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Full Length Research Paper

An alternative evaluation method for Likert type attitude scales: Rough set data analysis

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Likert scales are the most prevalent attitude scale. However, the fact that different items could be added up to produce same attitude values decreases the validity of the Likert scale results. Rough set data analysis could perhaps be conducted to overcome related limitations. Thus, this study seeks to investigate the employability of the rough set approach for the interpretation of Likert type attitude scales. Data was collected through a scale developed by using four sub-dimensions of the Fennema-Sherman mathematics attitude scale. In order to carry out the analysis, students were distributed to three groups of high, moderate and low in terms of their attitude to mathematics. Data was then converted to appropriate information tables for rough set analysis and lower and upper approximation sets were specified for these three groups. Accordingly, the potential membership of some students, who already belong to certain groups, to other groups was determined via rough sets. The mathematical value of to what extent each sub-dimension or any group can explain the total score was calculated. Through reduction of attributes, anxiety and usefulness sub-dimensions were found to be the indispensable sub-dimensions. The findings indicate that the rough set approximate can be used for the analysis of attitude scales.

Key words: Attitude scale, rough sets, data mining, approximations, data analysis.

INTRODUCTION

Education is, simply, the art of behavioural change (Erturk, 1972). Thus, it is essential to determine and interpret factors that influence behaviour. Attitudes are among the main determinants of behaviour (Tavşancıl, 2006). According to Collins, the relationship between attitude and behaviour assists social scientists in the challenging task of behaviour assessment (Tavşancıl, 2006). Knowledge of attitudes enables the knowledge of several related behaviours (Krech and Cructhfield, 1980). In this respect an important question to answer is: What is attitude?

Cognitive and emotional processes are indisputable components of learning and a reciprocal relationship exists between them. Emotions and expectations affect what is being learnt. Findings of a number of brain studies also indicate the significance of emotions in learning (Caine and Caine, 1994; Lackney, 1998). Emotions aroused by a topic can fluctuate in the process of learning. Emotions are expressed via attitudes. Even if the information is forgotten, attitudes and tendencies towards a topic would not (Stodolsky at al., 1991).

Attitudes are individually attributed emotions, beliefs and behavioural tendencies an individual has towards a specific abstract or concrete object (Baron and Byrne, 1977). As this definition suggests, attitude is not behaviour but a behavioural tendency. Attitude is the provisional state of the reaction in response to various stimuli or in other words, it is the tendency to react. An individual is not aware of his/her attitude towards a situation until she/he is required to react. Similarly, Bogardus (1947) stated that “behaviour is in a way a test for individual attitudes”. Therefore the investigation of behaviour is the best way to investigate an individual’s attitudes. However, behaviour as described here is distinct from directly observable behaviour. Attitudes are limited to potential behaviour. Learning, social judgement, consistency and functional theories are employed to explain the development of and variation in attitudes (Kağıtçıbaşı, 1999).

According to learning theories, an individual develops emotions and cognitions behind attitudes through learning processes such as stimulus-response, imitation and information transfer. When faced with a certain topic, the individual displays either a positive or negative emotion or reaction tendency towards the topic based on his/her knowledge of the topic and to the extent that the topic fulfils one's own needs. Interactions with a favourable situation, event or object will facilitate the development of positive attitudes.

To like or not to like and to enjoy or not to enjoy something necessitates making a judgement about it. Sherif and Hoyland (1961) applied the theory of judgement, which is one of the initial studies of academic psychology and which is also used in psychophysics experiments, to the area of attitude in social psychology within the framework of influential communication. This theory suggests that attitudes can be altered or can stay intact via mechanisms of assimilation or contrast. For the contrast mechanism, the chances to reject opposing views on the basis that they are different from one's views are higher.

On the other hand, for the assimilation mechanism, it is more likely to accept other views to be more similar to one's own views. To exemplify the mechanisms with an example from politics, Ali, who supports right-wing politics, would perceive an interaction around left wing political views as being more extreme left than Ahmet who has moderate political views. The contrast mechanism is in effect for the case of Ali. On the other hand, assimilation mechanism is in effect for Ahmet as he would perceive the same interaction to be more similar to his views.

One of the most discussed theoretical frameworks of attitude shift has been consistency. It is possible to talk about intra-consistency among the elements of an attitude and inter-consistency among different attitudes. The human thinking and behaviour are also directed towards consistency and away from inconsistency. Kağıtçıbaşı (1999) stated that inconsistency between attitude and behaviour is rarely observed or is a challenge to be explained through environmental factors.

Functional approach is another important theory which tries to explain the development and alterations of attitude. According to Kağıtçıbaşı (1999), this approach was defined by Smith et al. (1954) as "What are personal attitudes used for?". In other words, the individual develops an attitude for a specific reason. As reported by Allport (1956), the first study on attitude was carried out by Thurstone (1929) and subsequent research followed.

Assessing attitudes

The assessment of attitude has always been important, because knowledge of attitude allows one to predict and control behaviour (Eren, 2001; Krech and Cructhfield, 1980). However, as attitudes do not have a physical dimension, it is very difficult to scale them. Therefore, attitudes cannot be directly assessed. Information on individual thoughts, emotions and reaction tendencies are gathered instead (Thurstone, 1967).

For the assessment of attitudes, various methods such as observation, question lists, incomplete sentences and storytelling as well as various techniques such as choosing the wrong one and content analysis have been employed (Anderson, 1988; Arul, 2002). However, the most prominent and widespread method for the assessment of attitude has been attitude scales (Tavşancil, 2006). Several attitude scales are being used such as Bogardus social distance scale, Thurstone scale, Likert type attitude scale, Guttman scales, Osgood emotional meaning scale (Tavşancil, 2006).

Likert type attitude scale and the rationale behind the study

The most widely used scale among these is the Likert type attitude scale developed by Renis Likert (Likert, 1932). The reason for its widespread use is that these scales are relatively easier to develop and administer compared to other scales. On the other hand, the disadvantage of Likert type scales is that, responses to different statements can generate the same aggregate scores (Tavşancil, 2006). For instance, let's consider a Likert type attitude scale consisting of a number of subdimensions. The scores for these subdimensions could be varied: some could be low, while others could be high. Two students who have similar aggregate scores could have different sub-dimension scores. For example, let's say an attitude test has two dimensions A and B. A student could have a high score for sub-scale A and a low score for sub-scale B; and another student could have a low score for sub-scale A and a high score for sub-scale B. When aggregate scores are considered, both students could be said to possess moderate attitudes. However, is it possible to claim that these two students have the same attitudes because they seem to have the same aggregate scores while their sub-dimension scores are different? To what extent do sub-dimensions scores explain the total aggregate scores? How can we formulate and calculate it mathematically? These kinds of questions might entail abstract concepts without clear limits. Rough sets approach could be used for this kind of evaluation of attitude scales.

Vague concepts have preoccupied people's mind for centuries. Currently, the subject is very popular among computer engineers and mathematicians as well as philosophers and psychologists. Vague concepts are not easy to formulate using ordinary mathematical concepts since they may not include mathematically definite results. Thus, the necessity of alternative mathematical concepts is apparent for the mathematical modelling of vague concepts.

The idea of a vague or fuzzy concept is related to the Frege's (1904) boundary-line view. A concept is fuzzy if there are some objects which can not be classified either to the concept or to its complements but are members of the concept's boundary. The first successful approach to fuzziness was the notion of a fuzzy set proposed by Lotfi Zadeh in 1965. In this approach, sets are defined by partial membership in contrast to crisp membership used in classical definition of a set.

Another successful approach for fuzziness is the rough set (RS) theory defined by Pawlak (1982). After its introduction in the early 1980s, the theory of RS has been used as a mathematical tool for extracting knowledge from unclear and incomplete data (Pawlak, 1991; 1995). The theory assumes that initially there is necessary information or knowledge about the objects in the universe with which the objects can be divided into different groups (Hassanien, 2003). If we have exactly the same information about two objects, then they are said to be indiscernible (similar), that is, we cannot differentiate between them with available information.

The RS theory can be used for a variety of purposes such as finding relationships among data, interpreting the importance of attributes, discovering the patterns of data, learning common decision-making rules, reducing all redundant objects and attributes and seeking the minimum subset of attributes so as to attain a satisfactory classification (Hassanien, 2003). In addition, using possibly large and simplified patterns, the rough set reduction algorithms enable approximation of the decision classes (Kent, 1994; Lin and Cercone, 1997; Nings at al., 1995; Pawlak at al., 1995; Polkowski and Skowron, 1998; Skowron and Polkowski, 1998; Yorek and Narli, 2009; Zhong and Showron, 2000). The RS theory has many applications in a variety of fields such as artificial intelligence, machine learning, pattern recognition, decision support systems, data analysis and data mining (Hong at al., 2000; Shaari, 2009; Skowron and Polkowski, 1998; Slowinski, 1992; Wang at al., 2003; Ziarko, 1994). By using the concepts of lower and upper approximations in rough set theory, knowledge hidden in information systems may be unravelled and expressed in the form of decision rules (Intan and M. Mukaidono, 2002; Jagielska at al., 1999; Kryszkiewicz, 1998; Liu and Yu, 2009; Pawlak, 1991; Tsumoto, 1998).

Rough set theory which is applied in a variety of fields mentioned above has been considered as a valid data analysis method for attitude scales in education. Below you'll find an explanation on how an attitude scale, which was developed earlier, could be evaluated using rough set theory.

PRELIMINARIES

Rough set theory is a fundamental mathematical tool for studying uncertainty that may arise in various domains. For instance, suppose patients having a particular illness are the objects, then one has similar information about the patients, namely symptoms of the disease. Objects identified by the same information are indiscernible (similar) with respect to the available information about them. The relation generated from this indiscernibility is the mathematical basis of RS theory. A basic element of the theory of RS is to determine redundancies and dependencies among the available features of a problem. Rough set theory approximates a topic using lower and upper approximations. Therefore, using the learning algorithm of RS theory and from a decision table formed, one can obtain a set of values in an IF-THEN format (Pawlak, 1982; Skowron and Polkowski, 1998).

This section presents a review of some fundamental notions of rough set. Details of the theory of rough sets can be found in the literature (Pawlak, 1982, 1998; Pawlak at al., 1995; Polkowski and Skowron, 1998).

Indiscernibility relation

The mathematical mechanism of RS is obtained from the assumption that granularity can be expressed by partitions and their associated equivalence relations on the set of objects. This is called indiscernibilty relations. Lower and upper approximation sets are important concepts of rough set theory defined by the help of its equivalence relation.

Lower and upper approximations

A rough set approximates traditional sets using a pair of sets named the lower and upper approximation of the set. The lower and upper approximations of a set $A \subset IU$ are defined by the following equations respectively:

$$R_{low}(A) = \cup_{a \in IU} \{ R(a) : R(a) \subset A\} \quad (1)$$

$$R^{up}(A) = \cup_{a \in IU} \{ R(a) : R(a) \cap A \neq \varnothing\} \quad (2)$$

where $R(a)$ denotes the equivalence class of $a \in IU$.

Boundary region of A is defined as $B_R(A) = R^{up}(A) - R_{low}(A)$. It can be said that with respect to the attribute defined by the relation R, the set $R_{low}(A)$ is composed of elements which are certainly elements of the set A. Elements of the set $R^{up}(A)$ with respect to the attribute defined by the relation R, consist of elements which have the possibility of belonging to set A. On the basis of these definitions, set A is said to be crisp if the boundary region of A is empty and A is said to be rough if the boundary region of A is non-empty. This is shown schematically in Figure 1. Rough sets can be also characterized numerically by the following coefficient:

$$\alpha_R(X) = \frac{|\mathrm{Rlow(X)}|}{|\mathrm{Rup(X)}|} \quad (3)$$

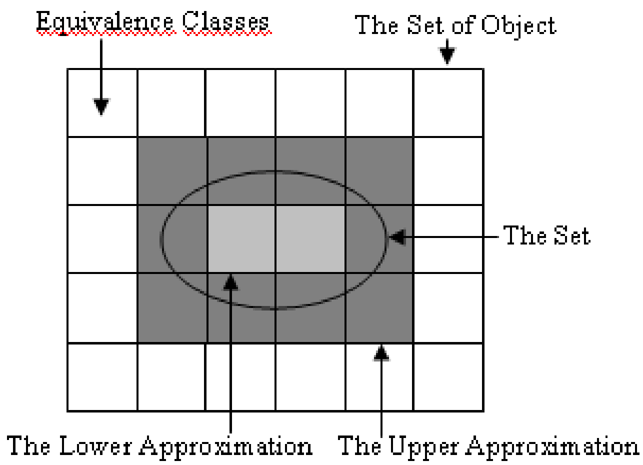

Figure 1. Schematic representation of a rough set

which is called accuracy of approximation, where |*X*| denotes the cardinality of *X*. Obviously $0 \le \alpha_R(X) \le 1$. If $\alpha_R(X) = 1$, *X* is crisp with respect to *R* (*X* is *precise* with respect to R), and otherwise, if $\alpha_R(X) < 1$, *X* is rough with respect to *R* (*X* is vague with respect to *R*)

Information systems

Knowledge representation in rough sets is done via information systems, which are a tabular form of an OBJECT → ATTRIBUTE relationship. More precisely, an information system, $\tau = \langle IU, \Omega, V_q, f_q \rangle_{q \in \Omega}$, where IU is a finite set of object, IU={x_1, x_2, …, x_n}, Ω is a finite set of attributes, the attributes in Ω are further classified into disjoint condition attributes A and decision attributes D, Ω=A∪D. For each q∈ Ω:

V_q is a set of attribute values for q
Each f_q: IU→ V_q is an information function which assigns particular values from domains of attributes to objects such that $f_q(x_i) \in V_q$ for all $x_i \in$ IU and q∈ Ω.

Dependency of attributes

Determining dependencies between attributes is another significant problem in data analysis. It is anticipated that if all values of attributes from Q are uniquely determined by values of attributes from P, then the set of attributes Q depends totally on set of attributes P, expressed as P ⇒ Q. In other words, if a functional dependency between values of Q and P exists, then Q depends totally on P.

The degree of dependency $\gamma(P,Q)$ of a set P of attributes with respect to a set Q of class labelling is define as:

$$\gamma(P,Q) = \frac{|POS_P(Q)|}{|IU|} \quad (4)$$

where the set of $POS_P(Q) = \bigcup_{X \in IU/R(Q)} R(P)_{low}(X)$ is called positive region, $|S|$ denotes the cardinality of set S, R(P) denotes the dependency relation of set P of attributes and IU/R(Q) denotes family of equivalence classes of the dependency relation of set Q of attributes.

The importance of P in mapping the dataset examples into Q is determined by the degree of dependency. If it is one, $\gamma(P,Q) = 1$, this means that Q is totally dependent on P and thus the attributes are essential. If it is zero, $\gamma(P,Q) = 0$, then Q is independent of the attributes in P and thus the attributes are not useful. Values of the degree of dependency between zero and one, $0 < \gamma(P,Q) < 1$, represent partial dependency, which means that some of the attributes in P may be useful or the dataset was inaccurate to begin with. In addition, the contradictions in some selected subset of the dataset can be determined by looking at the complement of $\gamma(P,Q)$.

Reduction of attributes

One of the important concepts of rough set theory is the reduction of attributes. The main question is whether some data from a data table could be removed while preserving its basic properties, that is, whether a table contains some superfluous data. For example, it is clear that if either attribute C or E is dropped in Table 1, a data set is obtained which is equivalent to the original one, in terms of approximations and dependencies. That is, in this case, accuracy of approximation and degree of dependencies are the same as in the original table although smaller set of attributes are used.

To clarify the above idea a couple of auxiliary notions would be appropriate. Let P be a subset of attributes set (IA) and let attribute “a” belong to P.

1. a is dispensable in P if IU/R(P) = IU/R(P − {a}); otherwise a is indispensable in P.
2. Set P is independent if all its attributes are indispensable.
3. Subset P' of P is a reduct of P if P' is independent and IU/R (P') = IU/R(P).

Thus a reduct is a set of attributes that preserves partition. It means that a reduct is the minimal subset of attributes that enables the same classification of elements of the universe as the whole set of attributes. In other words, attributes that do not belong to a reduct are superfluous with regard to classification of elements of

Table 1. Information system for attitude dataset.

	Condition attributes				Decision class
Object	**C**	**A**	**U**	**E**	**Dec**
x_1	3	3	3	3	High
x_2	3	3	3	3	High
x_3	3	2	2	3	High
x_4	3	2	2	3	High
x_5	3	2	3	3	High
x_6	3	2	3	3	High
x_7	2	2	3	3	High
x_8	2	2	3	3	High
x_9	2	2	3	3	High
x_{10}	2	2	3	3	High
x_{11}	2	2	3	3	High
x_{12}	2	2	3	3	Moderate
x_{13}	2	2	3	3	Moderate
x_{14}	2	3	2	2	Moderate
x_{15}	3	2	3	3	Moderate
x_{16}	2	2	2	2	Moderate
x_{17}	2	2	2	2	Moderate
x_{18}	1	2	1	1	Moderate
x_{19}	1	2	1	1	Low
x_{20}	1	1	1	1	Low

the universe. Reducts have several important properties. In what follows two of these are presented. First, a notion of a core of attributes is defined.

Let P be a subset of IA. The core of P is the set off all indispensable attributes of P.
The following is an important property, connecting the notion of the core and reducts

$$Core\ (P) = \bigcap \mathrm{Re}\ d\,(P)\,, \tag{5}$$

where Red(P) is the set off all reducts of P.

Because the core is the intersection of all reducts, it is included in every reduct, that is, each element of the core belongs to some reduct. Thus, in a sense, the core is the most important subset of attributes, because none of its elements can be eliminated without affecting the classification power of attributes.

METHODS

One of the most famous studies on attitude towards mathematics was carried out by Fennema and Sherman, 1977. The original scale consisted of 9 sub-dimensions. Each sub-dimension contained 12 items. It was reported that it is possible to use each sub-dimension of the scale separately on its own (Mulhern and Rae, 1998). For the current study, a 48 item scale composed of 4 sub-dimensions (confidence, anxiety, usefulness and effectance motivation) of the Fennema –Sherman Mathematics attitude scale was used as an example. The three-point Likert type scale was administered to 8th grade students. A random sample of 20 students was drawn from the whole sample and data collected from these students were included in analysis. Data was coded as 1 for 'I disagree', 2 for 'I moderately agree' and 3 for 'I agree'. After the process of recoding, average attitude scores were calculated for each dimension and for the total scale. Average scores for the 20 students are presented in Table 2.

A crucial point to note here is that due to the recoding process in data analysis, students with a high average anxiety score, as presented in Table 2, do not have mathematics anxiety. The students were then divided into three groups of low, moderate and high according to their sub-dimension and total attitude scores. Group span value, as the scale was 3-point Likert type, was calculated as 2/3≅0,66. Accordingly, group interval values were recorded in Table 3. Therefore, the categories the students belonged to in terms of their scores for sub-dimensions and total attitudes are presented in Table 4.

As presented in Table 4, students who have same sub-dimension categories can belong to different groups for their total attitude scores. For example although x_{11} and x_{12} have moderate attitudes in confidence and anxiety sub-dimensions and high attitudes in usefulness and effectance motivation sub-dimensions, they belong to different groups in terms of their total attitude scores. As four sub-dimensions explain for the total attitude score, for example while x_{12} belongs to the moderate attitude group, x_{12} could be claimed to have a potential high attitude. In the following sections, the effects of sub-dimension categories on the total attitude scores for the students as presented in Table 4 will be analysed based on rough set theory.

ROUGH SET ANALYSIS FOR PRESENT STUDY

Data in rough set and knowledge representation should be arranged as condition attributes and the decision attribute. The data in Table 4 were thus arranged accordingly. The sub-dimensions were identified as condition attributes while total attitude score was identified as the decision attribute. In terms of the decision attribute, the students belong to a "Low, Moderate or High" group. This is presented in Table 5. Likewise, the "attitude information system" in the present study is presented in Table 1.

Indiscernibility relation for present study

When the condition attributes in Table 1 is considered, equivalence relation R divides student set IU into the following equivalence classes:

IU/R ={{x_1, x_2}, {x_3, x_4}, {x_5, x_6, x_{15}}, {x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}}, {x_{14}}, {x_{16}, x_{17}}, {x_{18}, x_{19}},{x_{20}}}

Lower and upper approximation sets are important concepts of rough set theory defined by the help of its equivalence relation.

Lower and upper approximations for present study

In the present study, three separate rough sets can be formed. Lower and upper approximation sets which can

Table 2. Average scores for sub-dimensions and total.

Students	Confidence	Anxiety	Usefulness	Effectance motivation	Attitude (Total)
x_1	2.75	2.83	3	3	2.9
x_2	3	3	3	3	3
x_3	2.67	2.08	2,17	2.75	2.42
x_4	2.58	2.33	1,83	3	2.44
x_5	2.58	2.33	2,67	2.67	2.56
x_6	2.5	2.32	2,83	2.92	2.65
x_7	2.32	2,08	2,67	3	2.52
x_8	2.32	1.83	2,67	3	2.46
x_9	2.25	2.33	2,42	2.83	2.46
x_{10}	2.33	1.92	2,92	2.83	2.5
x_{11}	2.33	2.08	2,67	3	2.52
x_{12}	2.08	2.17	2,42	2.54	2.3
x_{13}	2.08	1.75	2,42	3	2.31
x_{14}	2.33	2.42	2,17	2.25	2.29
x_{15}	2.4	2	2,4	2.5	2.32
x_{16}	2.17	2.08	2,33	2	2.15
x_{17}	2	2	2	2	2
x_{18}	1.42	2.25	1,6	1.64	1.71
x_{19}	1.58	1.75	1,33	1.17	1.46
x_{20}	1.58	1.08	1,42	1	1.27

Table 3. Group boundary values.

Interval boundary value	1 - 1.66	1.67 - 2.33	2.34 - 3
Group	Low	Moderate	High

Table 4. Student categories in terms of sub-dimensions and total attitudes.

Students	Confidence	Anxiety	Usefulness	Effectance motivation	Attitude (Total)
x_1	High	High	High	High	High
x_2	High	High	High	High	High
x_3	High	Moderate	Moderate	High	High
x_4	High	Moderate	Moderate	High	High
x_5	High	Moderate	High	High	High
x_6	High	Moderate	High	High	High
x_7	Moderate	Moderate	High	High	High
x_8	Moderate	Moderate	High	High	High
x_9	Moderate	Moderate	High	High	High
x_{10}	Moderate	Moderate	High	High	High
x_{11}	Moderate	Moderate	High	High	High
x_{12}	Moderate	Moderate	High	High	Moderate
x_{13}	Moderate	Moderate	High	High	Moderate
x_{14}	Moderate	High	Moderate	Moderate	Moderate
x_{15}	High	Moderate	High	High	Moderate
x_{16}	Moderate	Moderate	Moderate	Moderate	Moderate
x_{17}	Moderate	Moderate	Moderate	Moderate	Moderate
x_{18}	Low	Moderate	Low	Low	Moderate
x_{19}	Low	Moderate	Low	Low	Low
x_{20}	Low	Low	Low	Low	Low

Table 5. Condition and decision attributes of attitude data.

Label	Attribute	Domain
C	Confidence	1 - 3
A	Anxiety	1 - 3
U	Usefulness	1 - 3
E	Effectance motivation	1 - 3
Dec	Class	Low, Moderate, High

identify students who certainly belong or has the possibility of belonging to any of the high, moderate and low attitude groups (abbreviated high, moderate, low) are determined.

Lower and upper approximations of high set

The set $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}$ is the set of students classified as high. Lower and upper approximations of this set are formed as the following:

$R_{low}(H)= \cup_{a\in IU} \{ R(a) : R(a) \subset H\}=\{x_1, x_2\}\cup\{x_3, x_4\}=\{ x_1, x_2, x_3, x_4\}$.

$R^{up}(H)= \cup_{a\in IU} \{ R(a) : R(a)\cap H\neq\emptyset\}=\{x_1, x_2\}\cup\{x_3, x_4\}\cup\{x_5, x_6, x_{15}\}\cup\{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}= \{ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{15}\}$.

Set of Utilitarian students can be schematized as in Figure 2. Figure 2 shows that in the upper approximation of the high set, there are elements $\{x_{12}, x_{13}, x_{15}\}$ which do not belong to the high set. While students who belong to set $\{x_{12}, x_{13}, x_{15}\}$ are moderate, since they are members of the set $R^{up}(H)$, they may be regarded as potentially high. Members of the lower approximation set $\{x_1, x_2, x_3, x_4\}$ are composed of students who are certainly high.

Lower and upper approximations of moderate set

Set of moderate and lower and upper approximation sets are formed as the following:

Moderate= $\{x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}\}$.
$R_{low}(M) = \cup_{a\in IU} \{ R(a) : R(a) \subset M\}=\{x_{14}\}\cup\{x_{16}, x_{17}\}= \{x_{14}, x_{16}, x_{17}\}$.

$R^{up}(M) = \cup_{a\in IU} \{ R(a) : R(a)\cap M\neq\emptyset\}=\{x_5, x_6, x_{15}\}\cup\{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}\cup\{x_{14}\}\cup \{x_{16}, x_{17}\}\cup\{x_{18}, x_{19}\}= \{x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}\}$.

Although the members of the set $\{x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{19}\}$ do not belong to moderate set, they are members of the upper approximation set. Therefore, these members are potentially moderate. Consequently, high $\{x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}$ and low $\{x_{19}\}$ students may be said to be potentially moderate. Lower approximation set $\{x_{14}, x_{16}, x_{17}\}$ is composed of students who are certainly moderate.

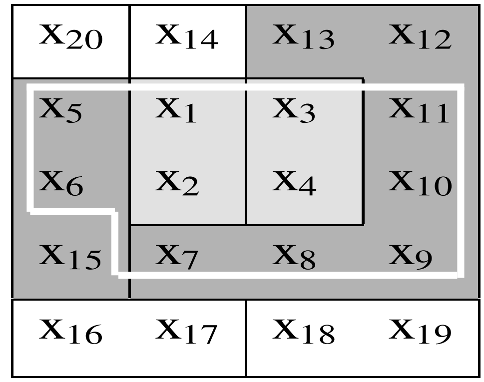

Figure 2. Rough symbolization of high set. White line, H; Dark and light grey region, $R^{up}(H)$; Light grey region, $R_{low}(H)$.

Lower and upper approximations of low set

Lower and upper approximations of the set $\{x_{19}, x_{20}\}$ which consists of low attitude students are presented below:

$R_{low}(L) = \cup_{a\in IU} \{R(a) : R(a) \subset L\}=\{x_{20}\}$

$R^{up}(L) = \cup_{a\in IU} \{ R(a) : R(a)\cap L\neq\emptyset\}=\{x_{18}, x_{19}\}\cup\{x_{20}\}= \{x_{18}, x_{19}, x_{20}\}$

The set of students who are not members of the low set but are members of the upper approximation set is $\{x_{18}\}$. Although this member is moderate, since it is a member of the upper approximation set, it can be regarded as potentially low. Since student x_{20} is a member of the lower approximation set according to the theory of rough set it is certainly low. Since the boundary sets of these three sets, $B_R(H)$, $B_R(M)$ and $B_R(L)$, are non-empty, the sets of H, M and L are rough sets. Also, the corresponding coefficients for the present study are calculated as the following:

$$\alpha_R(H) = \frac{4}{14} \cong 0{,}285\,,$$

$$\alpha_R(M) = \frac{3}{15} = 0{,}200\,,$$

$$\alpha_R(L) = \frac{1}{3} \cong 0{,}333$$

This shows that, sub dimensions of attitude scale partially explain total attitude score.

Moreover, high, moderate, low sets and the lower and upper approximations of these sets could be schematically represented as in Figure 3.

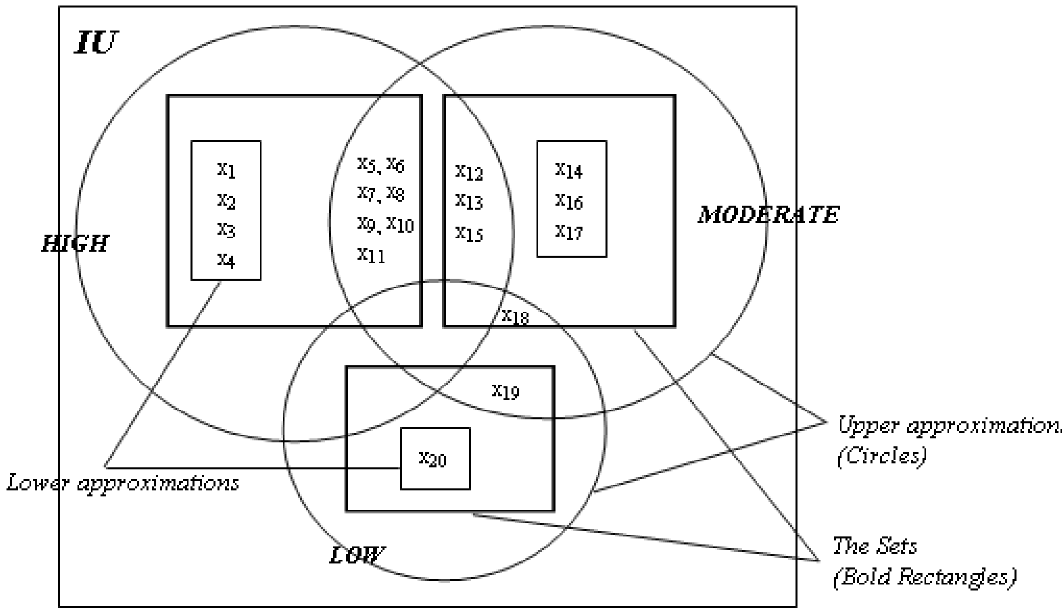

Figure 3. Schematic representation of interrelations between attitude groups.

Table 6. Approximation qualities.

Attributes	γ
C	0.000
A	0.050
U	0.000
E	0.150
C, A	0.200
C, U	0.250
C, E	0.150
A, U	0.200
A, E	0.300
U, E	0.250
C, A, U	0.400
C, A, E	0.300
C, U, E	0.250
A, U, E	0.400

Dependency of attributes for present study

In the present study, letting P = {C, A, U, E} and Q = {Dec}, positive region and the degree of dependency can be found asp:

$POS_P(Q) = \bigcup_{X \in IU/R(Q)} R(P)_{low}(X)$ ={ x_1, x_2, x_3, x_4}∪{x_{14}, x_{16}, x_{17}}∪{x_{20}}= {x_1, x_2, x_3, x_4, x_{14}, x_{16}, x_{17}, x_{20}},

$$\gamma(P,Q) = \frac{|POS_P(Q)|}{|IU|} = \frac{8}{20} = 0{,}400$$

This value represents to what extent C, A, U and E sub-dimensions jointly explain the total attitude score. Moreover, to what extent each sub-dimension on its own or any two or three sub-dimensions jointly explain for the total attitude score could be calculated. These values are presented in Table 6.

Reduction of attributes for present study

For the present study, when P = {C, A, U, E}, attributes C and E are dispensable. It is apparent that {C, A, U} and {A, U, E} sets are reducts of P. Therefore:

Red(P)={{C, A, U}, {A, U, E}}, $Core(P) = \bigcap \mathrm{Red}(P) = \{A, U\}$

Thus, A and U sub-dimensions could be considered as the indispensible sub-dimensions for this attitude scale.

Conclusion

Loslever and Lepoutre (2004) argue that humans have inherent multivariate and sophisticated behaviours. Thus, it may not always be straightforward to classify humans into categories of ‘crisp’ and ‘definite’.

In educational research, qualitative or quantitative, most commonly used statistical analysis procedures are descriptive statistics, t-test, ANOVA/MANOVA, correlation, regression and psychometric statistics (Hsu, 2005). The present study used a mathematical analysis approach to an educational research involving quantitative data. This could be considered as a new approach in quantitative data analysis. Consequently, the present study attempts to show the applicability of rough set theory in detailed analyses of quantitative and categorical data. The theory of rough sets is employed mostly in areas such as artificial intelligence, machine learning, pattern recognition, decision support systems, expert systems, data analysis and data mining.

Students who already belong to one of the three categories of attitude identified in this study may be included in any of the other categories and this possibility can be determined mathematically as explained in this paper. The degree of accuracy of approximation, $\alpha_R(X) = \frac{|\,\mathrm{Rlow(X)}\,|}{|\,\mathrm{Rup(X)}\,|}$, was calculated for each of three categories. The results indicate that there were 8 students who certainly belonged to any one of the typologies. No crisp boundaries could be drawn for the remaining 12 students. Therefore, it was concluded that 12 students displayed the characteristics of different categories and the boundaries of their attitudes toward mathematics were not clear. In addition, the degree of accuracy of approximation values, $\alpha_R(H) \cong 0{,}285$, $\alpha_R(M) = 0{,}200$, $\alpha_R(L) \cong 0{,}333$ were closer to zero which implied that the sets H, M and L were far from being exact. This also explained the case of 12 students. Furthermore, anxiety and usefulness sub-dimensions were observed to be indispensable, while confidence and effectance motivation sub-dimensions were observed to be superfluous sub-dimensions. Therefore, rough set analysis promises to be the only method that generates the results to reach these conclusions.

In conclusion, the results of the present study indicate that adopting the rough set approach to analyse educational research data for the investigation of attitudes, behaviours or beliefs could reveal more comprehensive information about the data and hence about the characteristics of the participants involved. Moreover, precise knowledge about learners' attitudes and beliefs could guide curricular and instructional studies.

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