note that all entries in the matrix are fuzzy numbers from Table 1, each element on the main diagonal is a fuzzy number $\tilde{1}$, and that entries in the upper and lower matrix triangles are reciprocals.

The weighting vector w of criteria matrix A was determined by using the following expression:

$$w_i = \sum_{j=1}^{M} a_{ij} \otimes \left[\sum_{k=1}^{M} \sum_{l=1}^{M} a_{kl} \right]^{-1}, \; i = 1, \ldots, M. \tag{16}$$

All $w_i, \; i = 1, \ldots, M,$ are normalised fuzzy numbers with medium values equalling 1. It should be noted that fuzzy extent could be defined as the result of fuzzy arithmetic, or by using the extension principle. The second is slightly more difficult, but would lead to reduced uncertainty. Each entry of this vector is the sum of elements in the related row of matrix A, divided by the sum of all its elements. For example for level of certainty $\beta = 0.5$:

$$w_1 = \frac{\tilde{1} + \tilde{3}^{-1} + 2 + \tilde{3} + 4}{\tilde{1} + \tilde{3}^{-1} + 2 + \tilde{3} + 4 + \tilde{3} + \tilde{1} + 2 + \tilde{3} + \tilde{5} + 2^{-1} + \tilde{3}^{-1} + \tilde{1} + \tilde{1} + \tilde{3} + \tilde{3}^{-1} + \tilde{3}^{-1} + \tilde{1} + \tilde{1} + 4 + 4^{-1} + \tilde{5}^{-1} + \tilde{3}^{-1} + 4^{-1} + \tilde{1}}$$
$$= (0.090, 0.265, 0.728)$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \begin{bmatrix} (0.090, 0.265, 0.728) \\ (0.101, 0.359, 0.985) \\ (0.053, 0.154, 0.557) \\ (0.069, 0.171, 0.471) \\ (0.025, 0.052, 0.214) \end{bmatrix}$$

Subsequently, through the use of fuzzy pairwise comparisons, the judgment matrices (Equation 14) for sub-criteria related to respective criteria were obtained. Related sub-criteria weighting vectors were calculated as defined by Equation 16 and using the following expression:

$$S_i = \sum_{j=1}^{m} \mu_i^j \left[\sum_{k=1}^{n} \sum_{l=1}^{m} \mu_k^l \right]^{-1} \tag{17}$$

Where S_i is the value of fuzzy synthetic extent with respect to the ith object. All $\mu_i^j \left(i = 1, \ldots, n; \; j = 1, \ldots, m \right)$ are triangular fuzzy numbers representing the performance of the object x_i with regard to each goal u_j:

$$A_A = \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ A_1 & \tilde{1} & \tilde{3} & 4 & \tilde{3}^{-1} & 2^{-1} & 2 \\ A_2 & \tilde{3}^{-1} & \tilde{1} & 2 & 2^{-1} & \tilde{3}^{-1} & \tilde{1} \\ A_3 & 4^{-1} & 2^{-1} & \tilde{1} & \tilde{3}^{-1} & \tilde{3}^{-1} & \tilde{1} \\ A_4 & \tilde{3} & 2 & \tilde{3} & \tilde{1} & 2 & 2 \\ A_5 & 2 & \tilde{3} & \tilde{3} & 2^{-1} & \tilde{1} & 2 \\ A_6 & \tilde{3}^{-1} & \tilde{1} & \tilde{1} & 2^{-1} & 2^{-1} & \tilde{1} \end{matrix} \Rightarrow w_A = \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \end{bmatrix} = \begin{bmatrix} (0.073, 0.226, 0.662) \\ (0.042, 0.108, 0.441) \\ (0.033, 0.071, 0.343) \\ (0.067, 0.271, 0.829) \\ (0.060, 0.219, 0.735) \\ (0.300, 0.104, 0.294) \end{bmatrix}$$

$$A_{C_2} = \begin{matrix} & C_{21} & C_{22} & C_{23} & C_{24} \\ C_{21} & \tilde{1} & 2 & 4 & \tilde{3} \\ C_{22} & 2^{-1} & \tilde{1} & 2 & 2 \\ C_{23} & 4^{-1} & 2^{-1} & \tilde{1} & \tilde{1} \\ C_{24} & \tilde{3}^{-1} & 2^{-1} & \tilde{1} & \tilde{1} \end{matrix} \Rightarrow w_{C_2} = \begin{bmatrix} w_{C_{21}} \\ w_{C_{22}} \\ w_{C_{23}} \\ w_{C_{24}} \end{bmatrix} = \begin{bmatrix} (0.153, 0.474, 1.165) \\ (0.085, 0.261, 0.728) \\ (0.064, 0.130, 0.534) \\ (0.047, 0.134, 0.437) \end{bmatrix}$$

$$A_{C_3} = \begin{matrix} & C_{31} & C_{32} & C_{33} & C_{34} \\ C_{31} & \tilde{1} & \tilde{5} & 2 & \tilde{3} \\ C_{32} & \tilde{5}^{-1} & \tilde{1} & \tilde{5}^{-1} & 4^{-1} \\ C_{33} & 2^{-1} & \tilde{5} & \tilde{1} & 2 \\ C_{34} & \tilde{3}^{-1} & 4 & 2^{-1} & \tilde{1} \end{matrix} \Rightarrow w_{C_3} = \begin{bmatrix} w_{C_{31}} \\ w_{C_{32}} \\ w_{C_{33}} \\ w_{C_{34}} \end{bmatrix} = \begin{bmatrix} (0.130, 0.408, 1.037) \\ (0.032, 0.061, 0.231) \\ (0.116, 0.315, 0.807) \\ (0.099, 0.216, 0.576) \end{bmatrix}$$

$$A_{C_4} = \begin{matrix} & C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{41} & \tilde{1} & \tilde{3} & 2 & \tilde{3} & 2^{-1} & 2^{-1} \\ C_{42} & \tilde{3}^{-1} & \tilde{1} & \tilde{1} & 2^{-1} & \tilde{3}^{-1} & \tilde{3}^{-1} \\ C_{43} & 2^{-1} & \tilde{1} & \tilde{1} & 2 & \tilde{1} & 2 \\ C_{44} & \tilde{3}^{-1} & 2 & 2^{-1} & \tilde{1} & \tilde{3}^{-1} & 2^{-1} \\ C_{45} & 2 & \tilde{3} & \tilde{1} & \tilde{3} & \tilde{1} & 2 \\ C_{46} & 2 & \tilde{3} & 2^{-1} & 2 & 2^{-1} & \tilde{1} \end{matrix} \Rightarrow w_{C_4} = \begin{bmatrix} w_{C_{41}} \\ w_{C_{42}} \\ w_{C_{43}} \\ w_{C_{44}} \\ w_{C_{45}} \\ w_{C_{46}} \end{bmatrix} = \begin{bmatrix} (0.053, 0.214, 0.711) \\ (0.033, 0.075, 0.395) \\ (0.053, 0.161, 0.553) \\ (0.035, 0.100, 0.395) \\ (0.061, 0.257, 0.789) \\ (0.053, 0.193, 0.632) \end{bmatrix}$$

$$A_{C_5} = \begin{matrix} & C_{51} & C_{52} & C_{53} & C_{54} \\ C_{51} & \tilde{1} & \tilde{3} & \tilde{3} & \tilde{5} \\ C_{52} & \tilde{3}^{-1} & \tilde{1} & 2 & 2 \\ C_{53} & \tilde{3}^{-1} & 2^{-1} & \tilde{1} & 4 \\ C_{54} & \tilde{5}^{-1} & 2^{-1} & 4^{-1} & \tilde{1} \end{matrix} \Rightarrow w_{C_5} = \begin{bmatrix} w_{C_{51}} \\ w_{C_{52}} \\ w_{C_{53}} \\ w_{C_{54}} \end{bmatrix} = \begin{bmatrix} (0.134, 0.478, 1.298) \\ (0.072, 0.212, 0.649) \\ (0.101, 0.232, 0.649) \\ (0.038, 0.078, 0.303) \end{bmatrix}$$

By fuzzy multiplication of the related sub-criteria weighting vectors and criteria weights the aggregated weights of the sub-criteria were obtained with respect to the goal. For the given criterion C_j, which splits into k_j sub-criteria, it is necessary to determine the relative importance of the sub-criteria with respect to this criterion. After that the fuzzy judgment matrix can be determined as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1k_j} \\ a_{21} & a_{22} & \dots & a_{2k_j} \\ \dots & \dots & \dots & \dots \\ a_{k_j1} & a_{k_j2} & \dots & a_{k_jk_j} \end{bmatrix} \quad (18)$$

The weights of sub-criteria with respect to given criterion are obtained again as fuzzy extents. Final sub-criteria weights are derived through the aggregation of the weights at two consecutive levels. Multiplying sub-criteria weights by respective criterion weight (Equation 16) gives:

$$w_j^p = \left(\sum_{l=1}^{k_j} a_{il} \left[\sum_{i=1}^{k_j} \sum_{j=1}^{k_j} a_{il} \right]^{-1} \right) \cdot w_j,\ i = 1,...,M,\ p = 1,...,k_j \quad (19)$$

Where w_j^p are the aggregated fuzzy weights of sub-criteria. They are entries of the weight vector 20 with the total length K.

$$W = \left(w_1^1, w_1^2, ..., w_1^{k_1}, w_2^1, w_2^2, ..., w_2^{k_2}, ..., w_j^1, w_j^2, ..., w_j^{k_j}, ..., w_M^1, w_M^2, ... w_M^{k_M} \right) \quad (20)$$

For example for level of certainty $\beta = 0.5$:

$$w_{C_1}^{'} = w_1 \cdot w_{C_1} = \begin{bmatrix} w_1 \cdot w_{C_{11}} \\ w_1 \cdot w_{C_{12}} \\ w_1 \cdot w_{C_{13}} \\ w_1 \cdot w_{C_{14}} \\ w_1 \cdot w_{C_{15}} \\ w_1 \cdot w_{C_{16}} \end{bmatrix} = \begin{bmatrix} (0.090, 0.265, 0.728) \cdot (0.073, 0.226, 0.662) \\ (0.090, 0.265, 0.728) \cdot (0.042, 0.108, 0.441) \\ (0.090, 0.265, 0.728) \cdot (0.033, 0.071, 0.343) \\ (0.090, 0.265, 0.728) \cdot (0.067, 0.271, 0.809) \\ (0.090, 0.265, 0.728) \cdot (0.060, 0.219, 0.735) \\ (0.090, 0.265, 0.728) \cdot (0.030, 0.104, 0.294) \end{bmatrix}$$

$$= \begin{bmatrix} (0.007, 0.060, 0.482) \\ (0.004, 0.029, 0.321) \\ (0.003, 0.019, 0.250) \\ (0.006, 0.072, 0.589) \\ (0.005, 0.058, 0.535) \\ (0.003, 0.028, 0.214) \end{bmatrix}$$

Several methods have been proposed to aggregate the decision maker's assessments. The most commonly used are the mean, median, max, min and mixed operators (Buckley, 1985). Additive synthesis has been assumed here and the final alternative performance weights with respect to overall goal are calculated by the summation of elements in the rows of the performance matrix

$$Z = \begin{bmatrix} x_{11} \cdot w_1 & x_{12} \cdot w_2 & .... & x_{1K} \cdot w_K \\ x_{21} \cdot w_2 & x_{22} \cdot w_2 & .... & x_{2K} \cdot w_K \\ ... & .... & .... & .... \\ x_{N1} \cdot w_1 & x_{11} \cdot w_1 & .... & x_{NK} \cdot w_K \end{bmatrix} \quad (21)$$

to obtain the following expression:

$$F_i = \sum_{j=1}^{K} x_{ij} \cdot w_j,\ i = 1, 2, .., N. \quad (22)$$

To finally rank the alternatives, the prioritisation of aggregated assessments is required. Since each F_i is a triangular fuzzy number, it is necessary to apply the method of ranking triangular fuzzy numbers. There are several methods that are able to do this; such as the centre of gravity method, the dominance measure method, the α-cut with interval synthesis method, and the total integral value method. The last, the total integral value method (Liou and Wang, 1992), is considered to be a good choice for performing the task efficiently and, therefore, has been proposed within this methodology. For the given triangular fuzzy number $A = (a_1, a_2, a_3)$, the total integral value is defined as:

$$\mathrm{I}_T^\lambda(\mathrm{A}) = \left[\lambda a_3 + a_2 + (1-\lambda) a_1 \right] 2^{-1},\ \lambda \in [0,1] \quad (23)$$

In Equation 23, λ represents an optimism index which expresses the decision maker's attitude towards risk. A larger value of λ indicates a higher degree of optimism. In practical applications, values 0, 0.5 and 1 are used respectively to represent the pessimistic, moderate and optimistic views of the decision maker. For given fuzzy numbers A and B it is said that if $\mathrm{I}_T^\lambda(\mathrm{A}) < \mathrm{I}_T^\lambda(\mathrm{B})$, then $A < B$; if $\mathrm{I}_T^\lambda(\mathrm{A}) = \mathrm{I}_T^\lambda(\mathrm{B})$ then $A = B$; and if $\mathrm{I}_T^\lambda(\mathrm{A}) > \mathrm{I}_T^\lambda(\mathrm{B})$, then $A > B$.

The final ranking of alternatives means to adopt certain level λ of optimism of the decision-maker, then to apply Equation 23 on fuzzy numbers Equation 22 and finally to rank alternatives regarding obtained values for

Table 2. Final rank of alternatives.

Level of certainty $\beta = 0.3$		Level of certainty $\beta = 0.5$		Level of certainty $\beta = 0.7$		Level of uncertainty $\beta = 1.0$		$\sqrt[n]{\prod_{k=1}^{n} W_k}$	
A_4	0,28421	A_4	0,28023	A_4	0,26333	A_4	0,21375	A_4	0,26048
A_2	0,27173	A_2	0,26822	A_2	0,25223	A_2	0,20483	A_2	0,24925
A_1	0,21069	A_1	0,20925	A_1	0,19757	A_1	0,16084	A_1	0,19459
A_3	0,07573	A_3	0,07672	A_3	0,07337	A_3	0,06019	A_3	0,07150

$I_T^{\lambda}(F_i),\ i = 1,...,N$. The best alternative from the set is represented as $f_{V_i} = \max\left(f_{V_i}\right), i = 1,..,A$ (Table 2).

CONCLUSION

AHP/ANP has been proven to be an efficient method in tackling a multi-criteria, decision making problem whatever its formulation and solving framework is – crisp or fuzzy. However, related versions of the method all suffer from shortcomings such as unbalanced ratios of estimations, the strong influence of subjective judgments on final results and exposure of the method to inconsistencies.

This paper presents an approach which aims to improve the application of the fuzzy version of AHP. By following the logic of the original (crisp) method in solving decision problems with criteria, sub-criteria and alternatives, a well-balanced fuzzy framework has been created. Unlike the works considered so far in which fuzzification of AHP/ANP method has been described, the model presented in this work takes into account the level of the decision maker's uncertainty and its influence upon weight of criteria. In this way, level of uncertainty which is used for assessment of linguistic expression is taken into account. Since large number of experts often takes part in the process of decision making, the model considers possibilities of synthesis of values of optimality criteria in case of group decision making. Decision making in group differs from individual decision maker on methodological and mathematics level. The model considers group synthesis with complete information and incomplete information.

Application of the described model is shown in the example of water management scenario in West Serbia. Five criteria with 24 sub-criteria have been used for assessing three different management plans. The proposed fuzzy decision making (FDM) approach has been verified as computationally efficient and stable. The derived results have been checked by an alternative (centre of gravity) method of defuzzification and the same ranking of management plans has been obtained. Finally, the standard and revised versions of AHP/ANP, which both use eigenvector method to derive weights of the decision elements, are used to check the consistency of the overall decision making process. The consistency was well below the tolerant limit, and again the final ranks of management plans were equal to those derived via the FDM approach.

Due to the satisfactory results of performed tests, the FDM approach can be considered to be flexible and robust. In particular, it has been recommended as a reliable support tool for use by decision makers in real situations, characterised by the uncertainty and imprecision of both the problem and the decision maker's expertise and cognitive abilities. One of the expected advantages of this is the ease of implementing the proposed method in a meeting with stakeholders.

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