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*Review*

# Redundant manipulators kinematics inversion

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**A robotic system is kinematically redundant when it possesses more degrees of freedom than those required to execute a given task. This paper reviews the well-known methods used to find the inverse kinematics of redundant manipulators. Because redundant manipulators have infinite solutions for their inverse kinematics, therefore the conventional method that was used to calculate the inverse kinematics of non-redundant manipulators cannot be used. The methods used to calculate the inverse kinematics of redundant manipulators are divided into three main categories in this paper: pseudoinverse, artificial intelligence and geometrical methods. These types have been explained, their advantages and disadvantages are also discussed in this paper.**

**Key words:** Redundant manipulator, inverse kinematics, pseudoinverse, artificial intelligence, geometrical method.

## INTRODUCTION

The area of robotics has been developed since the seventies and researchers have recognized that the addition of extra degrees of freedom to form a redundant arm overcomes the functional limitations of conventional non-redundant manipulators (Fernando and Tenreiro, 2002). Therefore, robotic systems can be divided in terms of kinematic structure into two classes: redundant and non-redundant (Graca and You-Liang, 1994). A manipulator is considered to be redundant if the number of degrees of freedom exceeds the dimension of the task space (Antonelli, 2008; Omrčen et al., 2007). And if the manipulator has a large or a finite degree of kinematic redundancy, this means that this manipulator is considered to be a hyper redundant manipulator (Gregory and Joel, 1994; Gregory, 1993). Unlike traditional rigid link robots, hyper redundant robot manipulators do not have rigid joints; also the increased number of degrees of freedom gives the manipulator some very useful properties. The manipulators are flexible, compliant, extremely dexterous and capable of dynamic adaptive manipulation in unstructured environments (Braganza et al., 2006). Hyper redundant manipulators can avoid obstacles and singularities and they are excellent candidates for optimization techniques. In most of the robotic applications, the task is to reach the goal and avoid obstacles at the same time (Atef and Mohamed, 2006). Robot kinematics generally includes forward and inverse kinematics at the position, velocity and acceleration level. These constructs are essential for the Cartesian control of serial manipulators (Chetan and Delbert, 1999). The forward kinematics (the determination of the end-effector position from known joint positions) is simple for most manipulators, and the inverse that is of more practical use because usually, the desired end-effector position is known and the corresponding joint positions have to be found to attain the desired end-effector position (Das et al., 1988). The inverse kinematics problem involves the existence and uniqueness of a solution, and effectiveness and efficiency of solution methods. The inverse kinematics problem is thus much more difficult to solve than the forward kinematics problem for serial-link manipulators (Youshen and Jun, 2001).

Inverse kinematics and control of redundant manipulators become increasingly complicated with each added degree of freedom. With a large number of redundant degrees of freedom, as in the so called ‘hyper redundant’ manipulators, the motion capabilities in highly cluttered environments can be enhanced to a great extent, but the corresponding path planning problem also

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becomes increasingly challenging (Dasgupta et al., 2009).

## PSEUDOINVERSE

Usually, kinematics of non redundant robots is solved by deriving analytical solutions for several manipulator configurations. The analytical solutions offer minimal numerical complexity together with special treatment of singularities (Manja and Tatjana, 1991; Manja, 1991). On the other hand, one of the problems in controlling redundant manipulators is a considerably increased computational complexity compared with non redundant robots. For a given workspace position, one solution has to be chosen among an infinite number of inverse kinematics solutions corresponding to all possible configurations (Ramdane-Cherif, 2002). Therefore, the classical methods cannot be used for solving their inverse kinematics. Many investigations have focused on the redundancy resolution of this type of manipulator based on the manipulator Jacobian pseudoinverse. Singularity avoidance, obstacle avoidance and keeping joint variables in their physical limitation are some examples of supplementary tasks (Farbod et al., 2002). A forward kinematic transformation is developed in its most simplified form, suitable for real-time control applications and the manipulator Jacobian is derived in Charles et al. (1990) using the vector cross product method. Using the developed forward kinematic transformation and quaternion representation of orientation matrices, computer simulation is performed to evaluate the efficiency of the Jacobian in converting joint velocities into Cartesian velocities and to investigate the accuracy of Jacobian pseudoinverse for various sampling times. In Kenneth et al. (1991), the inverse and pseudoinverse of the Jacobian of a serial-link spatial manipulator are expressed as operator forms that act on end-effector velocities to produce corresponding joint space velocities via link-to-link recursions. As it is common in the literature, a “spatial manipulator” is taken to mean a manipulator with at least 6 degrees-of- freedom that can fully position and orient its end-effector in Euclidean 3 space. A fast procedure for the computation of manipulator inverse kinematics and pseudoinverse robustness is presented in Mayorga et al. (1992). The approach is based on solving a linear algebraic system and evaluating the norm of the Jacobian matrix. This value is properly used in an original scheme that is also proposed for the appropriate robustness of the pseudoinverse matrix. In O’Neil and Chen (2000), the instability of the pseudoinverse acceleration controller is analyzed and shown to be related to the singular values of the Jacobian matrix and their rates of change, the corresponding growth of joint velocities and accelerations is characterized in terms of the smallest singular value of the Jacobian matrix of the kinematic function and a nearly-conserved quantity analogous to angular momentum.

The concept of kinematic manipulability ellipsoids is introduced as a measure of the capability of a manipulator for executing a specific task in a given configuration (Keith et al., 1995). When the end-effector of a redundant manipulator traces a closed path in the workspace, the path in the joint space is not necessarily closed (Shengwu and Shaheen, 1992). This phenomenon is known as the non-repeatability of the joint motions. These properties of manipulability and non-repeatability of the joint motions of redundant robots during pseudoinverse motion rate control has been known for many years. Fernando and Tenreiro (1998, 1999, 2000) presented the main aspects of the pseudoinverse application in the control of redundant manipulators. An alternative based on the least squares approximation to the manipulability optimization is also developed. These techniques are applied in the trajectory control of redundant and hyper redundant manipulators. The pseudoinverse schemes lead to non-optimal performances both from the manipulability and repeatability viewpoints. Tzafestas et al. (1996) presents a complete generalized solution to the inverse kinematics of redundant robots. It is obtained using an iterative technique for the velocity control which is suitable for redundant robots with a maximum of 11 (DOF). This technique which splits the Jacobian inversion problem into two sub-problems (wrist position and wrist orientation) provides an exact solution for the arm tip position and an approximate solution for the wrist orientation. In many typical industrial applications where point-to-point motion of robot manipulators is needed, such as assembly, material handling, etc., robot manipulators should be able to avoid obstacles in their workspace while performing the desired task. For this purpose, robots should possess at least one degree-of-freedom (DOF) more than the number required for the general free positioning, which means that they should be redundant (Nearchou and Aspragathos, 1996). If the path of an end-effector in a work space is already given, the computation of a feasible joint path sequence for a redundant manipulator is referred to as redundancy resolution. On the other hand, when the task is given as a specified goal point that the end-effector is to reach, the process of competing a feasible joint path sequence is referred to as the path planning problem (Erdinc, 2003, 2005; Erdinc et al., 2005).

The problem of singularities is solved by planning trajectories that avoid singular configurations. A solution to the problem of singularities for six-joint manipulators is presented in Stefano and Olav (1990). By using the method, the unachievable components of the commanded motion are removed while an exact inverse kinematic solution is used for the remaining motion components. At a singularity, there are directions in Cartesian space where a differential translation or rotation cannot be specified. The underlying idea is to identify these directions, and then to eliminate the corresponding components of the commanded motion when the manipulator becomes singular using a pseudoinverse of the manipulator Jacobian.

A new method is proposed in Alfonso and Jose (2003) to solve the motion planning problem of redundant manipulators for execution of specified operational trajectories. The critical case is considered in which the use of singularities is essential to accede to the complete desired path. The main idea of the method is that one minor of the Jacobian matrix would be controlled in such a way that the manipulator changes its posture by smooth crossing through one singularity. In Jesus et al. (2000), a broadly applicable approach for numerically obtaining the workspace and the singularity curves of a planar RRRRR-type manipulator is presented. The workspace generation is formulated as a direct kinematic problem involving only two branches which are mathematically defined and related with the manipulator's assembly configurations. For solving that problem, the analytical solution of two simple quadratic equations is found. A simple existence criterion is also obtained to detect the set of points forming the manipulator's workspace. On the other hand, the singularity curves are composed of sets of singular points. In order to obtain the singular points, the properties of the Jacobian matrix are used. In Reni (1997), a fast geometrical bounded method for the on-line obstacle avoidance of redundant manipulators is presented. The approach is based on formulating an inverse kinematics problem under an inexact context. This approach allows dealing with the avoidance of obstacles utilizing an appropriate and easy to calculate null space, geometrically bounded based vector; while the avoidance of singularities is attained by the proper pseudoinverse robustness. Here the computation of the inverse kinematics problem is accomplished by solving numerically a linear system which includes the vector for obstacle avoidance and a proper scheme for the proper pseudoinverse robustness. Leon and Bojan (2002) present a control algorithm for an on-line obstacle avoidance which allows a kinematically redundant manipulator to move in an unstructured environment without colliding with obstacles.

The presented approach is based on the redundancy resolution at the velocity level. The primary task is determined by the end-effector trajectories, and for the obstacle avoidance, the internal motion of the manipulator is used. The obstacle avoiding motion is defined in one-dimensional operational space and hence, the system has less singularity that makes the implementation easier. Instead of the exact pseudoinverse solution, an approximate one is proposed which is computationally more efficient and allows also considering many simultaneously active obstacles without any problem. The fast cycle times of the numerical implementation enable the use of the algorithm in real time control. Mirosław (2004) presents an approach to the problem of controlling a redundant manipulator so that its end-effector follows a prescribed geometric path and the manipulator simultaneously avoids collisions with moving obstacles in the work space. In addition, the constraints imposed on the robot controls are taken into account. Provided that a solution to the control problem of redundant manipulators exists, the Lyapunov stability theory is used to derive the trajectory generator. The approach offered does not require any inverse of robot kinematic equations. Instead, a transpose Jacobian matrix is used to generate robot motions. Under the pseudoinverse control, robots with kinematical redundancy exhibit an undesirable chaotic joint motion which leads to an erratic behavior. Maria et al. (2008) studies the complexity of fractional dynamics of the chaotic response. Fourier and wavelet analysis provides a deeper insight, helpful to know better the lack of the repeatability problem of redundant manipulators. This perspective for the study of the chaotic phenomena will permit the development of superior trajectory control algorithms.

In the case of hyper redundant manipulators with high number of degrees of freedom, the computational burden of pseudoinverse Jacobian becomes prohibitive, despite proposed improvements. Furthermore, most of the proposed schemes handle the inverse kinematic problem at the velocity level only (Farbod et al., 2002). Therefore many approaches are presented and they did not require calculation of the pseudoinverse of the Jacobian.

## Artificial intelligence

The coupling between robot’s axes makes even more difficult the design of a trajectory planning or a control system of the robot. To overcome these difficulties, several researchers have used soft computing techniques. For example, genetic algorithms are increasingly applied to motion planning and dynamic control of robotic systems. Neural networks and fuzzy control perhaps constitute the most applied artificial intelligence fields in control systems of industrial manipulators and mobile robots. Moreover, these artificial intelligence techniques have added a new dimension to many engineering fields to study (Amar et al., 2008).

## Neural network

In recent years, new interest in the neural network research has been generated to reduce the computational complexity of motion planning and control of manipulators. Several neural networks methods have mainly been studied by researcher to model a forward and inverse kinematics mapping for manipulators (Benallegue et al., 2002). And in trajectory control of robotic devices, neural networks provide a fast method of autonomously learning the relation between a set of output states and a set of input states. Since a neural network can only generalize within the range of information which has been presented to it, it will give an approximated solution near singularity points and therefore solve the singularity problem which has long been the major difficulty in implementing ‘resolved motion rate control’ (Amel et al., 2003). Ding and Tso (1998) presents a neural network based computational scheme for redundancy resolution of manipulators. The Tank-Hopfield (TH) network is adopted for pseudoinverse and inverse kinematics calculations and it can provide joint velocity and joint acceleration solutions within a time frame of the order of hundred nanoseconds. The connection weights of the network can be directly obtained from the known matrix, the Jacobian matrix and the end-effector velocity or the end-effector acceleration. Incorporating the TH network into the redundancy resolution scheme allows planning algorithms to be implemented in real time. A new neural network approach to robot arm kinematic control based on an iterative update of joint vector is presented in Sukhan and Rhee (1993). In the proposed method, the pseudoinverse of the gradient of a Lyapunov function is defined in the joint space to update the joint vector towards a solution. This paper established explicit convergence control schemes to achieve fast and stable convergence. Furthermore, the proposed method allowed direct incorporation of potential field approaches to obstacle avoidance into joint trajectory planning. Bao-Liang and Koji (1995) presents an approach to regularizing the inverse kinematics problem for redundant manipulators using neural network inversions. This approach is a four-phase procedure.

In the first phase, the configuration space and associated workspace are partitioned into a set of regions. In the second phase, a set of modular neural networks is trained on associated training data sets sampled over these regions to learn the forward kinematic function. In the third phase, the multiple inverse kinematic solutions for a desired end-effector position are obtained by inverting the corresponding modular neural networks. In the fourth phase, an "optimal" inverse kinematic solution is selected from the multiple solutions according to a given criterion. This approach has an important feature in comparison with existing methods, that is, both the inverse kinematic solutions located in the multiple solution branches and the ones that belong to the same solution branch can be found. An iterative method using the neural networks to solve the inverse kinematics problem for equal length links redundant manipulators is presented in Yahya et al. (2009). The training phase, calculating the neural networks weights is accomplished for a new proposed geometrical method to solve the problem of multi-solution caused by redundancy. The use of this geometrical method results in one solution among the infinite solutions of the inverse kinematics of the redundant manipulators. This method is very effective for avoiding the singularity problem because it guarantees that there is no lining up for two or more links. Another advantage for this method is that the angles between the links will be set between two maximum and minimum values. This means that the end-effector can reach any point on the desired path and the angles between the links will not be less than the minimum limit or more than the maximum limit, which makes this method effective for joint limits. The inverse kinematic of a constrained redundant robot manipulator is considered in Amar et al. (1995). An optimization procedure using neural network is formulated. It produced position, velocity and acceleration trajectories in joint space from position and orientation trajectories in Cartesian space and guarantees a good tracking of the desired end-effector trajectory. The redundancy is solved by minimizing a performance function. This new method gave an accurate solution with only a few iterations. In (Yunong and Jun, 2004), a recurrent neural network is developed and applied for kinematic control of redundant manipulators with obstacle avoidance capability. An improved problem formulation is proposed in the sense that the collision avoidance requirement is represented by dynamically-updated inequality constraints. In addition, physical constraints such as joint physical limits are also incorporated directly into the formulation. Based on the improved problem formulation, a dual neural network is developed for the online solution to collision-free inverse kinematics problem. Ramdane-Cherif et al. (1997) proposes an iterative method using a neural optimization network to solve the inverse kinematic problem for redundant and non-redundant manipulators.

The neural network weights are adapted in the direction of decreasing a Lyapunov function to move the end-effector to the desired position. This approach offers substantially better accuracy and avoids the computation of the Jacobian inverse and pseudoinverse Jacobian matrix. This method gave an accurate solution with only a few iterations per input point even in singular points and only required the computation of the direct kinematic functions. A new problem formulation is proposed in the sense that the collision avoidance scheme is described in Yunong and Wang (2003) by dynamically-updated inequality constraints and the physical constraints such as joint limits are also incorporated in the formulation. For real-time computation, the dual neural network is applied for the online solution of obstacle-avoidance inverse kinematic control problem. In Yunong et al. (2003), a dual neural network is proposed for online redundancy resolution of kinematically redundant manipulators. Physical constraints such as joint limits and joint velocity limits, together with the drift-free criterion as a secondary task are incorporated into the problem formulation of redundancy resolution. Compared to other recurrent neural networks, the dual neural network is piecewise linear and has much simpler architecture with only one layer of neurons.

## Fuzzy logic

The ‘fuzzy learning algorithm’ is a control algorithm which has been developed for the kinematic control of redundant robotic manipulators without any modeling of the manipulator itself. It is based on conventional kinematic control methods for manipulators combined with the techniques of fuzzy regression and fuzzy inferencing to learn the appropriate kinematic models based on actual trajectory data. Randy and You-Liang (1994) modifies the fuzzy regression formulation itself which is a linear programming problem to learn a fuzzy generalized inverse of the manipulator Jacobian which is normally a non-unique matrix. However, this paper imposed additional constraints in the fuzzy regression formulation, and modified the cost function to maximize the effect of the additional constraints such that the matrix that is learned is one which optimizes the subtask as well as executing the main task of trajectory tracking. In Gu et al. (1993), a new fuzzy logic approach with multi-criteria is proposed for the kinematic control of redundant manipulators. One first calculated a joint trajectory based on pseudoinverse and gradient vectors corresponding to each criterion. Then the desired one is selected among all obtained trajectories based on the relative importance degrees of criteria and the satisfaction degrees of a trajectory to all criteria by a fuzzy multi-criteria decision making process. An inverse kinematics solution that utilizes the differential relationship between the joint space and Cartesian space of redundant manipulators is proposed in Sung-Woo and Ju-Jang (1993, 1993b). This solution can be obtained using the pseudoinverse of the Jacobian matrix. However, the computation of the pseudoinverse is complex so that it is not easy to be implemented on a digital computer for on-line motion planning of the robot. As an alternative of the pseudoinverse solution, the inverse kinematics solution using the fuzzy logic is derived. We find the rough solution based on the gradient method and then refine it by introducing the fuzzy logic through the extension principle. Based on the fact that humans do not compute exact inverse kinematics, but can do precise positioning from heuristics, Yangsheng and Michael (1993) proposes an inverse kinematic mapping through fuzzy logic. The implementation of the proposed scheme had demonstrated that it is feasible for both redundant and non-redundant cases, and that it is very computationally efficient. The result provided sufficient precision and transient tracking error could be controlled based on a fuzzy adaptive scheme proposed in this paper. In Mohammad and Alireza (1999), the adaptive fuzzy logic (AFL) approach for solving the inverse kinematics of redundant robots in an environment with obstacles is presented. The obstacles are modeled as convex bodies. A fuzzy rule base that is updated via an adaptive law is used to solve the inverse kinematic problem. Additional rules are introduced to take care of the obstacles avoidance problem.

The proposed method has advantages such as high accuracy, simplicity of computations and generality for all redundant robots. An approach of choosing appropriate joint angles using fuzzy rules is presented in Rainer (1992). In this paper, both distances and joint corrections are denoted as fuzzy terms. In this way, various criteria for controlling a redundant arm are formulated linguistically by means of fuzzy IF ... THEN rules. A fuzzy-logic based method for avoiding joint limits and obstacles in kinematically redundant manipulators is presented in (Syed et al., 1996). A comparison is made with the gradient projection method (GPM) for avoiding joint limits and obstacles. The fuzzy logic based method automatically chooses an appropriate magnitude of self-motion using linguistic rules. The method is simple and is physically realizable. Neural network and fuzzy system are based on the mechanism of human brain. While the neural network simulates physiological features of human brain, fuzzy system simulates psychological features of human brain. To realize higher intelligence on the robotic systems, the emerging synthesis of various techniques is required as a whole system. New-fuzzy computing has been developed for overcoming their disadvantages. In general, the neural network part is used for its learning and classifying, while the fuzzy logic part is used for inference (Naoyuki et al., 1998). A neural optimization network is proposed in Woong et al. (1991) to control the redundant robot manipulators in an environment with the obstacle. The weightings of the network are adjusted by considering both the joint dexterity and the capability of collision avoidance of joint differential motion. The fuzzy rules are proposed to determine the capability of collision avoidance of each joint. Rene and Sandeep (2006) presents a novel, fast and simple procedure for obstacle and singularities avoidance via an inverse kinematics problem. Obstacle avoidance is achieved based on the calculation of an appropriate null space vector and a proper pseudoinverse perturbation helped avoid singularities effectively. The computation of the inverse kinematics is accomplished with the help of dully trained ‘adaptive neuro–fuzzy inference systems’, thus enabling the methodology to be applicable to all redundant robots operating in a sensor based real time environment.

In Amar et al. (2007), the problem of multi-objective trajectory planning is studied for redundant planar serial manipulators using a data-driven hybrid neuro-fuzzy system. A first, pre-processing step involved an offline planning generating a large dataset of multi-objective trajectories, covering mostly the robot workspace. The optimized criteria are travelling time, consumed energy and singularity avoidance. The offline planning is initialized through a cycloidal minimum time parameterized trajectory in joint space. This trajectory is then optimized using an augmented Lagrangian technique. The outcomes of this pre-processing step allow building a Tsukamoto neuro-fuzzy inference system to learn and capture the robot multi-objective dynamic behavior. In Samy et al. (2004), a back propagation neural network is presented for the inverse kinematics of redundant manipulators with joint limits; also a novel online inverse kinematics solution of redundant manipulators to avoid joint limits is presented in Samy et al. (2005). A Widrow-Hoff neural network with a learning algorithm derived by applying Lyapunov approach is introduced in this paper for this problem. Since the inverse kinematics has infinite number of joint angle vectors, a fuzzy neural network is designed in both (Samy et al., 2004, 2005) to provide an approximate value for that vector. This vector is fed into the neural network as a hint input vector to guide the output of the neural network within the self-motion. This fuzzy neural network is designed based on cooperatively controlling each joint angle of the manipulator. The joint velocity limits as well as the joint limits are incorporated into the method of these papers.

## Genetic algorithm

In the last 20 years, genetic algorithms have been applied in a plethora of fields such as: control, system identification, robotics, planning and scheduling, image processing, pattern recognition and speech recognition (solteiro et al., 2007). A genetic algorithm is a stochastic search algorithm that can optimize nonlinear or discrete functions, and is applied to optimize redundant parameters for the motion plan of a redundant manipulator (Takanori et al., 1997). A genetic algorithm to generate trajectories for robotic manipulators based on the direct kinematics is presented in Solteiro and Tenreiro (1999). The objective is to minimize the ripple in the time evolution of robot positions and velocities. Moreover, the manipulator is required to reach a predefined goal without colliding with obstacles in the workspace. Since the genetic algorithm uses merely the direct kinematics, the singularities did not constitute a problem. A novel genetic algorithm using a floating point representation is proposed in Lianfang et al. (2002) to search for the optimal end-effector trajectory for a redundant manipulator. An evaluation function is defined based on the total displacement of the end-effector, the total angular displacement of all the joints, the uniformity of Cartesian and joint space velocities. George-Christopher and Zenon (2008) investigates how to distribute in an optimum fashion the desired movement of the end-effector of an industrial robot with respect to the work-piece, when there are redundant degrees of freedom such as a positioning table. The desired motion is given as a series of acceleration functions in respective time intervals. The constraints of the optimization are the available acceleration limit of axes such as the table axes, the upper bounds to velocity and displacement of each axis and the avoidance of singular point areas of the robot as defined by its manufacturer. The optimization criterion has minimum total work for the motion. A genetic algorithm is used to solve the problem. Hiroki et al. (2004) introduces an optimization method for a trajectory planning of redundant manipulators which achieve a given task with high efficiency and apply the method to the wheelchair propulsion problem. Genetic algorithm is used to optimize the redundant variables of the manipulator. Additionally, the procedure of the method did not use any forward dynamics computation. This means that the procedure is more stable than conventional procedures of optimal control.

For dealing with the complexity in gaining inverse kinematics solution of 7-DOF manipulator, two approaches, the first one based on RBF neural network is proposed in Yugui et al. (2007) and the other based on genetic algorithm is proposed in Yugui et al. (2007b). To solve the problem of multi-solution caused by redundancy, a rule for a joint of “best compliance” based on weighted ‘least square method’ is supposed at the beginning of these papers, which makes the multi-solution a mono-one with application of the genetic algorithm to search for all global optimum solutions. In Yonggang and Wei (2006), the adaptive simulated annealing genetic algorithm is presented by integrating the advantages of adaptive mechanics, simulated annealing algorithm and simple genetic algorithm. More successful results are obtained in manipulator trajectory planning using adaptive simulated annealing genetic algorithm compared with simple genetic algorithm. The experiment results showed that the method can be used in manipulator trajectory planning effectively and will shed new light on how to bring the redundancy into full play to improve the movement of manipulator. A new approach to solve the inverse kinematics problem of redundant robot manipulators in environments cluttered with obstacles is presented in Andreas (1998). The physical problem is formulated as an optimization problem under constraints, and solved via a modified genetic algorithm. The modified genetic algorithm searched for successive robot configurations in the entire free space so that the robot moves its end-effector from an initial placement to a final desired. The objective of this optimization is to minimize simultaneously the end-effector's positional error and the robot's joint displacements.

## MISCELLANEOUS AND GEOMETRICAL APPROACHES

There are other references which presented different methods for controlling hyper redundant manipulators and are not based on the pseudoinverse Jacobian or on the artificial intelligence methods. Some of them are geometrical method: Mohamed et al. (2009) describes a new geometrical method to solve the inverse kinematic problem for hyper redundant manipulators. The proposed method finds one solution from these infinite solutions. By using this method, the angles between the adjacent links are set to be the same which makes controlling the movement of these links easier and makes a lining up of two or more joint axes not possible, that is very effective for avoiding singularities. A computationally efficient, kinematic optimal control scheme is presented in Rajiv et al. (1991) for (7-DOF) manipulators. This scheme used the gradient projection optimization method in the framework of resolved motion rate control and it did not require calculation of the pseudoinverse of the Jacobian. An efficient formulation for determining joint velocities for given Cartesian components of linear and angular end-effector velocities is obtained. Thus, this control scheme is well suited for real-time implementation which is essential if the end-effector trajectory is continuously modified based on sensory feedback. A new geometrical method is proposed in Samer et al. (2009) to solve the problem of multi-solution caused by redundancy. The proposed method finds one solution to the inverse kinematics of redundant or hyper redundant manipulators from these infinite solutions. The most important advantage of this method is that the angles between the adjacent links are the same which avoid the singularity. This method can be used for the hyper redundant equal length links planar manipulators. Samer et al. (2009b) proposes a method for the motion planning of the redundant planar manipulators. The main idea is to find a smooth path consisting of points close enough to each other and geometrically compute their inverse kinematics. The proposed method will generate a single solution, which makes the problem of finding one solution among the infinite solutions caused by redundancy is avoided. In Samer et al. (2011), a new geometrical method for the inverse kinematics of hyper redundant manipulators is presented. The basic idea is that for a smooth path consisting of points close enough to each other, the proposed method is used to find the configuration of the end-effector on these points. The manipulability measure of manipulators has been calculated to show how much freedom the manipulator configuration has that is, how far the manipulability value is from zero (singularity configuration).

The advantage of the proposed method is that since angles between the adjacent links are the same, it makes the lining up of two or more joint axes impossible; therefore, this can be exploited for the avoidance of interior singularities. Minimum and maximum reach of the end-effector is calculated using the proposed method. The workspace of the manipulator is calculated as well. Aimed at the inverse kinematics of hyper redundant planar manipulators, a new and simple geometrical method is proposed in Li (2006). The proposed method can obtain an optimal solution of the inverse manipulators with fewer computations. Some disadvantages of the existing methods are overcome by the proposed method. The proposed method can be applied to any planar manipulators with n-links serially connected by revolute joints.

## Conclusion

In this paper, various methods of inverse kinematics and planning have been reviewed and discussed. The iteration solutions and artificial intelligence methods often require more computations and they do not guarantee convergence to the correct solution. Furthermore, there is no indication of how to choose the correct solution for a particular arm configuration. In the case of hyper redundant manipulators with high degrees of freedom, the computational burden of pseudoinverse Jacobian becomes prohibitive despite proposed improvements. Furthermore, most of the proposed schemes handle the inverse kinematic problem at the velocity level only. Therefore, among these many schemes, the geometrical method for path planning is preferred because of its simplicity, power saving and reduced computations.

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