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Full Length Research Paper

Design of a hybrid type-2 fuzzy logic/proportional-integral controller for single-phase three-wire inverter system

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This paper presents the design of a hybrid type-2 fuzzy logic (T2FL) / proportional-integral (PI) controller proposed to solve harmonic and balancing problems in the output voltage of a single-phase three-wire inverter system. The inverter system consists of a converter circuit with three legs and an auxiliary LC filter, and a voltage-based feedback control algorithm including a differential-mode and common-mode controllers. In this algorithm, a T2FL controller and a PI controller whose gain coefficients are optimized by particle swarm optimization (PSO) algorithm are used as the differential-mode controller and common-mode controller, respectively. To evaluate the effects on system performance of the proposed hybrid controller, the inverter is also controlled using traditional PI/PI controller and hybrid type-1 fuzzy logic (T1FL) / PI controller. Simulation results show that the hybrid T2FL/PI controlled inverter has faster dynamic response in the case of step load changes, and gives a better sinusoidal alternating current (AC) output voltage and a lower neutral line voltage under unbalanced linear and non-linear load conditions than the both conventional PI/PI and hybrid T1FL/PI controlled inverters.

Key words: Proportional-integral controller, type-1 fuzzy logic controller, type-2 fuzzy logic controller, hybrid controller, particle swarm optimization algorithm, single phase three-wire inverter.

INTRODUCTION

Nowadays, single phase three-wire inverter systems have been widely used in various low power applications such as uninterruptible power supply (UPS), photovoltaic power conditioner and battery energy storage system (BESS). In particular, these systems are preferred for home and office applications since they have two separate voltage levels, small volume, low weight and low cost. The main challenge for single phase three-wire inverter systems is to cope successfully with the control problems related to sinusoidal wave shaping, load regulation and keeping voltage balance between the two sub-outputs (Chiang and Liaw, 1994a; Liaw and Chiang, 1994b; Yeong-Chau, 2003). In the past, many advanced control techniques was proposed to overcome these problems. The techniques developed by Liaw et al. (1993) and Divan et al. (1990) can lead to load insensitive voltage regulating control performance, but it seems that they are not suitable for handling nonlinear loads. Kawamura and Yokoyama proposed some real-

Abbreviations: PI, Proportional-integral; **T2FL,** type-2 fuzzy logic; **PSO,** particle swarm optimization; **T1FL,** type-1 fuzzy logic; **AC,** alternating current; **UPS,** uninterruptible power supply; **BESS,** battery energy storage system; **GA,** genetic algorithm; **DC,** direct current; **PWM,** pulse width modulation; **NB,** negative big; **NS,** negative small; **Z,** zero; **PS,** positive small; **PB,** positive big; **NVB,** negative very big; **NM,** negative medium; **PM,** positive medium; **PVB,** positive very big; **COS,** center of set; **IAE,** integrated absolute error; **ITSE,** integrated of time weight square error; **ISE,** integrated of squared error; **VS,** voltage sags.

time digital feedback control approaches to overcome the problems caused by nonlinear loads (Kawamura, 1990). Sampling a head preview control with on-line parameter estimation was proposed for producing low harmonic and fast transient response sinusoidal voltage ((Gokhale et al., 1985). On the other hand, Vokosavic et al. (1990) developed a switching algorithm using output-filter state feedback to improve the waveform of inverter voltage and also to reduce its output impedance. One of the most popular controller applications used in control of industrial power systems is conventional PI controllers because of their simple structures, easy designs, and inexpensive costs (Li, 1998; Er and Sun, 2001).

However, due to their fixed proportional gains and integral time constants, the performance of PI controllers are affected by parameter variations and load disturbances, especially under nonlinear load conditions. So far, several tuning methods such as Ziegler-Nichols rule, Cohen-Coon rule and so on (Ziegler and Nichols, 1942; Astrom and Hagglund, 1995; Halevi et al., 1997) have been proposed for setting the gain parameters of these controllers, but they did not provide the desired effect on controller performance because they rely on a minimum amount of dynamic information about the controlled process. Developing a controller structure that can tune its own parameters online will provide a significant contribution to solve these problems. Recently, genetic algorithm (GA) and PSO algorithm has been frequently used as a gain scheduler in traditional controllers to obtain the desired performance from the controlled system (Gaing, 2004; Mukherjee and Ghoshal, 2007; Jain and Nigam, 2008; Atacak and Bay, 2009; Griffin, 2003). GA is a stochastic global search optimization technique based on the mechanisms of natural selection. The advantage of tuning with GA is the ability of choosing controller gains which optimize system performance based on multi-objective criterion without tripping in a local minima solution (Gadoue et al., 2005); Moghadasian and Alenasser, 2011). PSO is a heuristic-based algorithm for finding optimal regions of complex search spaces through the interaction of individuals in a population of particles (Clerc, 2002). When compared with GA, the main advantage of PSO is that its paradigm can be implemented in simple form of computer codes and is computationally inexpensive in terms of both memory requirements and speed. Therefore, it has been more widely used in many areas than GA (Cui et al., 2010; Zhou et al., 2006). Another type of controller proposed for control problems in nonlinear systems is T1FL controller. This controller has been successfully applied to many different fields, especially when the system is too complex for analysis by conventional mathematical techniques (John and Langari, 1998; King and Mamdani, 1977).

While designing a T1FL controller, an expert's experience and knowledge are needed to determine parameters related to the rule base and membership functions. Because the parameters refer to the different meanings for different experts, the occurrence of uncertainty problems is inevitable in this controller (Karnik et al., 1999). In order to cope with these problems in type-1 fuzzy systems, Zadeh (1975) introduced the concept of a new class sets named as type 2 fuzzy sets in 1975. These sets are an extension of type-1 fuzzy sets where uncertainty is represented by the addition of an extra dimension, which gives more degrees of freedom for better representation of uncertainty when compared to type-1 fuzzy sets. In recent years, there has been increasing interest in type-2 fuzzy based control. Researches on this field have shown that type-2 fuzzy systems have a better effect on the performance of the controlled system than type-1 fuzzy systems in circumstances, where it is difficult to determine parameters associated with the rule base and membership functions (Hagras, 2004; Sepulveda et al., 2007; Lin et al., 2009; Atacak and Bay, 2010; Mendez et al., 2010; Li et al., 2010). In this study, a hybrid controller that combines the advantages of T2FL controllers with PSO-based PI controllers is proposed to minimize harmonic and voltage imbalancing problems in a single-phase three-wire inverter system. The organization of this study is as follows: In Section 2, the structure and model of single-phase three-wire inverter system, and the design of controllers are given. The results obtained from simulation studies are presented in section 3. In the last section, the performance of the proposed controller is evaluated by using these results.

MATERIALS AND METHODS

Structure and model of the single phase three-wire inverter system

Figure 1 shows the power circuit of the single-phase three-wire inverter system which consists of a direct current (DC) /AC converter with three-leg power switches, LC output filters, and loads in the output terminals. In this circuit configuration, the power switches in legs "a" and "c" are used to shape and regulate the main output voltage between A and B terminals, while those in leg "b" allow the compensation of the voltage imbalances caused by the unbalanced loads connected to split-phase outputs "AN" and "BN". The performance of inverter system is tested under linear and nonlinear load conditions. In the inverter outputs, a resistor and a diode rectifier with input inductance, output capacitance and output resistive load are used as linear and nonlinear loads, respectively. The circuit can be used to feed both the 110V load and 220 V load because it has two split-phase outputs. This inverter topology is the same as the three phase six-switch inverter but in this topology, middle phase leg is connected to the ground instead of a phase of the output with a capacitor, in order to make the simulation studies of system, it is necessary to obtain the dynamic equations that describe the inverter and loads. For this purpose, as the first process, the equivalent circuit model of single-phase three-wire inverter is created. As in many power electronics systems, inverter exhibits a nonlinear characteristic due to constantly changing

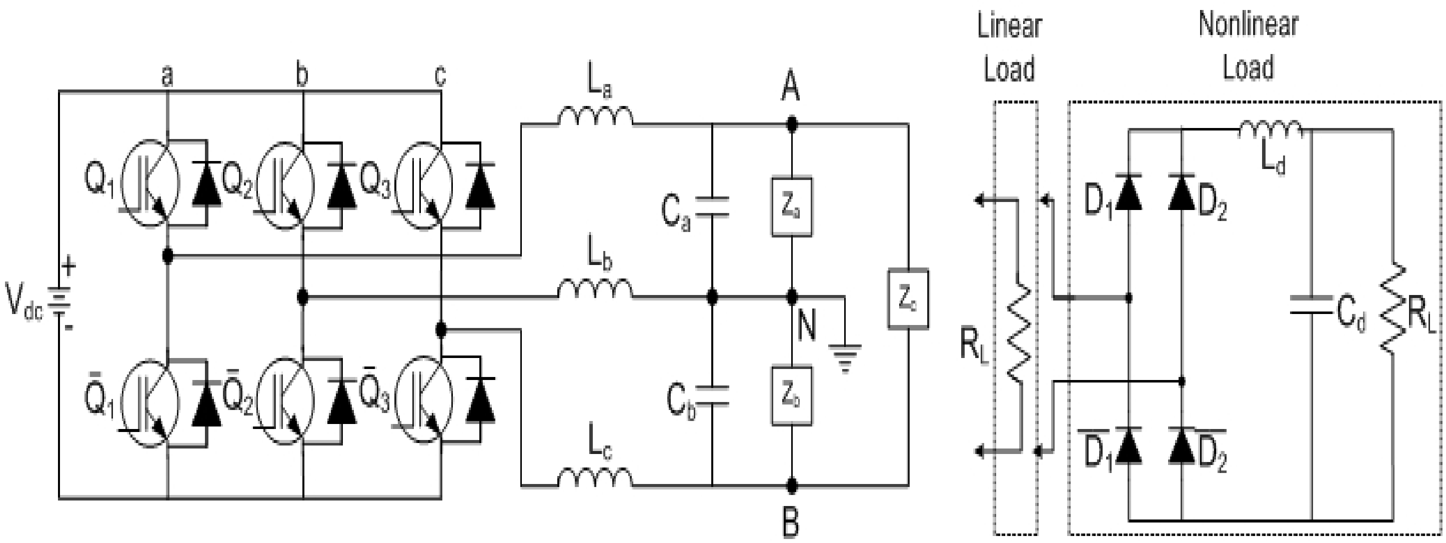

Figure 1. Power circuit of the single-phase three-wire inverter system.

system parameters. In modeling of the systems with nonlinear characteristic, switching model is not preferred much because of disadvantages such as compromising the numerical convergence of solution when introduced in a large system, long simulation time and interactions which can lead to instability or fault cases when the switching-power system is integrated together.

Thus, the best approach for creating a model of these type systems is to use an average model and to switch converter at significantly higher frequency than its fundamental frequency. While deriving the average model of single-phase three-wire inverter, it is assumed that the diodes and IGBT's in the power circuit are ideal. $Q1-Q3$ and $\bar{Q}1 - \bar{Q}3$ IGBT's work in a symmetrical with each other due to bipolar PWM switching scheme used for switching IGBTs. The model obtained from single-phase three-wire inverter system is shown in Figure 2. The most important difference between this model circuit and the power circuit given in the Figure 1 is interchange of IGBTs with their average models. Where C_A and C_B are the output filter capacitors, L_a, L_b and L_c are the output filter inductors, r_a, r_b and r_c are the inductor series resistances, V_{dc} is the inverter input voltage, and V_{AN}, V_{BN} and V_{AB} are the inverter output voltages. S_a, S_b and S_c are the switching functions which represent the turn-on and turn-off states of IGBTs in each leg. The state-space equations describing system dynamics can be found by applying Krammer's rule to the equations which are obtained through the direct application of Kirchoff's current and voltage laws to the equivalent circuit given in Figure 2.

$$\frac{di_a}{dt} = \frac{-(L_b+L_c).V_{AN} + L_b.V_{BN} - (L_b+L_c).r_a.i_a - L_b.r_c.i_a + L_c.r_b.i_b - L_b.r_c.i_b + (L_b+L_c).V_{dc}.S_a - L_c.V_{dc}.S_b - L_b.V_{dc}.S_c}{L_a.L_b + L_a.L_c + L_b.L_c} \quad (1)$$

$$\frac{di_b}{dt} = \frac{L_c.V_{AN} + L_a.V_{BN} + L_c.r_a.i_a - L_a.r_c.i_a - (L_a+L_c).r_b.i_b - L_a.r_c.i_b - L_c.V_{dc}.S_a + (L_a+L_c).V_{dc}.S_b - L_a.V_{dc}.S_c}{L_a.L_b + L_a.L_c + L_b.L_c} \quad (2)$$

For the output load with resistor, the state-space equations belonging to the split-phase outputs of inverter are given as follow.

$$\frac{dV_{AN}}{dt} = -\frac{V_{AN}}{R_L.C_A} + \frac{i_a}{C_A} \quad (3)$$

$$\frac{dV_{BN}}{dt} = \frac{-V_{BN}}{R_L.C_B} - \frac{(i_a+i_b)}{C_B} \quad (4)$$

For the output load with diode rectifier, the derived state-space equations of split-phase outputs and this load can be written as follow.

$$\begin{aligned} &\frac{di_{da}}{dt} = \frac{|V_{AN}| - V_{CLa}}{L_{da}} \quad \text{if}(|V_{AN}| > V_{CLa}), && i_{Lda} = i_{da} \quad \text{if}(V_{AN} > 0 \ \textit{then}) \\ &\frac{di_{da}}{dt} = 0 \quad \text{otherwise} && i_{Lda} = -i_{da} \quad \text{elseif}(V_{AN} < 0 \ \textit{then}) \\ & && i_{Lda} = 0 \quad \textit{othewise} \end{aligned} \quad (5)$$

$$\begin{aligned} &\frac{di_{db}}{dt} = \frac{|V_{BN}| - V_{CLb}}{L_{db}} \quad \text{if}(|V_{BN}| > V_{CLb}), && i_{Ldb} = i_{db} \quad \text{if}(V_{BN} > 0 \ \textit{then}) \\ &\frac{di_{db}}{dt} = 0 \quad \text{otherwise} && i_{Ldb} = -i_{db} \quad \text{elseif}(V_{BN} < 0 \ \textit{then}) \\ & && i_{Ldb} = 0 \quad \textit{othewise} \end{aligned} \quad (6)$$

$$\frac{dV_{Cda}}{dt} = \frac{I_{da} - {}^{V_{Cda}}\!/_{R_{La}}}{C_{da}}, \qquad \frac{dV_{Cdb}}{dt} = \frac{I_{db} - {}^{V_{Cdb}}\!/_{R_{Lb}}}{C_{db}} \quad (7)$$

$$\frac{dV_{AN}}{dt} = \frac{i_a - i_{Lda}}{C_A}, \qquad \frac{dV_{BN}}{dt} = -\frac{(i_a + i_b + i_{Lda})}{C_B} \quad (8)$$

The voltages dropped across diodes are neglected above the equations because they are assumed ideal.

Design of the controllers

The control of inverter is carried out by using a voltage-based

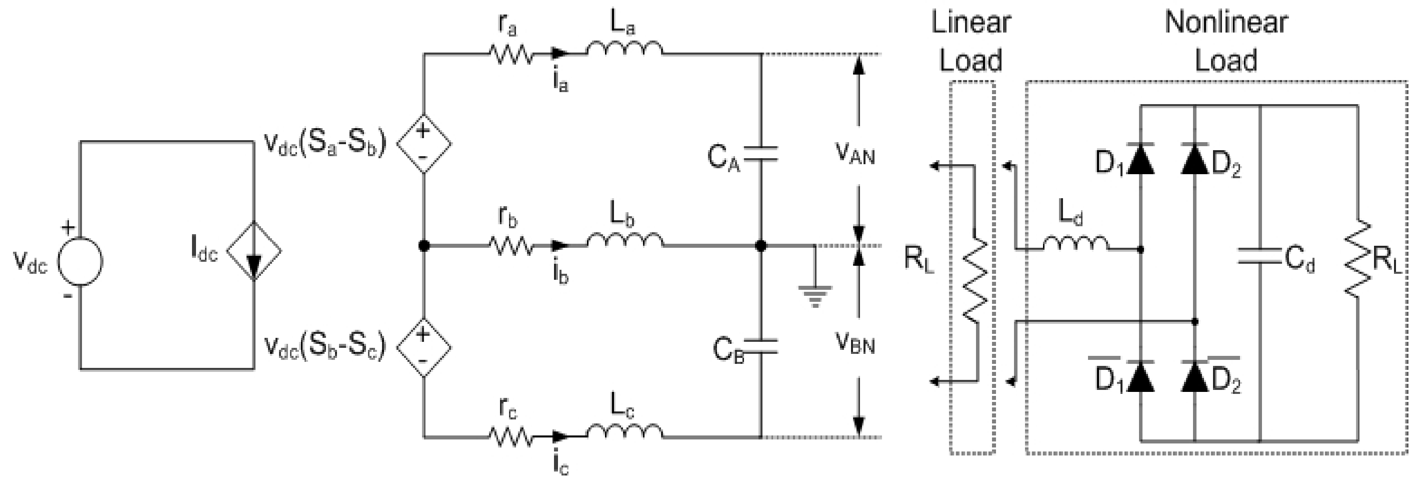

Figure 2. Average model of the single-phase three-wire inverter system.

feedback control algorithm. As shown in Figure 3, the proposed algorithm consists of a differential-mode controller, a common-mode controller and a bipolar pulse width modulation (PWM) generator. From the point of control functions to both controllers, the main task of differential mode controller is to produce the main AC voltage with good voltage regulating characteristics, whereas the common mode controller is used to maintain the voltage balance between two split-phase outputs even when the loads connected to these outputs are unbalanced. Outputs of these controllers provide the production of PWM signals which are required to switch IGBTs in the differential mode sub-system comprising legs "a" and "c", and the common mode sub-system including leg "b". This function is achieved by the bipolar PWM generator comparing these outputs with the carrier signal, which is often either a saw tooth or triangular wave form. Since the inverter is controlled with respect to the main output voltage V_{AB} and neutral line voltage V_N, the input variables for both controllers consist of the error and change in the error concerning these voltages. These variables can be given as the main voltage error e_d and change in the error Δe_d for the differential-mode controller, and the neutral line voltage error e_c and change in the error Δe_c for the common-mode controller. Because the input equations for both controllers resemble each other, only ones for the differential-mode controller are defined.

$$e_d(k) = V_{dref}(k) - V_{AB}(k) \tag{9}$$

$$\Delta e_d(k) = e_d(k) - e_d(k-1) \tag{10}$$

Where $V_{dref}(k)$ is the amplitude of reference output voltage at the k^{th} sampling instant and $e_d(k-1)$ is the amplitude of output voltage error at the $(k\text{-}1)^{th}$ sampling instant. The main output voltage to be obtained from A and B terminals is an AC voltage of 220V/50 Hz. Therefore, the reference variable for differential-mode controller is assigned as a sinusoidal function with the mentioned values. This variable for common-mode controller is taken as 0 V, not to be any voltage drop across the neutral line.

Hybrid T1FL/PI controller

Block diagram of the hybrid T1FL/PI controller used for the control of single-phase three-wire inverter is shown in Figure 4. The controller scheme mainly consists of a T1FL controller structure working as the differential-mode controller together with a discrete PI controller used as common-mode controller. T1FL controller performs the forming and regulating functions of main output voltage by using its four sections: fuzzifier, rule base, inference engine, and defuzzifier. In this structure, the fuzzifier translates crisp inputs into linguistic values using a membership function. According to knowledge of the control rules and the linguistic variable definition, a fuzzy control action is derived in the inference section. In the last section, the inferred fuzzy control action is converted to a crisp control action by the defuzzifier section. The output of T1FL controller determines change in the reference waveform required for the first input of bipolar PWM generator. As shown in the following equation, this output can be calculated by adding the previous value of reference waveform $u_d(k-1)$ to the calculated value of change in reference waveform $\Delta u_d(k)$.

$$u_d(k) = u_d(k-1) + \Delta u_d(k) \tag{11}$$

The basic criteria that determine the performance of T1FL controllers are their membership functions and rule base parameters. Thus, in the design procedures of T1FL controllers, defining these parameters according to the characteristics of the controlled system has a vital importance in terms of their performances.

The membership functions for the output error and change in the output error consist of Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB) fuzzy sets as shown in Figure 5a. The type of membership function for these sets is taken as a triangular form because it is the simplest and most

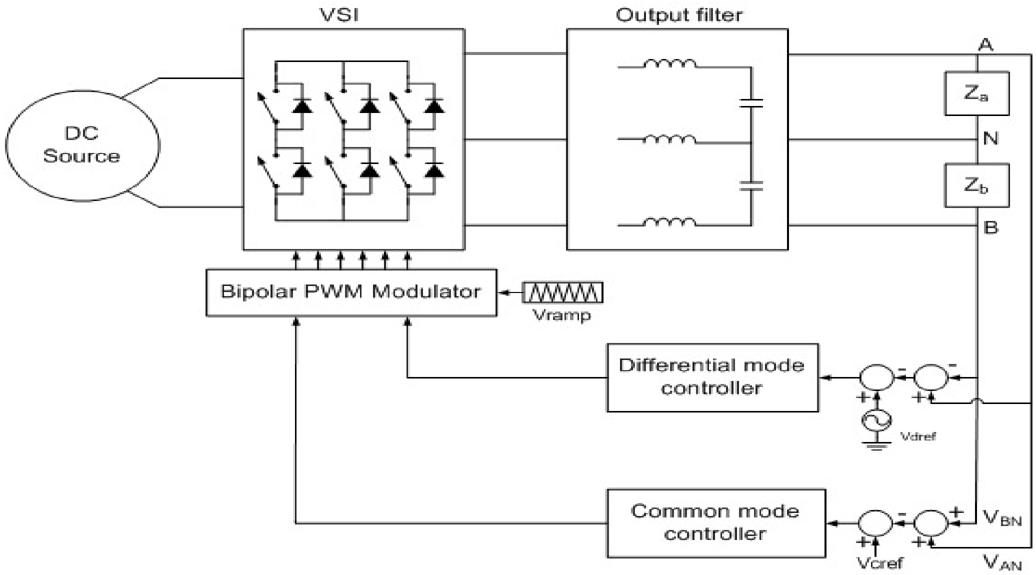

Figure 3. Control algorithm used in control of the single-phase three-wire inverter system.

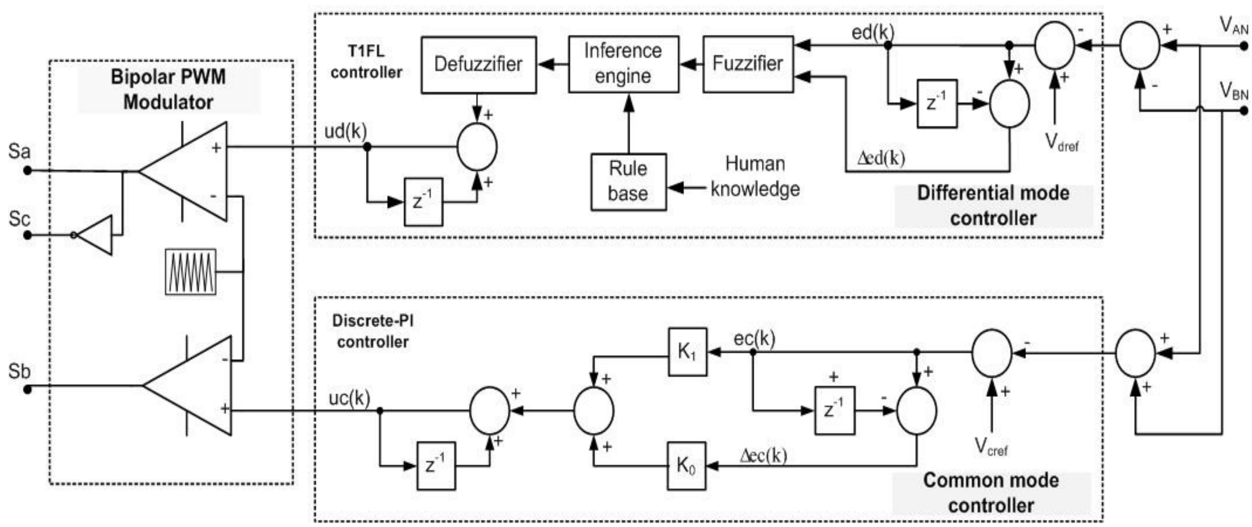

Figure 4. Block diagram of the hybrid T1FL/PI controller used in the control of the single-phase three-wire inverter.

efficient form for many applications. As shown in Figure 5b, singleton membership function is used for the output membership functions due to easy calculation. The value of the input and output variables is normalized in [-1, 1] by using suitable scale factors. The fuzzy control rules for the controller are obtained by analyzing the behavior of the controlled system. While deriving these rules from the dynamic behavior of the system, the following two concepts are taken into account to achieve a better controller performance: (1) in order to provide the stability of the system on the working point, the controller must be exactly maintain the system output when the output error and change in the output error is zero: (2) the controller must quickly correct the deviation occurring in the system output depending on the output error and change in the output error whenever the output voltage deviates from the reference value. Each fuzzy rule of T1FL controller is in form: R^i: IF e_d is $F^i_{e_d}$ and Δe_d is $F^i_{\Delta e_d}$ THEN Δu_d is w^i. Where $F^i_{e_d}$ and $F^i_{\Delta e_d}$ are fuzzy sets in their universe of discourse and w^i is a fuzzy singleton. Table 1 shows the 25 fuzzy control rules derived from the above concepts. Although there have been several fuzzy inference method such as Mamdani, Larsen, and Takagi Sugeno fuzzy implications, the inference operations, connected to rules, is

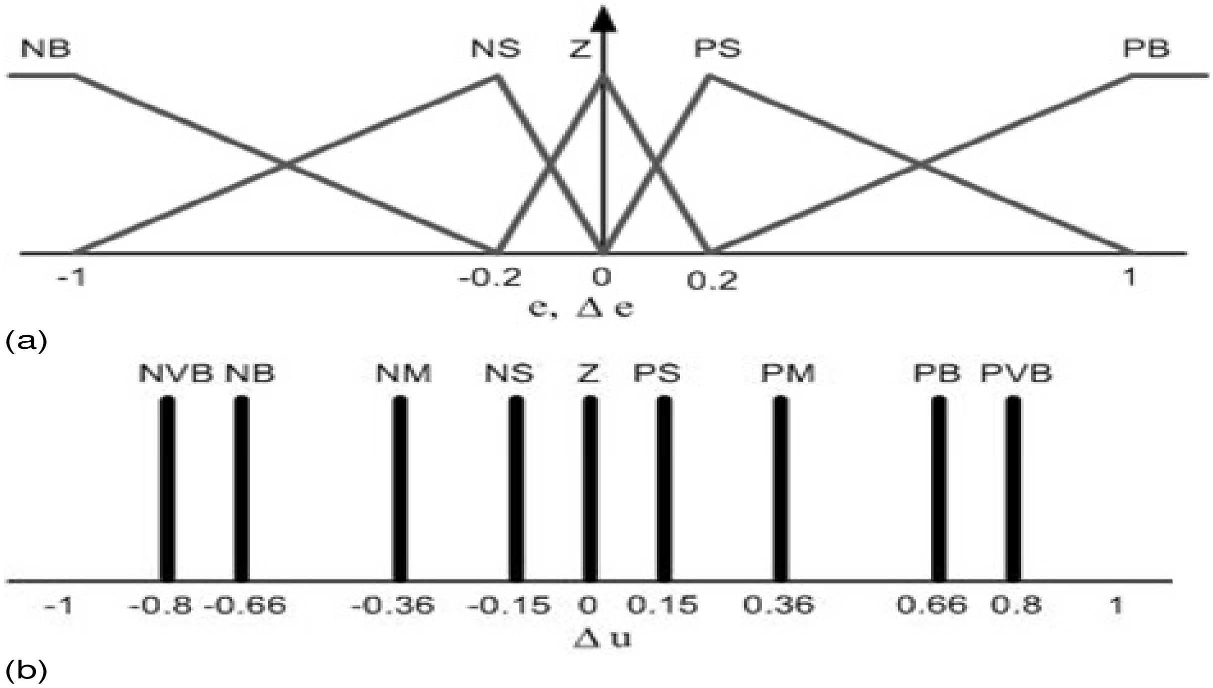

Figure 5. Input and output membership functions used in the T1FL controller; **a** membership functions for input variables; **b** membership functions for output variable.

Table 1. Fuzzy control rules used in the T1FL controller.

		e				
		NB	**NS**	**Z**	**PS**	**PB**
Δe	NB	NVB	NB	NO	NS	Z
	NS	NB	NO	NS	Z	PS
	Z	NO	NS	Z	PS	PO
	PS	NS	Z	PS	PO	PB
	PB	Z	PS	PO	PB	PVB

performed by using Mamdani's MIN implication method whose equation is given as follow.

$$z_d^i = \min\left\{\mu_{F_{e_d}^i}(e_d), \mu_{F_{\Delta e_d}^i}(\Delta e_d)\right\}.w_d{}^i = f_d^i.w_d^i \qquad (12)$$

Where z_d is the fuzzy representation output of change in control effort, w_d is the degree of change in reference waveform, f_d is the weighting factor of change in reference waveform, and $\mu_{F_{ed}}$ and $\mu_{F_{\Delta ed}}$ are the membership degrees of F_{e_d} and $F_{\Delta e_d}$ fuzzy sets. To obtain the analogue reference value of bipolar PWM generator from the controller output, the fuzzy inference results should be subject to the defuzzification operation. The center of gravity method, which calculates the center of area of the inference mechanism output distribution, is used as defuzzification strategy in this study.

$$\Delta u_d = \frac{\sum_{i=1}^{N} f_d^i.w_d^i}{\sum_{i=1}^{N} w_d^i} \qquad (13)$$

In the hybrid configuration, the PI controller is used to produce PWM signals required for the voltage balance between the split-phase outputs of inverter. The output equation of a conventional PI controller in the time domain can be defined as follow.

$$u_c(t) = K_p * e(t) + K_i * \int_0^t e(\tau).d\tau \qquad (14)$$

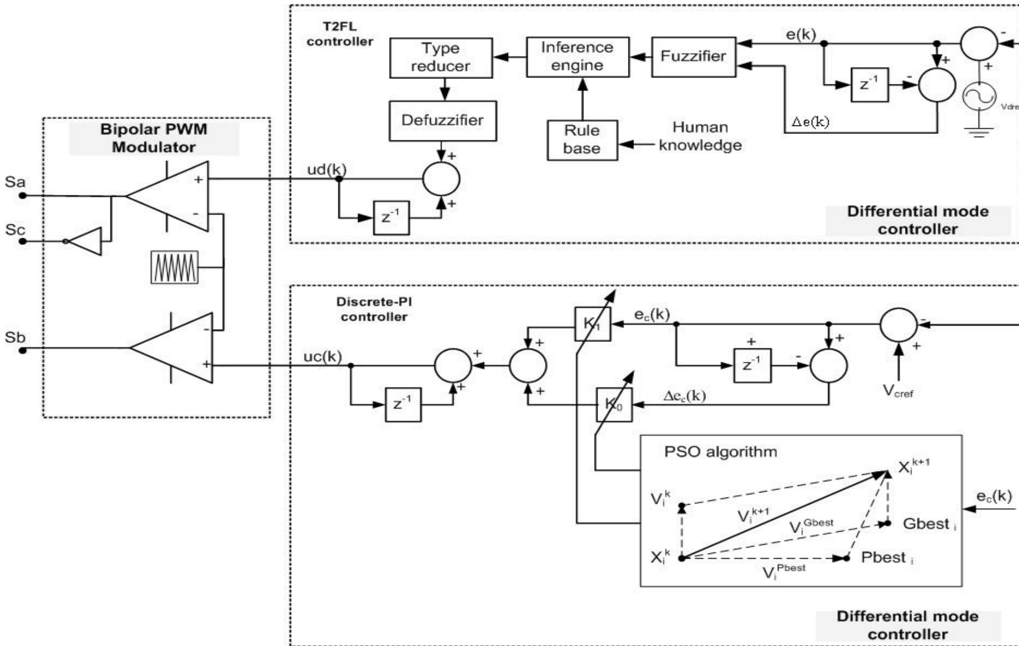

Figure 6. Block diagram of the hybrid T2FL/PSO-based PI controller proposed for control of the single-phase three-wire inverter.

Generally, in computer and microcontroller-based applications, the discrete time equations of PI controllers is used instead of the time-domain equations due to the realization of discrete time processes in these environments and the need of much processing time in integration process. The output equation of a PI controller in the discrete time domain, which is derived by applying the bilinear transform and inverse z transform to its output equation in the frequency domain, can be given as follows.

$$u_c(k) = u_c(k-1) + [K_p - \frac{K_i.T}{2}].\Delta e_c(k) + K_i.T.e_c(k) \quad (15)$$

Where, $u_c(k)$ and $u_c(k-1)$ are the control outputs of discrete PI controller at the k^{th} and $(k-1)^{th}$ sampling instants, respectively and T is the sampling period. To simplify the controller output equation, $[K_p - \frac{K_i.T}{2}]$ and $K_i.T$ are taken as K_0 and K_1, respectively. The K_p and K_i coefficients which determines the PI controller performance are obtained as 1 and 298 by using Ziegler-Nichols method.

Hybrid T2FL/PSO-based PI controller

Block diagram of the hybrid T2FL/PSO-based PI controller which is proposed to obtain a better voltage regulation and voltage balance from the inverter output is shown in Figure 6. In the proposed controller configuration, the T2FL and PSO-based PI controllers are used instead of the T1FL and fixed-gain PI controllers, respectively, in order to eliminate uncertainty and performance problems in these controllers. From the functional point of view, the both controllers used in the proposed hybrid controller are the same as the controllers in the hybrid T1FL/PI controller. T2FL controller is quite similar to T1FL controller, but there are two major differences: (1) the output processing of T2FL controller includes type reducer which converts the type-2 fuzzy output sets into type-1 sets; (2) T2FL controller uses the type-2 fuzzy sets in the fuzzification and inference processing. These characteristics allow T2FL controller to capture more information about uncertainties concerning fuzzy rules and sets than T1FL controller (Karnik et al., 1999; Liang and Mendel, 2000). As seen from the block diagram shown in Figure 6, the controller contains five sections which are fuzzifier, rule base, inference engine, type-reducer and defuzzifier. During the fuzzification process, all input data are converted into appropriate fuzzy values through type-2 fuzzy sets. According to the knowledge of the control rules and the values obtained from the fuzzifier output, a type-2 fuzzy control action is derived in the inference engine. The inference engine outputs are then processed by the type-reducer which combines the output sets and then performs a centroid calculation, which leads to type-1fuzzy sets called type-reduced sets (Mendel, 2001). Finally, the defuzzifier obtains the PWM signals used to work the differential mode sub-circuit by converting the type-reduced outputs to crisp value.

While designing the T2FL controller for the control of the mentioned sub-circuit, interval tip-2 triangular fuzzy sets is used in the fuzzification process of controller inputs e_d and Δe_d because they can enable the real-time operation of a dynamic system by means of their easy use and less computational function. These are defined by using 5 membership sets with the linguistic labels: NB, NS, Z, PS and PB, as shown in Figure 7a. For easy computation, output membership functions are described as singleton upper and lower control actions, which means that the upper and lower bounds of the sets are used in the inference process without being fuzzified. The output universe is divided into 9 singleton type-2

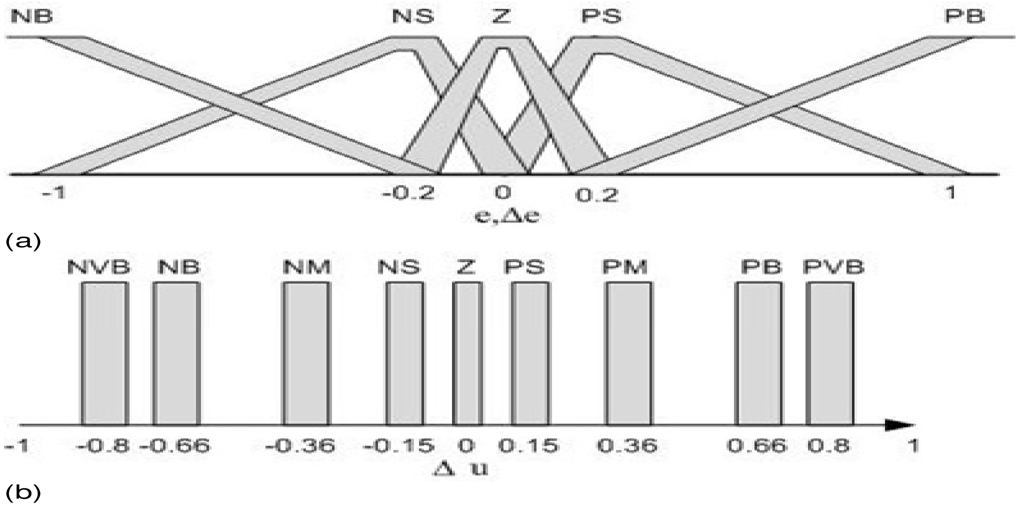

Figure 7. Type-2 membership functions for the T2FL controlled system; **a** membership functions for the input variables; **b** membership functions for the output variable.

fuzzy sets and labeled with Negative Very Big (NVB), NB, Negative Medium (NM), NS, Z, PS, Positive Medium (PM), PB and Positive Very Big (PVB). Figure 7b shows the membership functions defined for the controller output. The rule structure of T2FL controller resembles that of the T1FL controller, however the antecedents and consequents in the T2FL controller are represented by type-2 fuzzy sets. The rule structure used in this controller is given as follow:

R^i: IF e_d is $\tilde{F}^i_{e_d}$ and Δe_d is $\tilde{F}^i_{\Delta e_d}$ THEN Δu_d is $[w_l^i, w_r^i]$ (16)

Where $\tilde{F}^i_{e_d}$ and $\tilde{F}^i_{\Delta e_d}$ are the type-2 fuzzy sets defined for the both input variables, and w_l^i and w_r^i are the control outputs obtained from system actions. Since five interval type-2 fuzzy sets are assigned to each input, twenty five fuzzy rules are needed to designate both the upper and lower bounds of T2FL controller. Tables 2 and 3 show the upper and lower bounds of the fuzzy rules used in control of the differential mode sub-circuit.

The subscripts *u* and *l* given in Tables 2 and 3 represent the lower and upper bounds of the output fuzzy sets. The inference engine combines the fired rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. In the inference engine, antecedents in the rules are connected using the Meet operation, the membership grades in the input sets are combined with those in the output sets using the extended sup-star composition, and multiple rules are combined using the Join operation. Similar to type-1 fuzzy system, the firing strength can be obtained by the following inference process:

$$F^i = [\overline{f}^i, \underline{f}^i] \qquad (17)$$

$\overline{f}^i$ and $\underline{f}^i$ can be written as follow:

$$\overline{f}^i = \overline{\mu}_{\tilde{F}^i_{e_d}}(e_d) * \overline{\mu}_{\tilde{F}^i_{\Delta e_d}}(\Delta e_d) \qquad (18)$$

$$\underline{f}^i = \underline{\mu}_{\tilde{F}^i_{e_d}}(e_d) * \underline{\mu}_{\tilde{F}^i_{\Delta e_d}}(\Delta e_d) \qquad (19)$$

Where $\overline{\mu}$ and $\underline{\mu}$ denote the grade of upper and lower membership functions, respectively. Symbol $*$ is the t- norm operator (Atacak and Bay, 2010). In this study, the meet operation is realized by using a minimum t-norm operator. In T2FL controllers, the output of type-2 inference engine needs to be type reduced so that defuzzification procedure can be performed. Although there are many kind of type-reduction methods such as centroid, center of sets, height and modified height, the center of set (COS) type-reduction method is used in these procedure due to its reasonable computational complexity. For the T2FL controller, the type-reduced set can be expressed as follow:

$$\Delta u_{\cos} = \int_{w^1\in[w_l^1,w_u^1]}\cdots\int_{w^M\in[w_l^M,w_u^M]}\int_{f^1\in[\overline{f}^1,\underline{f}^1]}\cdots\int_{f^M\in[\overline{f}^M,\underline{f}^M]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i.w^i}{\sum_{i=1}^{M} f^i} = [\Delta u_l, \Delta u_r] \qquad (20)$$

Table 2. Upper bounds of the fuzzy rules for the T2FL controller.

		e				
		NB	**NS**	**Z**	**PS**	**PB**
	NB	NVB_u	NB_u	NM_u	NS_u	Z_u
Δ**e**	NS	NB_u	NM_u	NS_u	Z_u	PS_u
	Z	NM_u	NS_u	Z_u	PS_u	PM_u
	PS	NS_u	Z_u	PS_u	PM_u	PB_u
	PB	Z_u	PS_u	PM_u	PB_u	PVB_u

Table 3. Lower bounds of the fuzzy rules for the T2FL controller.

		e				
		NB	**NS**	**Z**	**PS**	**PB**
	NB	NVB_l	NB_l	NM_l	NS_l	Z_l
	NS	NB_l	NM_l	NS_l	Z_l	PS_l
Δ**e**	Z	NM_l	NS_l	Z_l	PS_l	PM_l
	PS	NS_l	Z_l	PS_l	PM_l	PB_l
	PB	Z_l	PS_l	PM_l	PB_l	PVB_l

Where $\Delta u_{\cos}$ an

interval type-1 fuzzy set with is left most point Δu_l and right most point Δu_r, which are calculated by using the Karnik–Mendel iterative procedure (Karnik and Mendel, 2001). The last processing step in T2FL controller is the defuzzification procedure, where a crisp output of the controller can be calculated by getting the average of Δu_l and Δu_r.

$$\Delta u_d = \frac{\Delta u_l + \Delta u_r}{2} \tag{21}$$

In order to achieve a better voltage balance between the split-phase outputs of inverter, PSO algorithm is adapted to discrete time PI controller for online tuning of its gains K_p and K_i. PSO is a swarm intelligence optimization technique that finds optimal solution depending on the behavior of particles in a multidimensional search space. In this study, the problem to be solved is to determine the optimal gain coefficients of PI controller, which eliminate the voltage drop on the neutral line when an unbalanced load is connected to the split phase outputs of inverter. Thus, this algorithm uses a two-dimensional solution space consisting of the particles K_p and K_i. Studies made on the inverter system showed that 20 particles for each dimension of solution space could provide a good controller performance. PSO is based on information-sharing of the candidate particles while reaching the optimal solution. Each particle modifies its position using the information such as current position and velocity, distance between the current position and best position of a particle, distance between the current position of a particle and best position of the group. The mathematical equations used to update the velocity and position information of particles can be given as follow.

$$v_{i,d}^{k+1} = w^k.v_{i,d}^k + c_1.r_{1,d}.(K_{i,d}best - K_{i,d}^k) + c_2.r_{2,d}.(K_{gd}best - K_{i,d}^k) \tag{22}$$

$$K_{i,d}^{k+1} = K_{i,d}^k + v_{i,d}^{k+1} \tag{23}$$

where $v_{i,d}$ is i^{th} particle velocity in the d^{th} dimension, w is inertia weight factor, c_1, c_2 are acceleration constants, $r_{1,d}, r_{2,d}$ are random numbers between 0 and 1in the d^{th} dimension, $K_{i,d}$ is i^{th} particle position in the d^{th} dimension, $K_{i,d}best$ is best position of the i^{th} particle, and $K_{gd}best$ is best position of the group in the d^{th} dimension. c_1 and c_2 are constants that affect the velocity of a particle depending on experience in itself and swarm, respectively. Researches related to the algorithm showed that the sum of these coefficients should be larger than 4 to achieve a better performance. The inertia weight factor w decides the effect on the current velocity of previous one. The following weigh function is usually utilized in velocity update function (Willjuice et al., 2007).

$$w^k = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}}.iter \tag{24}$$

Where $w_{\max}$ is initial weight, $w_{\min}$ is final weight, $iter_{\max}$ is maximum iteration number, and $iter$ is current iteration number. Practical studies showed that the initial weight and final weight should be 0.9 and 0.4, respectively. The parameter values of the PSO algorithm used in this study are given in Table 4. As in the other evolutionary algorithms, PSO algorithm uses the fitness functions to evaluate the fitness of a particle. Generally, the performance criteria commonly used in PI controller design are integrated absolute error (IAE), the integrated of time weight square error (ITSE) and integrated of squared error (ISE) which can be evaluated analytically in the frequency domain (Nasri et al., 2007). In this study, a fitness function based on ISE is used to evaluate the fitness of particles due to its response with relatively small overshoot.

$$f_i = \frac{1}{1 + \alpha.\sum e_{ci}^2} \tag{25}$$

Where f_i and α define the fitness value of i^{th} particle and the weight factor related to ISE criteria, respectively.

SIMULATION RESULTS

To verify the effectiveness on the system performance of the proposed hybrid controller, the control system have also been performed by using both PI/PI and hybrid

Table 4. Parameters of the PSO algorithm.

Property	Values
Number of particle [Ki,Kp]	[20, 20]
Inertia weight factor [wmax,wmin]	[0.9 0.3]
Acceleration constants [c1,c2]	[2.01, 2.05]
Number of iteration itermax	50

Table 5. Inverter parameters used in the simulation studies.

Inverter parameter		Parameter value
Input voltage V_{DC}		330 V_{DC}
Output voltages $V_{AN} - V_{BN} / V_{AB}$		110 V_{RMS} / 220 V_{RMS}
Frequency of the output voltages		50 Hz
Switching frequency		40 KHz
Inductances of the filter inductors $L_a - L_b - L_c$		1.8 mH
Series resistances of the filter inductors $r_a - r_b - r_c$		20 mΩ
Capacitances of the filter capacitors $C_a - C_b$		20 uF
Linear load parameters	$Z_{AN} / Z_{BN} / Z_{AB}$	10 Ω/100 Ω/20 Ω
	$Z_{AN} : L_{da} / C_{da} / R_{La}$	400uH/4000uF/10 Ω
Nonlinear load parameters	$Z_{BN} : L_{db} / C_{db} / R_{Lb}$	400uH/4000uF/100 Ω
	$Z_{AB} : L_{d} / C_{d} / R_{L}$	400uH/4000uF/20 Ω

T1FL/PI controllers. For this purpose, a program which contains the dynamic model of the whole system was written in C programming language, and simulation studies were carried out through this program by using the inverter parameters given in Table 5 and the controller parameters derived in design procedures. The simulation results of the PI/PI controlled inverter under unbalanced linear load conditions are shown in Figure 8. Figure 8a and b depict that PI/PI controller can achieve a 215.95V sine wave with the THD of 1.24% from the main output of inverter. It is clear from Figure 8c that a voltage with a peak to peak value of 3.79V occurs on the neutral line. The simulation results of the PI/PI controlled inverter system under unbalanced nonlinear load conditions are shown in Figure 9. From the harmonic spectrum and wave form given in Figure 8b and c, it can be seen that the THD of main output voltage and neutral line voltage are 2.27% and 5.39 V, respectively. Figure 10 shows the simulation results of the hybrid T1FL/PI controlled inverter under unbalanced linear load conditions.

As seen in Figure 10a, b c, the main output voltage of inverter is a 216.42 V sine wave with THD of 0.72%, and the peak to peak value of neutral line voltage is 3.72 V. Figure 11 illustrates the simulation results of the hybrid T1FL/PI controlled inverter under unbalanced nonlinear load conditions. From Figure 11b and c, we can see that the THD of main output voltage and the peak to peak value of neutral line voltage are 1.73% and 4.45 V, respectively. The simulation results of the hybrid T2FL/PSO-based PI controlled inverter under unbalanced linear load conditions are shown in Figure 12. It can be seen from Figure 12b and c that the THD of output voltage and the neutral line voltage are 0.38% and 0.52 V, respectively. For the inverter controlled by the same controller under unbalanced nonlinear load conditions, these results are obtained as 0.67% and 1.01 V, as shown in Figure 13. Figure 14 shows the dynamical behaviors of the inverter controlled by the mentioned three controllers in the case of step load changes from 100 to 10 Ω. In that case, the voltage sags (VS) of 140.6 V, 128.9 V and 20 V occur in the main output voltages of the PI/PI, hybrid T1FL/PI and hybrid T2FL/PSO-based PI controlled inverters, respectively. For this load change, the transient response of the PI/PI

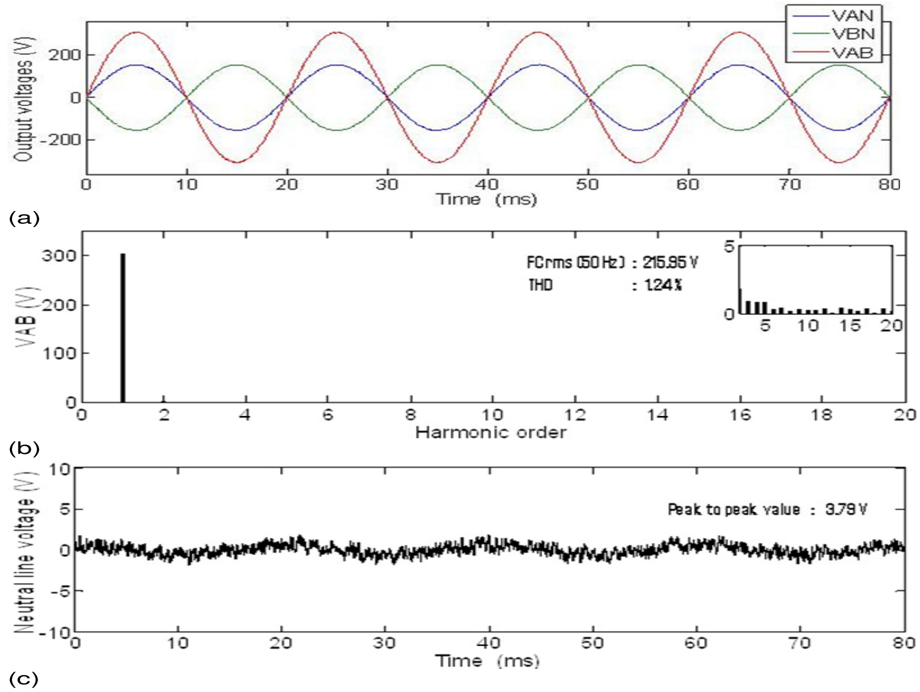

Figure 8. Simulation results of the PI/PI controlled inverter system under unbalanced linear load conditions; **a**. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c.** Neutral line voltage.

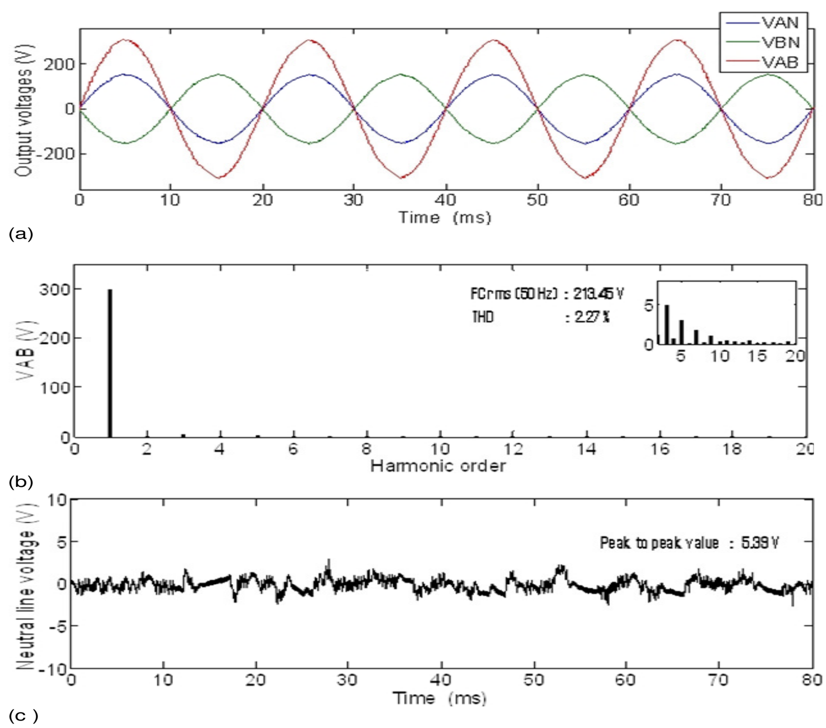

Figure 9. Simulation results of the PI/PI controlled inverter system under unbalanced nonlinear load conditions; **a**. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c**. Neutral line voltage.

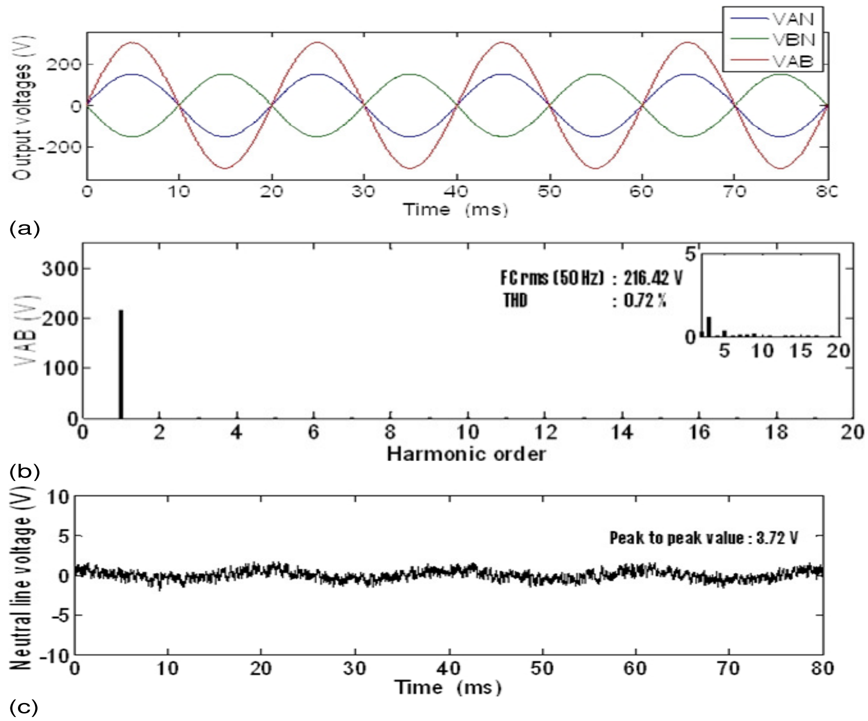

Figure 10. Simulation results of the hybrid T1FL/PI controlled inverter system under unbalanced linear load conditions; **a**. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c**. Neutral line voltage.

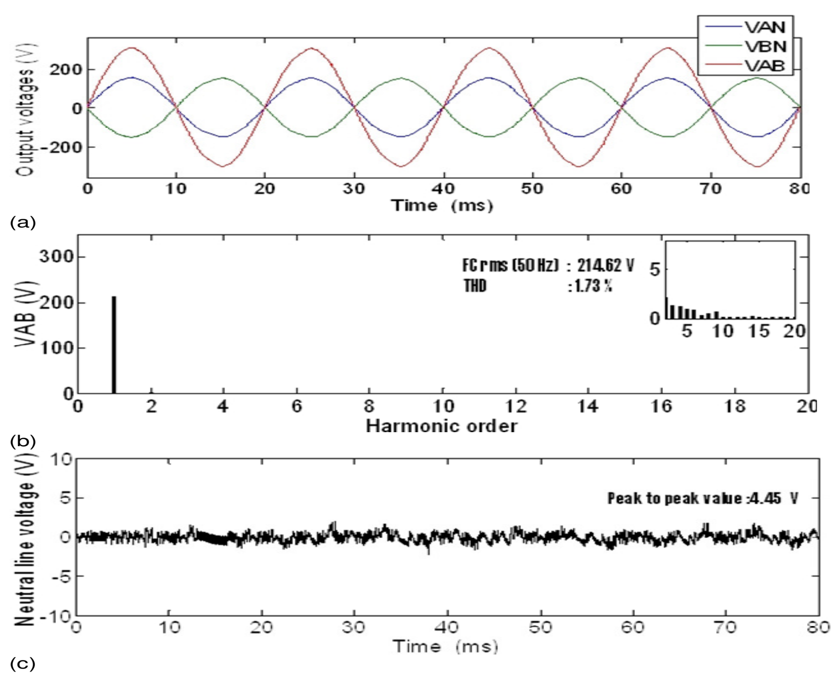

Figure 11. Simulation results of the hybrid T1FL/PI controlled inverter system under unbalanced nonlinear load conditions; **a**. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c**. Neutral line voltage.

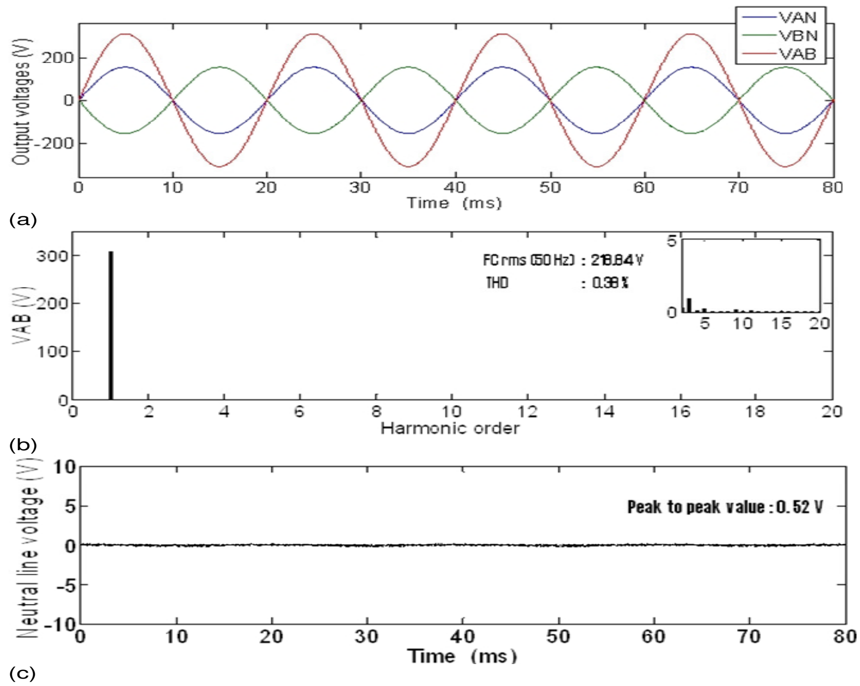

Figure 12. Simulation results of the hybrid T2FL/PSO-based PI controlled inverter system under the unbalanced linear load conditions; **a**. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c**. Neutral line voltage.

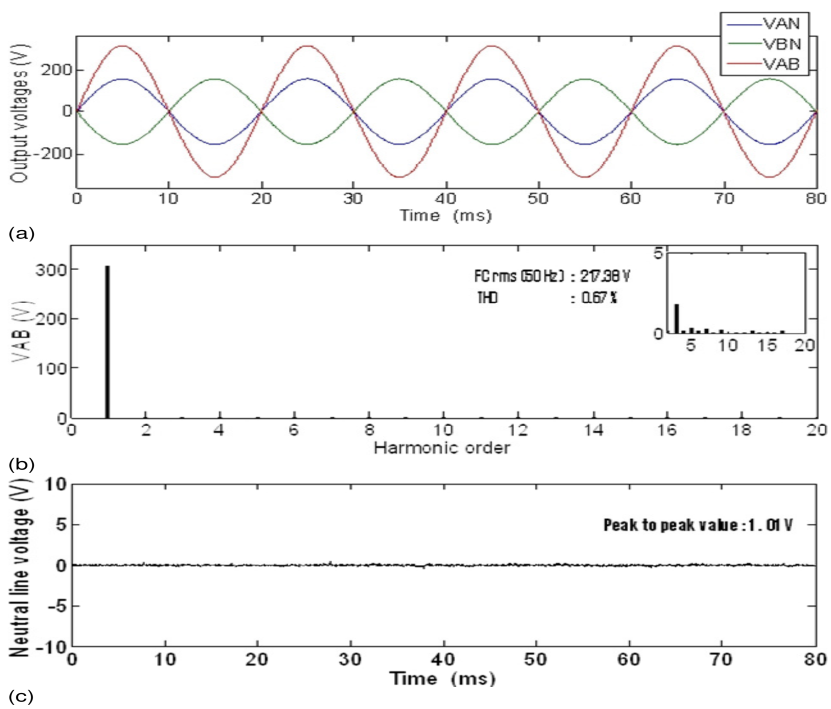

Figure 13. Simulation results of the hybrid T2FL/PSO-based PI controlled inverter system under the unbalanced nonlinear load conditions; a. Output voltages; **b**. Harmonic spectrum of the main output voltage; **c**. Neutral line voltage.

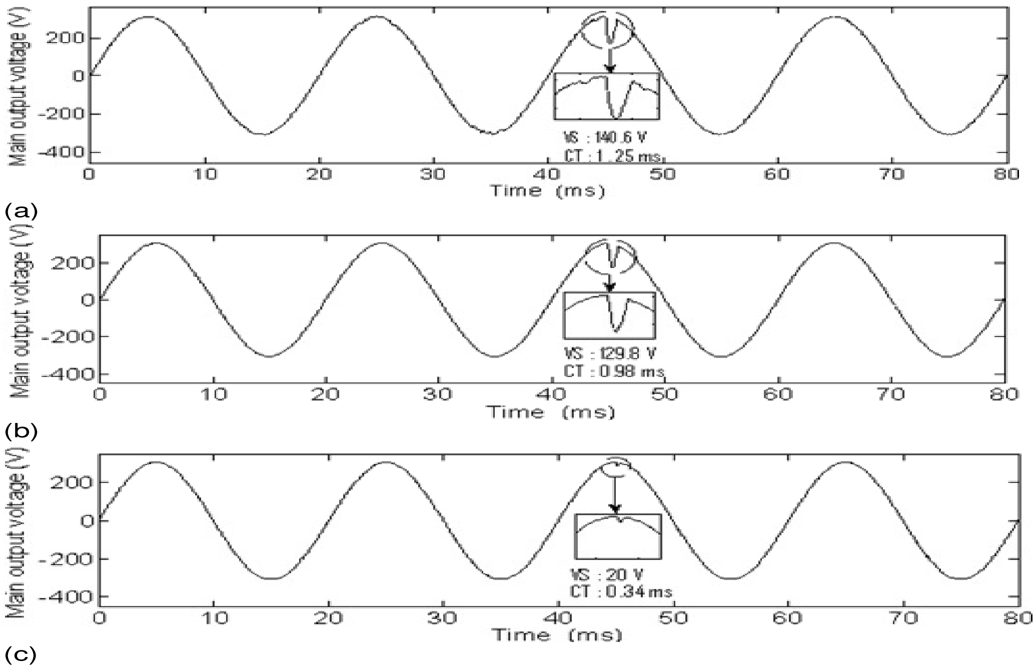

Figure 14. Dynamical behaviors of the inverter controlled by three controllers in the case of step load changes from light-load to full-load; **a**. PI/PI controlled inverter; **b**. Hybrid T1FL/PI controlled inverter; **c**. Hybrid T2FL/PSO-based PI controlled inverter.

controlled inverter is 1.25 ms while that of the hybrid T1FL/PI controlled inverter is 0.98 ms. In the same load change, the transient responses of the inverter controlled by the proposed controller is obtained as 0.34 ms.

DISCUSSION

The performance analysis of the inverter system controlled by PI/PI controller, hybrid T1FL/PI controller and hybrid T2FL/PSO-based PI controller was made on its output voltage harmonics, neutral line voltage and transient response in the case of step load changes. The system performance under the unbalanced load conditions was evaluated by using the analysis results of output voltage harmonics and neutral line voltage. The transient response results were used to evaluate the dynamic response of system when the resistive load connected to the main output of inverter changed from 100 Ω to 10Ω. From the simulation results under the unbalanced linear load conditions, it can be understood that hybrid T2FL/PSO-based PI controller allows an improvement of 0.86% in the elimination of harmonics and a decrease of 3.27 V on the neutral line voltage, when compared with PI/PI controller. The same performance criteria for the proposed controller in comparison with hybrid T1FL/PI controller were obtained as 0.34 and 3.20 V, respectively. The simulation results under the unbalanced nonlinear load conditions showed that hybrid T2FL/PSO-based PI controller achieved a better harmonic elimination of 1.6% in the inverter output voltage and a lower neutral line voltage of 4.38 V than PI/PI controller. Under these conditions, when compared to the performances of hybrid T1FL/PI controller and proposed controller with each other, it was seen that hybrid T2FL/PSO-based PI controller obtained an improvement of 1.06% in the elimination of harmonics and a decrease of 3.44 V on the neutral line voltage.

It also had, respectively, 4 and 3 times faster dynamic response in the correction of voltage sags arising from the load change than PI/PI and hybrid T1FL/PI controllers. The major factor in providing the better performance of the proposed controller for step load changes and harmonic elimination than hybrid T1FL/PI controller is that T2FL controller in this structure uses type-2 fuzzy sets which can model uncertainties in the T1FL controller, associated with the rule base and membership functions. From these results, it can be concluded that hybrid T2FL/PSO-based PI controller has fairly robust control performance for troubleshooting related to voltage unbalances and harmonics encountered in single-phase three-wire inverter systems, when compared with PI/PI and hybrid T1FL/PI controllers. Although T2FL controller is fairly useful for the control of dynamic systems as in this study, its main drawback is a large amount of computations needed to arrive at a conclusion. In practical applications, this computational complexity leads to more memory requirement and higher speed for the processor to be used in control of system.

Conclusions

In this study, a hybrid T2FL/PSO-based PI controller was proposed for the control of a single-phase three-wire inverter in order to perform a better voltage balance between split-phase outputs and obtain an AC output voltage with lower THD rate. To test the performance of the proposed controller, the inverter was also controlled by using both discrete PI/PI and hybrid T1FL/PI controllers. The validity of hybrid T2FL/PSO-based PI controller was verified by means of simulations studies. The simulation results showed that hybrid T2FL/PSO-based PI controller achieved better performance criteria than PI/PI and hybrid T1FL/PI controllers. When compared with the performances of PI/PI and hybrid T1FL/PI controllers in terms of the output voltage harmonics, split phase voltage balances and transient responses in the case of step load changes, it was observed that hybrid T1FL/PI controller provided a better improvement on the output voltage harmonics and transient response of inverter, but a little improvement on split-phase voltage balance than PI/PI controller. However, the proposed controller had fairly robust effect on the inverter in terms of the mentioned performance criteria than both PI/PI and hybrid T1FL/PI controllers under unbalanced linear and nonlinear load conditions. As a result, simulation results verified that the proposed controller is effective and it is very feasible for single-phase three-wire inverter since it contains a controller structure both that eliminates the uncertainties in T1FL controllers and adjusts its own parameters according to the dynamic behavior of system.

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