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Analytical solution of the steady state condensation film on the inclined rotating disk by a new hybrid method

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In this paper, a similarity transformation is used to reduce the three-dimensional steady state condensation film on an inclined rotating disk by a set of nonlinear boundary value problems. This problem is solved using a new hybrid technique based on differential transform method (DTM) and Iterative Newton's Method (INM). The differential equations and its boundary conditions are transformed to a set of algebraic equations, and the Taylor series of solution is calculated. After finding Jacobian matrix, the unknown parameters computed using Multi-Variable Iterative Newton's Method. These techniques are used to obtain an approximate solution of the problem. In this solution, there is no need to restrictive assumptions or linearization. The results compared with the numerical solution of the problem, and a good accuracy of the proposed hybrid method observed. Finally, the velocity and temperature profiles demonstrated for different values of problem parameters.

Key words: Condensation film, rotating disk, nonlinear boundary value problem, differential transform method, iterative Newton's method, Jacobian matrix.

INTRODUCTION

Usually scientific problems and phenomena in our world are essentially nonlinear and modeled by the nonlinear differential equations. Most of them do not have an exact analytical solution. So, numerical and approximate methods are used by researchers to solve such equations. The numerical methods are often costly and time consuming to get a complete form of results, because it gives the solution at the discrete points. Furthermore, in the numerical solution the stability and convergence should be considered to avoid divergence or inappropriate results.

Approximate techniques like Decomposition Method (DM), Homotopy Analysis Method (HAM), Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM) are good substitutes for numerical methods. During the recent years, some of the nonlinear engineering problems have been solved using some of these methods, such as HAM (Rashidi et al., 2008; Dinarvand

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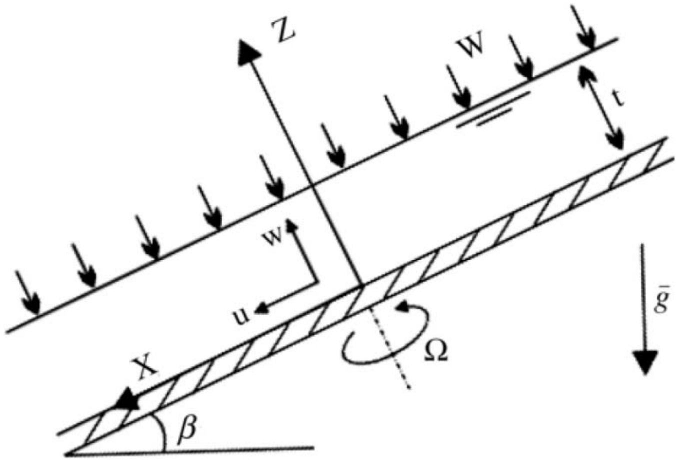

Figure 1. The schematic diagram of the problem (Rashidi and Dinarvand, 2009).

et al., 2009; Rashidi et al., 2008, 2009; Liao, 2009; Ziabakhsh and Domairry, 2009; Abbasbandy and Hayat, 2009), HPM (Raftari and Yildirim, 2010; Esmaeilpour and Ganji, 2007; Fathizadeh and Rashidi, 2009; Bararnia et al., 2012), VIM (Mohyud-Din et al., 2010; Rashidi and Shahmohamadi, 2009; Wazwaz, 2007) and DM (Wazwaz 2006; Alizadeh et al., 2009; Kechil and Hashim, 2007). In most of the researches, some modifications introduced to overcome the nonlinearity. Also, the fractional differential equations are investigated using some approximate solution in Yang (2012), Yang and Baleanu (2013) and Yang et al. (2013a, b).

Differential transform method is also one of the other approximate methods to solve differential equations. This method was introduced by Zhou (1986) to solve initial value problems in analysis of the electrical circuits. After that, DTM applied to differential algebraic equation (Ayaz, 2004; Liu and Song, 2007) partial differential equation (Ayaz, 2003, 2004; Ravi-Kanth and Aruna, 2008; Yang et al., 2006; Chang and Chang, 2009), integral equation (Odibat, 2008; Arikoglu and Ozkol, 2005, 2008), ordinary differential equation (Mosayebidorcheh, 2014; Mosayebidorcheh and Mosayebidorcheh, 2012; Torabi and Aziz, 2012; Joneidi et al., 2009) and fractional differential equation (Nazari and Shahmorad, 2010; Odibat et al., 2008; Erturk et al., 2008; Arikoglu and Ozkol, 2007). The method is an iterative technique to find the Taylor series solution of the problem. In this method, there is no need to the high calculation cost to determine the coefficients of Taylor series.

Removing a condensate liquid from a cooled, saturated vapor is applicable in engineering phenomena. First work with this subject is done by Von Karman (1921) about a rotating disk in an infinite fluid. The motion of the condensate film using centrifugal forces on a cooled rotating disk is considered by Sparrow and Gregg (1959). They transformed the Navier-Stokes equations into a system of nonlinear boundary value problems and numerically integrated for some finite film thicknesses. Their work was extended by considering vapor drag by Beckett et al. (1973) and considering suction on the plate by Chary and Sarma (1976). The problem is also related to chemical vapor deposition, when a thin fluid film is deposited on a cooled rotating disk (Jensen, 1991).

The main goal of this paper is to present an analytical approximate solution of the steady three-dimensional problem of condensation film on the inclined rotating disk. This problem was studied by Wang (2007) and Rashidi and Dinarvand (2009).

MATHEMATICAL FORMULATION

Consider a disk rotating in its plane with angular velocity Ω (Figure 1). The angel between disk and horizontal axis is β. A fluid film with the thickness t is formed by spraying, with a velocity W. Assume the disk radius is large compared to the thickness of fluid film. So, the end effects can be neglected. The gravity acceleration $\bar{g}$ acts downward. The temperatures on the disk and on the film are T_w and T_0, respectively. We can consider the pressure as a function of z only, because the ambient pressure on the film surface is constant p_0. Ignoring the viscous dissipation term, the continuity, momentum and energy equations for the steady state are given in the following form:

$$u_x + v_y + w_z = 0, \tag{1}$$

$$uu_x + vu_y + wu_z = \nu\left(u_{xx} + u_{yy} + u_{zz}\right) + \bar{g}\sin\beta, \tag{2}$$

$$uv_x + vv_y + wv_z = \nu\left(v_{xx} + v_{yy} + v_{zz}\right), \tag{3}$$

$$uw_x + vw_y + ww_z = \nu\left(w_{xx} + w_{yy} + w_{zz}\right) - \bar{g}\cos\beta - p_z/\rho, \tag{4}$$

$$uT_x + vT_y + wT_z = \alpha\left(T_{xx} + T_{yy} + T_{zz}\right). \tag{5}$$

In these equations u, v and w denote the velocity components in the x, y and z directions, respectively and T indicates the temperature, ρ, ν and α are the density, kinematic viscosity and thermal diffusivity of the fluid, respectively. Assuming zero shears on the film surface and zero slip on disk, the following boundary conditions exist

$$\begin{aligned} &u = -\Omega y, \quad v = \Omega x, \quad w = 0, \quad T = T_w \quad \text{at } z = 0, \\ &u_z = 0, \quad v_z = 0, \quad w = -W, \quad T = T_0, \quad p = p_0 \quad \text{at } z = t. \end{aligned} \tag{6}$$

The following transforms used for this problem (Wang, 2007).

$$\begin{aligned} u &= -\Omega y g(\eta) + \Omega x f'(\eta) + \bar{g} p(\eta)\sin\beta/\Omega, \\ v &= \Omega x g(\eta) + \Omega y f'(\eta) + \bar{g} s(\eta)\sin\beta/\Omega, \\ w &= -2\sqrt{\Omega\nu}\, f(\eta), \\ T &= (T_0 - T_w)\theta(\eta) + T_w, \end{aligned} \tag{7}$$

Where

$$\eta = z\sqrt{\Omega/\nu}. \tag{8}$$

The conservation law (Equation (1)) is automatically satisfied. The Equations (2) to (5) can be written as follow:

$$f''' - (f')^2 + g^2 + 2f\,f'' = 0, \tag{9}$$

$$g'' - 2g\,f' + 2f\,g' = 0, \tag{10}$$

$$p'' - p\,f' + sg + 2f\,p' + 1 = 0, \tag{11}$$

$$s'' - g\,p - s\,f' + 2f\,s' = 0, \tag{12}$$

$$\theta'' + 2\,\mathrm{Pr}\,f\,\theta' = 0. \tag{13}$$

Where $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number. The boundary conditions for Equations (9) to (13) are

$$\begin{aligned}
&f(0)=0, \quad f'(0)=0, \quad f''(\delta)=0,\\
&g(0)=1, \quad g'(\delta)=0,\\
&p(0)=0, \quad p'(\delta)=0,\\
&s(0)=0, \quad s'(\delta)=0,\\
&\theta(0)=0, \quad \theta(\delta)=1.
\end{aligned} \tag{14}$$

Where δ is constant normalized thickness

$$\delta = t\sqrt{\Omega/\nu}. \tag{15}$$

DIFFERENTIAL TRANSFORM METHOD

The differential transform is defined as follows:

$$X(k) = \frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}, \tag{16}$$

where, $x(t)$ is an arbitrary function, and $X(k)$ is the transformed function. The inverse transformation is as follows:

$$x(t) = \sum_{k=0}^{\infty} X(k)(t-t_0)^k. \tag{17}$$

Substituting Equation (16) into Equation (17), we have

$$x(t) = \sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}. \tag{18}$$

The function $x(t)$ is usually considered as a series with limited terms and Equation (17), can be rewritten as:

$$x(t) \approx \sum_{k=0}^{m} X(k)(t-t_0)^k. \tag{19}$$

Where, m represents the number of Taylor series' components. Usually, through elevating this value, we can increase the accuracy of the solution. Some properties of the DTM are shown in Table 1. These properties are extracted from Equations (16) and (17).

SOLUTION OF THE PROBLEM

Here, we try to solve the Equations (9) to (13) using a new hybrid technique. The solution consists of two stages, first through mathematical relations and applying DTM, the Taylor series of solution is found. After that, the iterative Newton's method applied to obtain the unknown parameters.

Applying DTM

Each boundary value problem (Equations (9) to (13)) can be transformed to an initial value problem with the replacement of the unknown initial conditions instead of the boundary conditions at end.

$$f''(0)=a_1, \quad g'(0)=a_2, \quad p'(0)=a_3, \quad s'(0)=a_4, \quad \theta'(0)=a_5. \tag{20}$$

By applying the DTM on Equations (9) to (13) at $\eta = 0$, the following recursive relations obtained for calculating the series solutions' coefficients

$$\begin{aligned}
F(k+3) = \frac{1}{(k+1)(k+2)(k+3)}\Bigg\{&\sum_{r=0}^{k}(r+1)F(r+1)(k-r+1)F(k-r+1)\\
&-\sum_{r=0}^{k}G(r)G(k-r) - 2\sum_{r=0}^{k}(r+1)(r+2)F(r+2)F(k-r)\Bigg\},
\end{aligned} \tag{21}$$

$$G(k+2) = \frac{2}{(k+1)(k+2)}\left\{\sum_{r=0}^{k}(r+1)F(r+1)G(k-r) - \sum_{r=0}^{k}(r+1)G(r+1)F(k-r)\right\}, \tag{22}$$

$$\begin{aligned}
P(k+2) = \frac{1}{(k+1)(k+2)}\Bigg\{&\sum_{r=0}^{k}(r+1)F(r+1)P(k-r)\\
&-\sum_{r=0}^{k}S(r)G(k-r) - 2\sum_{r=0}^{k}(r+1)P(r+1)F(k-r) - \delta(k)\Bigg\},
\end{aligned} \tag{23}$$

$$\begin{aligned}
S(k+2) = \frac{1}{(k+1)(k+2)}\Bigg\{&\sum_{r=0}^{k}G(r)P(k-r)\\
&+\sum_{r=0}^{k}(r+1)F(r+1)S(k-r) - 2\sum_{r=0}^{k}(r+1)S(r+1)F(k-r)\Bigg\},
\end{aligned} \tag{24}$$

$$\Theta(k+2) = \frac{1}{(k+1)(k+2)}\left\{-2\,\mathrm{Pr}\sum_{r=0}^{k}(r+1)\Theta(r+1)F(k-r)\right\}. \tag{25}$$

The differential transform of the conditions at $\eta = 0$ in Equations (14) and (20) is:

$$\begin{aligned}
&F(0)=0, \quad G(0)=1, \quad P(0)=0, \quad S(0)=0, \quad \Theta(0)=0,\\
&F(1)=0, \quad G(1)=a_2, \quad P(1)=a_3, \quad S(1)=a_4, \quad \Theta(1)=a_5,\\
&F(2)=a_1.
\end{aligned} \tag{26}$$

Table 1. The properties of the DTM.

Original function	Transformed function
$f(t)=g(t)\pm h(t)$	$F(k)=G(k)\pm H(k)$
$f(t)=cg(t)$	$F(k)=cG(k)$
$f(t)=\dfrac{d^n g(t)}{dt^n}$	$F(k)=\dfrac{(k+n)!}{k!}G(k+n)$
$f(t)=g(t)h(t)$	$F(k)=\sum_{r=0}^{k}G(r)H(k-r)$
$f(t)=t^n$	$F(k)=\delta(k-n)=\begin{cases}1 & if\ k=n\\ 0 & if\ k\neq n\end{cases}$

Substituting Equation (26) into Equations (21) to (25) for *k=0,1,...* , we have:

$$f(\eta)=\frac{a_1}{2}\eta^2-\frac{1}{6}\eta^3-\frac{1}{12}a_2\eta^4-\frac{1}{60}a_2^2\eta^5-\frac{1}{360}a_1\eta^6+\left(\frac{1}{2520}-\frac{1}{630}a_2a_1\right)\eta^7+\cdots, \quad (27)$$

$$g(\eta)=1+a_2\eta-\frac{1}{3}a_1\eta^3+\left(\frac{1}{12}a_1a_2-\frac{1}{12}\right)\eta^4-\frac{1}{15}a_2\eta^5-\left(\frac{1}{90}a_1^2+\frac{1}{45}a_2^2\right)\eta^6+\cdots, \quad (28)$$

$$p(\eta)=a_3\eta-\frac{1}{2}\eta^2-\frac{1}{6}a_4\eta^3-\frac{1}{12}a_2a_4\eta^4+\left(\frac{1}{40}a_1-\frac{1}{60}a_3\right)\eta^5-\left(\frac{1}{720}+\frac{1}{72}a_2a_3\right)\eta^6+\cdots, \quad (29)$$

$$s(\eta)=a_4\eta+\frac{1}{6}a_3\eta^3+\left(\frac{1}{12}a_2a_3-\frac{1}{24}\right)\eta^4-\left(\frac{1}{60}a_4+\frac{1}{40}a_2\right)\eta^5-\frac{1}{72}a_2a_4\eta^6+\cdots, \quad (30)$$

$$\theta(\eta)=a_5\eta-\frac{1}{1200}a_1a_5\eta^4+\frac{1}{6000}a_5\eta^5+\frac{1}{18000}a_2a_5\eta^6+\frac{1}{126000}\left(a_1^2+a_2^2\right)a_5\eta^7+\cdots. \quad (31)$$

Applying Iterative Newton's method

Now, we have to obtain the unknown parameters from the boundary conditions at the end (Equation (14)) and substituting $\eta=\delta$ in Equations (27) to (31). Regarding this, we define the following residual functions to minimize them for obtaining the unknown parameters:

$$\begin{aligned}
R_1 &= f''(\delta,a_1,a_2,a_3,a_4,a_5)=\sum_{k=2}^{m}k(k-1)F(k)\delta^{k-2},\\
R_2 &= g'(\delta,a_1,a_2,a_3,a_4,a_5)=\sum_{k=1}^{m}kG(k)\delta^{k-1},\\
R_3 &= p'(\delta,a_1,a_2,a_3,a_4,a_5)=\sum_{k=1}^{m}kP(k)\delta^{k-1},\\
R_4 &= s'(\delta,a_1,a_2,a_3,a_4,a_5)=\sum_{k=1}^{m}kS(k)\delta^{k-1},\\
R_5 &= \theta(\delta,a_1,a_2,a_3,a_4,a_5)-1=\sum_{k=0}^{m}\Theta(k)\delta^{k}-1.
\end{aligned} \quad (32)$$

The above functions must be zero to get the values a_1 to a_5. To obtain the roots of the Equation (32), we use the following multi-variable iterative Newton's method:

$$\begin{bmatrix}a_1\\a_2\\a_3\\a_4\\a_5\end{bmatrix}_{n+1}=\begin{bmatrix}a_1\\a_2\\a_3\\a_4\\a_5\end{bmatrix}_{n}-\begin{bmatrix}\frac{\partial R_1}{\partial a_1} & \cdots & \frac{\partial R_1}{\partial a_5}\\ \vdots & \ddots & \vdots\\ \frac{\partial R_5}{\partial a_1} & \cdots & \frac{\partial R_5}{\partial a_5}\end{bmatrix}_{n}^{-1}\begin{bmatrix}R_1\\R_2\\R_3\\R_4\\R_5\end{bmatrix}_{n}, \qquad n=0,1,2,\cdots. \quad (33)$$

After guessing the initial values for a_1 to a_5, we have to calculate the residual vector (*R*) and Jacobian Matrix ($\frac{\partial R_i}{\partial a_j}$). The residual vector can be obtained by substituting $(a_1,...,a_5)^n$ in Equation (32). The elements of the Jacobian matrix in Equation (33) can be computed by differentiating analytically with respect to a_1 to a_5 and then substituting $(a_1,...,a_5)^n$ in that equation. The Jacobian elements are presented in the appendix A for $\delta=1$ and *Pr*=1.

RESULTS

The accuracy chosen for computing a_1 to a_5 by Newton's method was 10^{-6}. Figures 2 and 3 demonstrate graphical representation of the presented results and numerical solution to show the efficiency and accuracy of the hybrid proposed method. In these figures, the present results compared with the numerical solution by the Runge-Kutta method. The approximate solution of the problem is presented in Table 2 for *Pr*=1. The values of the unknown parameters a_1,a_2,a_3,a_4 and a_5 presented in Table 3 for *Pr*=5 and different thickness numbers. These values can be substituted in Equations (27) to (31) to obtain the approximate solution of the problem. All of the initial guesses for a_1 to a_5 considered 1. The history of the convergence is shown in Figure 4 for a special case. As we can see in Figure 6 the problem converged rapidly

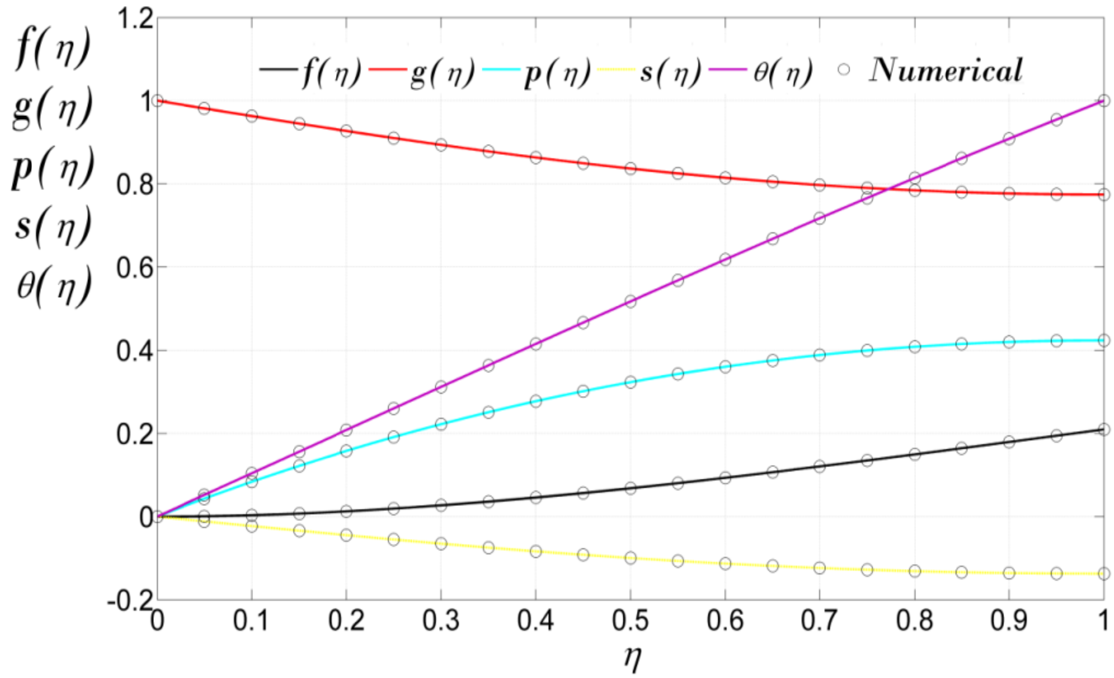

Figure 2. The profiles $f(\eta), g(\eta), p(\eta), s(\eta)$ and $\theta(\eta)$ when $\delta = 1$ and *Pr*=1.

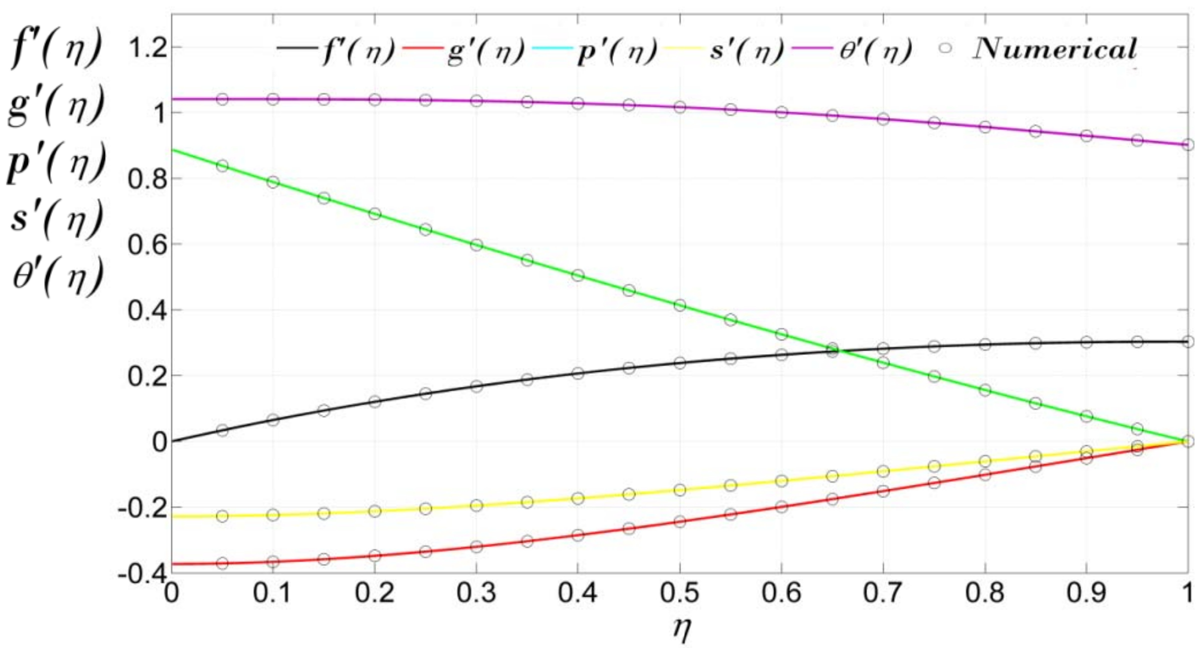

Figure 3. The profiles $f'(\eta), g'(\eta), p'(\eta), s'(\eta)$ and $\theta'(\eta)$ when $\delta = 1$ and *Pr*=1.

with only 5 iterations. This is because the Jacobian matrix obtained by differentiating analytically with respect to a_1 to a_5.

DISCUSSION

Most physical problems in fluid mechanics and heat transfer usually converted to a system of boundary value

Table 2. The approximate solutions $f(\eta), g(\eta), p(\eta), s(\eta)$ and $\theta(\eta)$ when *Pr*=1.

Solutions	Approximate solution
$\delta = 0.5$	$f(\eta) = 0.2412\eta^2 - 0.1667\eta^3 + 0.0066\eta^4 - 0.0001\eta^5 - 0.0013\eta^6 + 0.0004\eta^7 - 0.0000\eta^8$
	$g(\eta) = 1 - 0.0787\eta + 0.1608\eta^3 - 0.0865\eta^4 + 0.0052\eta^5 - 0.0027\eta^6 + 0.0016\eta^7 - 0.0004\eta^8$
	$s(\eta) = -0.0401\eta + 0.0823\eta^3 - 0.0449\eta^4 + 0.0026\eta^5 - 0.0000\eta^6 - 0.0003\eta^7 + 0.0000\eta^8$
	$p(\eta) = 0.4941\eta - 0.5\eta^2 + 0.0067\eta^3 - 0.0002\eta^4 + 0.0038\eta^5 - 0.0008\eta^6 - 0.0001\eta^7 - 0.0004\eta^8$
	$\theta(\eta) = 2.0080\eta - 0.0807\eta^4 + 0.0335\eta^5 - 0.0009\eta^6 + 0.0037\eta^7 - 0.0033\eta^8$
$\delta = 1$	$f(\eta) = 0.3489\eta^2 - 0.1667\eta^3 + 0.0311\eta^4 - 0.0023\eta^5 - 0.0019\eta^6 + 0.0008\eta^7 - 0.0001\eta^8$
	$g(\eta) = 1 - 0.3720\eta + 0.2355\eta^3 - 0.1052\eta^4 + 0.0248\eta^5 - 0.0086\eta^6 + 0.0031\eta^7 - 0.0009\eta^8$
	$p(\eta) = 0.8933\eta - 0.5\eta^2 + 0.0380\eta^3 - 0.0071\eta^4 + 0.0028\eta^5 + 0.0032\eta^6 - 0.0008\eta^7 - 0.0008\eta^8$
	$s(\eta) = -0.2281\eta + 0.1489\eta^3 - 0.0693\eta^4 + 0.0131\eta^5 - 0.0012\eta^6 + 0.0004\eta^7 - 0.0003\eta^8$
	$\theta(\eta) = 1.0445\eta - 0.0615\eta^4 + 0.0174\eta^5 - 0.0022\eta^6 + 0.0042\eta^7 - 0.0025\eta^8$

Table 3. The values of a_1, a_2, a_3, a_4 and a_5 obtained by iterative Newton's method when *Pr*=5.

Solutions	a_1	a_2	a_3	a_4	a_5
$\delta = 0.1$	0.0999	-0.0006	0.0999	-0.0003	10.0003
$\delta = 0.2$	0.1998	-0.0053	0.1999	-0.0027	5.0026
$\delta = 0.3$	0.2986	-0.0178	0.2995	-0.0089	3.3422
$\delta = 0.4$	0.3940	-0.0416	0.3980	-0.0210	2.5209
$\delta = 0.5$	0.4825	-0.0787	0.4941	-0.0401	2.0399
$\delta = 0.6$	0.5594	-0.1285	0.5862	-0.0667	1.7329
$\delta = 0.7$	0.6208	-0.1879	0.6726	-0.1003	1.5279
$\delta = 0.8$	0.6647	-0.2519	0.7522	-0.1395	1.3885
$\delta = 0.9$	0.6921	-0.3149	0.8249	-0.1823	1.2956
$\delta = 1$	0.7057	-0.3730	0.8910	-0.2272	1.2423

problems (BVPs) and it is essential to find the powerful analytical, approximate and numerical methods for solving this type of differential equations. The steady state condensation film on the inclined rotating disk is one of the mechanical problems which governing equations of it can be formulated as nonlinear system of boundary value problems. Here, a hybrid procedure is proposed to solve the differential equations of problem. This technique is based on differential transform method and Newton’s iterative method as a combination of analytical and numerical methods. The results of this technique can be obtained as a polynomial function (Taylor series with limited terms). This is one of the advantages of method. The rapid convergence of solution is also significant.

Conclusion

In this paper, a similarity transformation is used to reduce the governing equations of condensation film on an inclined rotating disk by a set of nonlinear boundary value problems. This problem is solved using a new hybrid technique based on differential transform method (DTM) and Iterative Newton's Method (INM). Using the method,

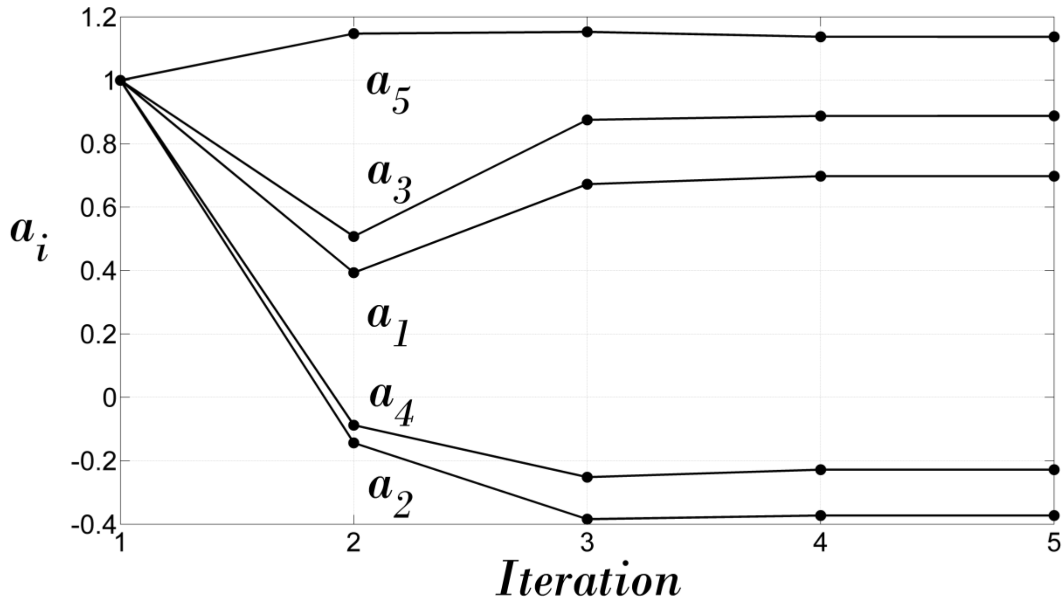

Figure 4. The history of the iterative Newton's method when $\delta = 1$ and *Pr*=5.

the differential equations and boundary conditions are transformed into a recurrence set of equations. After finding Jacobian matrix, the unknown parameters computed using multi-variable iterative Newton's method. Finally, the approximate solution is obtained. The main objective of the present research paper is to introduce a powerful and simple technique as a high accuracy and efficient method for solving a set of nonlinear boundary value problems. The accuracy and efficiency of proposed technique is verified by the numerical results.

NOMENCLATURE

DTM: Differential transformation method
$\bar{g}$: Gravity acceleration
p_0 : Pressure on the film surface
Pr : Prandtl number
T: Temperature
T_w : Disk temperature
T_0 : Film temperature
t: Thickness
u: Velocity component in *x* direction
v: Velocity component in *y* direction
w: Velocity component in *z* direction

Greek symbols

Ω : Angular velocity
β : Angel between disk and horizontal disk
ρ : Density
ν : Kinematic viscosity
α : Thermal diffusivity
δ : normalized thickness

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Appendix A

Here, the elements of the Jacobian matrix $\partial R_i / \partial a_j$ presented for $\delta = 1$ and *Pr*=1:

$$J(1,1) = \frac{1}{180} a_2^2 - \frac{1}{15} a_2 + \frac{11}{12} + \cdots,$$
$$J(1,2) = \frac{1}{90} a_1 a_2 - \frac{1}{15} a_1 - \frac{2}{3} a_2 - \frac{44}{45} + \cdots,$$
$$J(1,3) = 0,$$
$$J(1,4) = 0,$$
$$J(1,5) = 0,$$
$$J(2,1) = -\frac{1}{18} a_1 a_2 - \frac{2}{15} a_1 + \frac{221}{630} a_2 + \frac{46}{45} + \cdots,$$
$$J(2,2) = -\frac{1}{15} a_2^2 - \frac{1}{36} a_1^2 - \frac{4}{15} a_2 + \frac{221}{630} a_1 + \frac{2}{3} + \cdots,$$
$$J(2,3) = 0,$$
$$J(2,4) = 0,$$
$$J(2,5) = 0,$$
$$J(3,1) = -\frac{1}{35} a_1 + \frac{1}{36} a_4 a_2 + \frac{1}{8} + \cdots,$$
$$J(3,2) = -\frac{1}{3} a_4 - \frac{1}{12} a_3 + \frac{1}{36} a_4 a_1 - \frac{2}{45} a_2 a_3 + \frac{1}{105} a_2 + \frac{1}{90} + \cdots,$$
$$J(3,3) = -\frac{1}{45} a_2^2 - \frac{1}{12} a_2 + \frac{11}{12} + \cdots,$$
$$J(3,4) = \frac{1}{36} a_1 a_2 - \frac{1}{3} a_2 - \frac{179}{360} + \cdots,$$
$$J(3,5) = 0,$$
$$J(4,1) = \frac{1}{84} a_2 - \frac{1}{36} a_2 a_3 - \frac{1}{360} + \cdots,$$
$$J(4,2) = \frac{1}{3} a_3 - \frac{1}{12} a_4 - \frac{2}{45} a_4 a_2 - \frac{1}{36} a_1 a_3 + \frac{1}{84} a_1 - \frac{1}{8} + \cdots,$$
$$J(4,3) = \frac{1}{3} a_2 - \frac{1}{36} a_1 a_2 + \frac{179}{360} + \cdots,$$
$$J(4,4) = -\frac{1}{12} a_2 - \frac{1}{45} a_2^2 + \frac{11}{12} + \cdots,$$
$$J(4,5) = 0,$$
$$J(5,1) = \frac{1}{63} a_5 a_1 - \frac{437}{5040} a_5 + \cdots,$$
$$J(5,2) = \frac{1}{630} a_5 a_2 + \frac{1}{180} a_5 + \cdots,$$
$$J(5,3) = 0,$$
$$J(5,4) = 0,$$
$$J(5,5) = -\frac{437}{5040} a_1 + \frac{1}{180} a_2 + \frac{1}{1260} a_2^2 + \frac{1}{126} a_1^2 + \frac{61}{60} + \cdots.$$