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Full Length Research Paper

A new model in stress analysis: Quaternions

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A quaternion is an ordered combination of four real numbers. It can also be expressed as a sum of a scalar and a vector in three dimensional Euclidian space. Quaternion algebra allows the division of vectors. Since the current vector algebra does not allow the division of vectors; in many branches of mechanics, when a vectoral quantity falls to the denominator of an expression, the general tendency is to use its magnitude rather than its vector character. Once the vector division is defined by the quaternion algebra, it becomes possible to redrive the equations that have vector quantities in their denominators. In this study, basic equations of stress in strength of materials are reviewed according to the rules of quaternion algebra. It is shown that this new point of view brings a more powerful and consistent system. The normal stress becomes a scalar quantity. Area and moment of inertia of a cross-section becomes a vectoral quantity. Direction of shearing stresses change and becomes a more consistent convention especially in torsion problems. The most striking results are obtained in shear flow.

Key words: Quaternion, normal stress, shear stress, shear flow.

INTRODUCTİON

The use of vectors has a variety of applications in many branches of mechanics. For example, position, displacement, velocity, acceleration, and force are vectors according to their definitions. Besides, some simple operations defined on vectors, bring a powerful algebra. The resultant of forces acting to a point, can be obtained by using the addition of vectors, according to the parallelogram law. The vector product of lever arm and a force gives the moment of a force with respect to a point; while scalar product of force and the path gives the work done by this force.

As it is seen in the above examples, vector algebra brings a powerful tool in the branches related to physics. However, it becomes unsufficient when a vectoral quantity falls to the denominator of an arithmetical expression, since the division of vectors is not defined in the current vector algebra. The general tendency is to use the magnitude of the vector in the denominator. This type of solution brings some defects to the spirit of the problem, of course.

MATERIALS AND METHODS

Quaternion algebra

A quaternion is defined by the combination of four real numbers:

$$\mathsf{A} = (a_0, a_1, a_2, a_3) \tag{1}$$

where a_0, a_1, a_2, a_3, are real numbers and A is a quaternion. Capital boldface Arial letters denote quaternions. It is possible to consider a quaternion as a combination of a vector and a scalar. A quaternion A = (a_0, a_1, a_2, a_3) can be expressed in the form (Kosenko,1998):

$$\mathsf{A} = (a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3) \tag{2}$$

where e_1, e_2, and e_3 are the orthogonal unit vectors in a three dimensional Cartesian coordinate system. Boldface letters denote vectors. Equation (2) can be written in the form:

$$\mathsf{A} = (\alpha + \mathbf{u}) \tag{3}$$

where α is a scalar and u is a vector in three dimensional Euclidian space. The conjugate of the quaternion given in equation (3) is

$$\mathsf{A}^* = (\alpha - \mathbf{u}) \tag{4}$$

The norm of a quaternion is:

$$\|\mathsf{A}\| = a_0^2 + a_1^2 + a_2^2 + a_3^2 \tag{5}$$

Zero and unit quaternions are defined as:

$$\mathsf{0} = (0 + \mathbf{0}), \qquad \mathsf{1} = (1 + \mathbf{0}) \tag{6}$$

The product of two quaternions of A = (α + u) and B = (β + v), (where β is a scalar and v is a vector) is:

$$\mathbf{A}\,\mathbf{B} = (\alpha + \mathbf{u})\,(\beta + \mathbf{v}) = (\alpha\,\beta - \mathbf{u}\cdot\mathbf{v} + \alpha\,\mathbf{v} + \beta\,\mathbf{u} + \mathbf{u}\times\mathbf{v}) \tag{7}$$

where $(\cdot)$ stands for the dot product and $(\times)$ stands for the cross product defined for the vectors. Then the norm of a quaternion given in Equation (5) can be given as the following as well:

$$\|\mathbf{A}\| = \mathbf{A}\,\mathbf{A}^{*} \tag{8}$$

The inverse A^{-1} of a quaternion A should satisfy the condition:

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{1} \tag{9}$$

Equations (8) and (9) give us the inverse of a quaternion

$$\mathbf{A}^{-1} = \frac{\mathbf{A}^{*}}{\|\mathbf{A}\|} \tag{10}$$

which can be written in the form:

$$\mathbf{A}^{-1} = \frac{(\alpha - \mathbf{u})}{\alpha^{2} + \mathbf{u}\cdot\mathbf{u}} \tag{11}$$

Division of two quaternions can be expressed in two types, left division and right division respevtively:

$$\left.\frac{\mathbf{A}}{\mathbf{B}}\right|_{L} = \mathbf{B}^{-1}\mathbf{A} \quad \text{and} \quad \left.\frac{\mathbf{A}}{\mathbf{B}}\right|_{R} = \mathbf{A}\,\mathbf{B}^{-1} \tag{12}$$

The difference between left division and right division comes from the fact that the quaternion product is not commutative because of the cross product in Equation (7). Then, with the definition given in Equation (3) any vector u can be interprated as a vector quaternion whose scalar part is zero:

$$\mathbf{A} = (0 + \mathbf{u}) \tag{13}$$

Once any vector in three dimensional Euclidian space is expressed as a quaternion, then one can speak about the inverse of vectors and the division of vectors becomes possible. By using Equations (12) and (13) and choosing left division, division of two unit vectors gives unity if they are in the same direction and gives their cross product if they are perpendicular to each other.

Unit vector division quaternion

Okay (2010) defined division of two orthonormal vectors in three dimensional Euclidian space, as follows:

$$\left.\frac{\mathbf{e}_i}{\mathbf{e}_j}\right|_{L} = \mathbf{Q}_{ij} \quad \text{and} \quad \left.\frac{\mathbf{e}_i}{\mathbf{e}_j}\right|_{R} = \mathbf{Q}_{ji} \tag{14}$$

Q*ij* is the unit vector division quaternion and is defined as:

$$\mathbf{Q}_{ij} = \begin{cases} (1 + \mathbf{0}) & \text{if } i = j \\ (0 + \mathbf{e}_i \times \mathbf{e}_j) & \text{if } i \neq j \end{cases} \tag{15}$$

Analysis of cross-sectional properties

The properties of a cross section that are used in the elementary equations of strength of materials are its area, its first moment (sometimes called statical moment), its moment of inertia (sometimes called second moment) and its polar moment of inertia. All those physical quantities are treated to be scalar in many strength of materials books (Hibbeler, 1994; Popov, 1999; Shanley, 1957; Pytel and Singer, 1987). Actually none of them are scalar according to their physical definitions. Before starting the analysis of cross sectional properties, let us set the Cartesian coordinate system in our problems let x_1 be the axis along the axis of the beam, x_2 be directed upwards and x_3 be in the direction of the cross product of $\mathbf{e}_1 \times \mathbf{e}_2$.

The inclination of the area plays an important role in the analysis of the problems. Besides the angle between the area and the applied force becomes very important when the stress on this area is calculated. Even though the direction of the force does not change, the character of the stress changes when the inclination of the area changes.

The area of a cross section can easily be interprated as a vector which is in the same direction with the outnormal of the mentioned surface and its magitude is the amount of the area itself. This type of thinking can also be handled by recalling the definition of an area of a parallellogram as the cross product of the vectors forming itself (Beer and Johnston, 1996). Therefore the cross sectional area of a beam which is settled in a Cartesian Coordinate frame, in the way defined above, is:

$$\mathbf{A} = A\mathbf{e}_1 \tag{16}$$

We can emphasise the vector character of an area by Equation (16). Although, this type of definition is used by many authors (Halliday and Resnick, 1974; Eringen, 1967); they do not use them in this form when area falls to the denominator of an equation.

The first moment of an area is defined as (Beer and Johnston, 1996):

$$Q_z = \int_A y\,dA \tag{17}$$

where Q_z is the first moment of a section with repect to its horizontal centroidal axis, y is the distance of the differential area element from the horizontal centroidal axis and *A* is the area of the mentioned section. All the quantities in Equation (17) are treated to be scalar whereas they are not. Now, let us assign their vectoral characters and apply the product of two quaternions given in Equation (7), since all vectors can be treated as vector quaternions. The right-hand side of Equation (17) becomes:

$$\int_A y\,dA = \int_A \mathbf{y}\,d\mathbf{A} = \int_A y\,\mathbf{e}_2\,dA\,\mathbf{e}_1 \tag{18}$$

Equation (18) shows that first moment of an area is a vector in e_3 direction, but negative sense:

$$\mathbf{Q}_z = -Q_z\,\mathbf{e}_3 \tag{19}$$

Now, let us apply the same procedure for moment of inertia. Moment of inertia of a cross section with respect to its centroidal horizontal axis is defined as (Beer and Johnston, 1996):

$$I_z = \int_A y^2\,dA \tag{20}$$

Repeating the same procedure as it is done in first moment; the right hand side of Equation (20) becomes:

$$\int_A y^2\,dA = \int_A \mathbf{y}\,\mathbf{y}\,d\mathbf{A} = \int_A y\mathbf{e}_2\;y\mathbf{e}_2\;dA\,\mathbf{e}_1 \tag{21}$$

According to Equation (21) the moment of inertia of a cross section comes out to be a vector in the same direction with the longitudinal axis of the beam:

$$\mathbf{I}_z = I_z\,\mathbf{e}_1 \tag{22}$$

It should be noted that the indices in Equations (19) and (22) do not denote any components but show that the moments are taken with respect to the axis in the subscript. With the same analysis as it is performed for moment of inertia, polar moment of inertia of a cross section, also comes out to be vector in the same direction with the moment of inertia:

$$\mathbf{J} = J\,\mathbf{e}_1 \tag{23}$$

whose magnitude can be obtained from the equation (Beer and Johnston, 1996):

$$J = \int_A r^2\,dA \tag{24}$$

where *r* is the distance between the differential element *dA* and centroid of the cross section.

In this section, we saw that cross sectional properties of an area are all vectoral quantities. Moment of inertia,polar moment of inertia and area itself are in the same direction with the longitudinal axis of the beam and first moment is a vector in the horizontal transverse direction passing through the centroid of the cross section.

Normal stress in axial loading

The vectoral character of normal stress is not clearly identified in the textbooks written by different authors. According to Beer and Johnston, (1996) the normal stress in a member of cross sectional area *A* subjected to an axial load P is obtained by dividing the magnitude *P* of the load by the area *A*:

$$\sigma = \frac{P}{A} \tag{25}$$

This definition is given by many other authors (Hibbeler, 1994; Popov, 1999; Shanley, 1957; Pytel and Singer, 1987). Although, Equation (25) defines the normas stress to be scalar, it is not stated in those references that whether the normal stress is vectoral or scalar. Besides, many authors show the normal stress by arrows in the figures and speak about the direction of the normal stress, even its definition is a scalar quantity.Higdon states that:

> "Since stress is not a vector, the laws of vector addition do not apply the stresses that act on different planes." (Higdon et al., 1985).

On the other hand, from Continuum Mechanics point of view, stress acting a point is defined as a vector (Eringen, 1967):

$$\mathbf{t}_k = t_{kl}\,\mathbf{i}_l \tag{26}$$

where t_k is the stress vector acting on the surface whose normal is the unit vector i_k (or e_k in our text), t_{kl} is the stress tensor and $\mathbf{i}_l$ is the unit vector in *l* direction. Whether the normal stress is defined as a vector or a scalar the sign convention for the normal stress is always the same. If the member is pulled by the force which is the tension case, it is trested to be positive; vice versa, if it is compressed by the force, that is the compression case, it is treated to be negative. This type of sign convention brings an important inconsistancy that, even though the stress vectors are in opposite directions on the opposite faces of an element, they are treated to have the same sign according to the sign convention given above.

One the other hand; if we want to find the normal stress on the right hand side of a beam subjected to a tensile

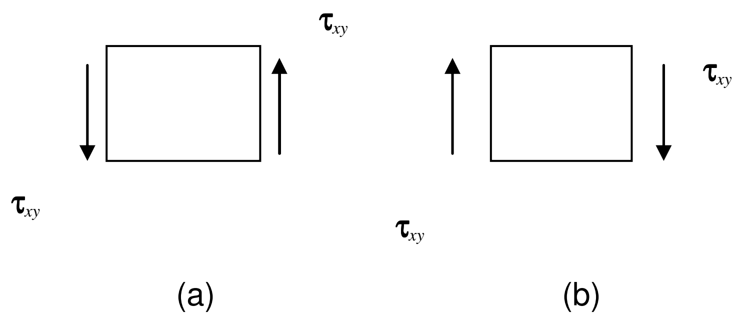

Figure 1. Sign convention for shearing stress.

axial load and whose logitudinal axis is x_1, for this case, the axial load and the cross sectional area are vectors in the same direction with the longitudinal axis of the beam. Then the normal stress on this face is calculated by the help of unit vector division quaternion to be:

$$\sigma = \frac{\mathbf{P}}{\mathbf{A}} = \frac{P\,\mathbf{e}_1}{A\,\mathbf{e}_1} = \frac{P}{A}\mathbf{Q}_{11} = \frac{P}{A} \quad (27)$$

Equation (27) shows that normal stress is scalar quantity. If we want to find the stress acting on the left hand side of the same beam; for this time both the force vector and the area vector become negative and the stress becomes:

$$\sigma = \frac{\mathbf{P}}{\mathbf{A}} = \frac{P(-\mathbf{e}_1)}{A(-\mathbf{e}_1)} = \frac{P}{A}\mathbf{Q}_{11} = \frac{P}{A} \quad (28)$$

Thus a more consistent result is obtained for the sign convention problems. Same consistency would be obtained for the compression case as well.

The simple shear

The shearing stress due to the transverse loading is defined as the magnitude of the shear force divided by the cross sectional area A (Beer and Johnston, 1992):

$$\tau = \frac{V}{A} \quad (29)$$

The same inconsistencies exist in the shearing stress problem as one has in normal stress problems. Although, its definition given as a scalar quantity, they are shown by using arrows in the figures. Another inconsistency exists in the sign convention of the shearing stress, as well. The positive and negative shearing stress cases are given in (Figure 1).

The positive shear stress is given by case (a) and negative shear stress is given by case (b). For example, for the positive case, the shearing stress acting on the right face is directed upwards and the one acting on the left face is directed downwards. However they are both named to be positive although they have opposite signs. When the quaternion algebra is used, the shearing stress is obtained as a vector:

$$\boldsymbol{\tau} = \frac{\mathbf{V}}{\mathbf{A}} = \frac{V\mathbf{e}_2}{A\mathbf{e}_1} = \frac{V}{A}\mathbf{Q}_{21} = \frac{V}{A}\,\mathbf{e}_3 \quad (30)$$

As it is seen from Equation (30) that the shearing stress is a vector which is both perpendicular to the force vector and the area vector. Although, this seems to be strange, it can easily be observed that each type of shear loading introduces a rotation on the applied element as it is seen from (Figure 1). The same result would be obtained if the left hand side of the element is considered by using quaternion algebra. Similar results can be recieved for the negative loading that is given in case (b) in (Figure 1). The quaternion model for stress gives rather good results for axial loading and transverse shar problems, especially from sign convention point of view.

Stresses on an oblique plane

The prismatic bar shown in (Figure 2) is loaded by a tensile force P and we will consider the stresses acting on the oblique plane on the right hand side of the bar that makes an angle θ with the vertical.
Let the cross sectional area on the left to be A_0, then the area of the oblique plane becomes:

$$A_\theta = \frac{A_0}{\cos\theta} \quad (31)$$

If we want to find the stresses on the inclined plane by using classical approach, first we try to find the components of the internal force P. The perpendicular and the parallel components to the mentioned plane are

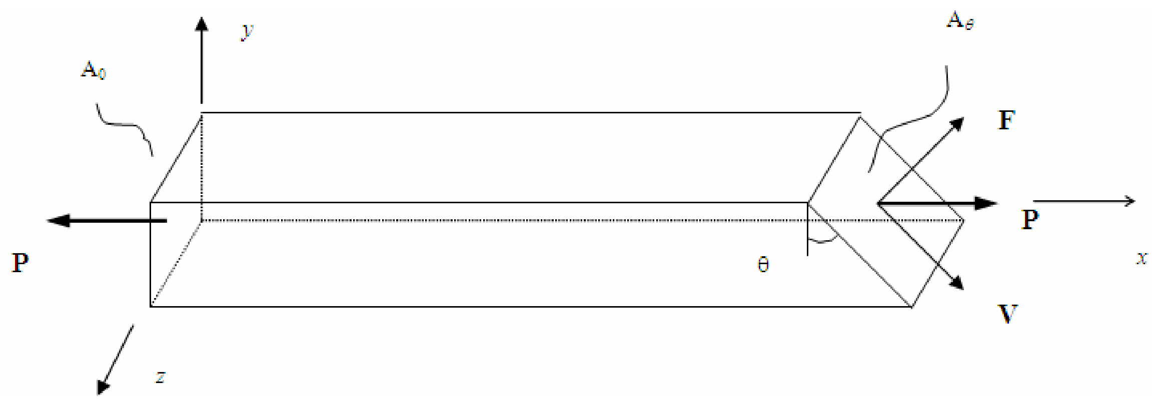

Figure 2. Internal forces acting on an oblique plane of an axially loaded member.

respectively:

$$\mathbf{F}=\mathbf{P}\cos\theta \ , \quad \mathbf{V}=\mathbf{P}\sin\theta \tag{32}$$

Then the normal stress acting on this inclined plane can be obtained as:

$$\sigma = \frac{\mathbf{F}}{A_\theta} = \frac{\mathbf{P}\cos\theta}{A_0 / \cos\theta} = \frac{\mathbf{P}}{A_0}\cos^2\theta \tag{33}$$

and similarly the shearing stress is obtained:

$$\tau = \frac{\mathbf{V}}{A_\theta} = \frac{\mathbf{P}\sin\theta}{A_0 / \cos\theta} = \frac{\mathbf{P}}{A_0}\sin\theta\cos\theta \tag{34}$$

If we want to solve the same problem by using quaternion algebra, we do not need to find the components of the internal force P since it already lies on the *x*-axis. But we have to find the components of the area A_θ since it is vector and it does not lie on any principal plane of rectangular coordinates. Directions of the vectors in (Figure 2) are:

$$\mathbf{P}=P\,\mathbf{e}_1 \tag{35}$$

$$\mathbf{A}_\theta = A_1\mathbf{e}_1 + A_2\mathbf{e}_2 \tag{36}$$

where

$$\frac{A_1}{A}=\cos\theta \qquad \frac{A_2}{A}=\sin\theta \tag{37}$$

Then the stresses on the inclined plane can be found by dividing the force vector P by the area vector A_θ by the help of unit vector division quaternion:

$$\frac{\mathbf{P}}{\mathbf{A}} = \frac{1}{\mathrm{A}^2}\mathbf{Q}_{ji}\ A_i P_j = \ = \frac{P\,A_1}{A_1^2+A_2^2} + \frac{P\,A_2}{A_1^2+A_2^2}\mathbf{e}_3 \tag{38}$$

Therefore stress tensor is a quaternion. We can rewrite equation (38) in the following form:

$$\sigma = \left(\frac{P}{A_x}\frac{A_x^2}{A^2} + \frac{P}{A_x}\frac{A_x}{A}\frac{A_y}{A}\mathbf{k} \right) \tag{39}$$

We may see Equation (39) is exactly the same with the Equations (33) and (34) if we substitute Equation (37) into the mentioned equations. The only difference is taking the normal stress as scalar.

Bending stresses

Normal stress caused by pure bending is given in all engineering books in the same way. If a bending moment in the horizontal (*z*) direction is applied to a cross section as shown in (Figure 3), then the normal stress at any point *A* is given in Equation (40) (Beer and Johnston, 1992). The double arrow in *z* direction in (Figure 3) shows the direction of the bending moment $\mathbf{M}_z$.

$$\sigma_x = -\frac{M_z}{I_z}y \tag{40}$$

Here, σ_x is a vector in *x* direction (according to classical approaches that do not obey Quaternion algebra), M_z is a vector in *z* direction and no clarification is given about the character of *y*. Moment of inertia I_z is treated as a scalar of course, since it appears in the denominator.

If the distance from the neutral axis *y* is treated as a vector; then the normal stress becomes a vector in *x*

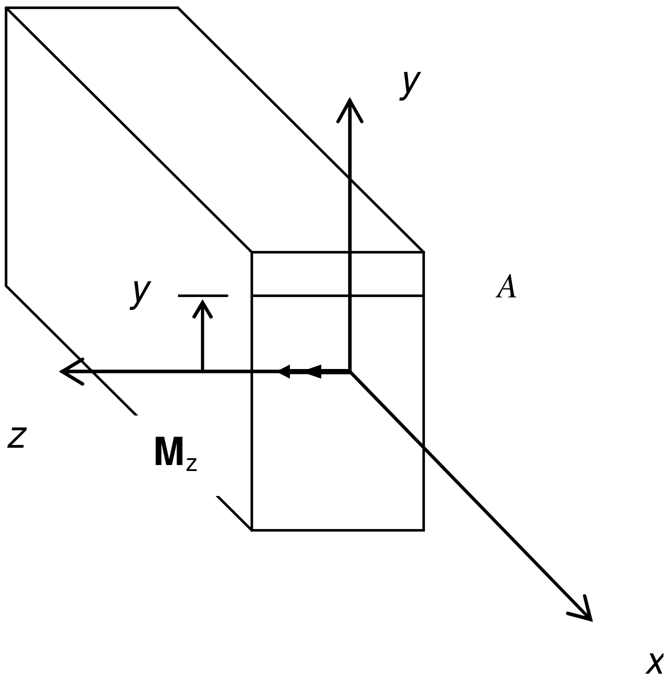

Figure 3. Bending moment acting at a rectangular cross-section.

direction. This is not a contradiction with the way they follow. However, it is not understood that the product in Equation (40) is a vector product and the result comes out just in the opposite sign if we follow the order in the mentioned equation. The order of multipliers is important in the cross product, as we remember from our preliminary vector algebra. On the other hand, if *y* is treated as scalar, the how can one obtain a vector in *x* direction by multiplying a vector in *z* direction by some scalars (that are the distance *y* and the moment of inertia I_z)? Knowing that the moment of inertia is vector in *x* direction according to Equation (22) and substituting the vector characters of physical quantities, then the normal stress due to the bending moments is found as:

$$\sigma_x = \frac{\mathbf{M}_z}{\mathbf{I}_z}\mathbf{y} = \frac{M_z}{I_z}\left(\frac{\mathbf{e}_3}{\mathbf{e}_1}\right) y\,\mathbf{e}_2 = \frac{M_z}{I_z}\,\mathbf{Q}_{31}\, y\mathbf{e}_2 = -\frac{M_z}{I_z}\, y \qquad (41)$$

Equation (41) shows that the stress due to a bending action is a scalar. This is something that we are expecting, since the bending stresses are normal stresses.

Torsional stresses

The maximum shearing stress due to torsional loading (Figure 4) occurs at outmost fibres of a circular shaft and is given by the equation (Beer and Johnston, 1992):

$$\tau_{\max} = \frac{T\,c}{J} \qquad (41)$$

where *T* is the magnitude of the applied torque, *c* is the radius of the circular shaft and *J* is the polar moment of inertia of the cross section of the shaft. No direction is given to the shearing stresses due to the torsional loading (Beer and Johnston, 1992).

However, some authors show the direction of the shearing stresseses according to torsional loading in their figures, but they can not give them their names by giving appropriate indices (Popov, 1999; Hibbeler, 1994).

When quaternion algebra is used to determine the shearing stresses according to torsion, we will use the fact that the applied torque T and the polar moment of inertia J are both vectors in the positive direction of x_1 (which is *x* in Figure 4) and the radius of the shaft c is also a vector. Then the shearing stress becomes:

$$\tau = \frac{\mathbf{Tc}}{\mathbf{J}} = \frac{T\mathbf{e}_1\, c\,\mathbf{u}_c}{J\mathbf{e}_1} = \frac{T}{J}\mathbf{Q}_{11}\, c\,\mathbf{u}_c = \frac{Tc}{J}\,\mathbf{u}_c \qquad (42)$$

where u_c is the unit vector directed along the radius vector **c** and *c* is the magnitude (that is the length) of the radius vector c. The physical meaning of Equation (42) can be explained in (Figure 5).

Figure 5 is the exaggerated view of infinitesimal element *A* in (Figure 4). Since shearing stresses occur in the shafts that are subjected to torsion; the shear forces formed by this process is shown on the edges of the mentioned infinitesimal element. These forces are denoted by *V* in (Figure 5). As it is mentioned earlier, they form a couple and cause a twist in the direction perpendicular to this surface. This is the shearing stress τ_{max} which is consistent with the shearing stress presented in Equation (30). The figure is also consistent with Equation (42).

Longitudinal shearing stress due to transverse loading

The longitudinal shearing stress caused by transverse loading is given by the equation (Beer and Johnston, 1992):

$$\tau_{xy} = \frac{VQ}{It} \qquad (43)$$

where *V* is the shearing force acting on the cross section, *Q* is the first moment of the area that shearing stress is acting, *I* is the moment of inertia of the section and *t* is the thickness or the width of the section. Then τ_{xy} is the shearing stress acting at a certain point. All the physical quantities on the right hand side of Equation (43) are

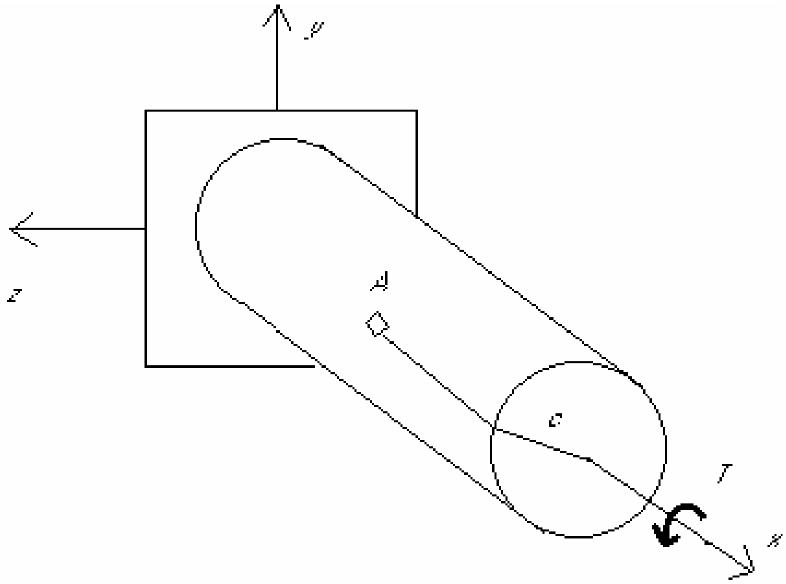

Figure 4. A shaft subjected to torque *T*.

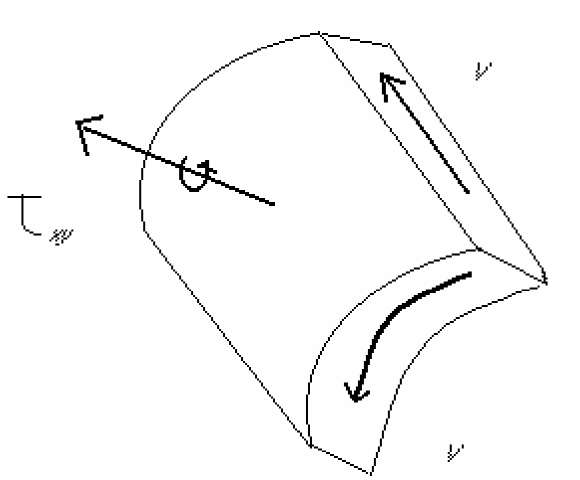

Figure 5. Torsional stress on the outer face of a shaft.

vectoral and by using quaternion algebra, the longitudinal shearing stress becomes:

$$\boldsymbol{\tau}_{xy} = \frac{\mathbf{VQ}}{\mathbf{It}} = \frac{V(-\mathbf{e}_2)(-Q)(\mathbf{e}_3)}{I\,\mathbf{e}_1\;t\,\mathbf{e}_3} = \frac{VQ}{It}\mathbf{e}_3 \qquad (44)$$

It is seen from Equation (44) that the longitudinal shearing stress is a vector in x_3 direction. This is something expected since the shearing forces making the couple, in transverse loading; occur in x_1 direction.

Up to now, it is shown that, the shearing stresses according to any type of loading; simple, torsional or transverse, can consistently be handled by using quaternion algebra. Especially the directions always give the correct manner.

Shear flow

Shear flow is defined as the shear force per unit length. The equation for shear flow is given as (Beer and Johnston, 1992):

$$q = \frac{VQ}{I} \qquad (45)$$

Although, the shear flow is given in scalar form in Equation (45), sense of *q* is pronounced and said to be the same as the sence of the shear force V in vertical parts (Beer and Johnston, 1992). Besides, when the shear flow *q* is sketched in a figure, it is designated by small arrows following each other as it is shown in (Figure 6). These two facts show that shear flow is accepted to be a vector. When the shear flow is accepted to be a vector, the following questions can not be answered:

(1) How can the direction of shear flow change at point *B* of (Figure 6a)? In other words, what changes the definition of shear flow - which is given in Equation (45) that gives a horizontal flow in the flanges and a vertical flow in the web?
(2) Two equal and opposite shear flows come to point *G* from points *E* and *E'* in Figure (6b). If they are vectors, they should cancel each other. However, in the mentioned figure, they are summing up and even both of

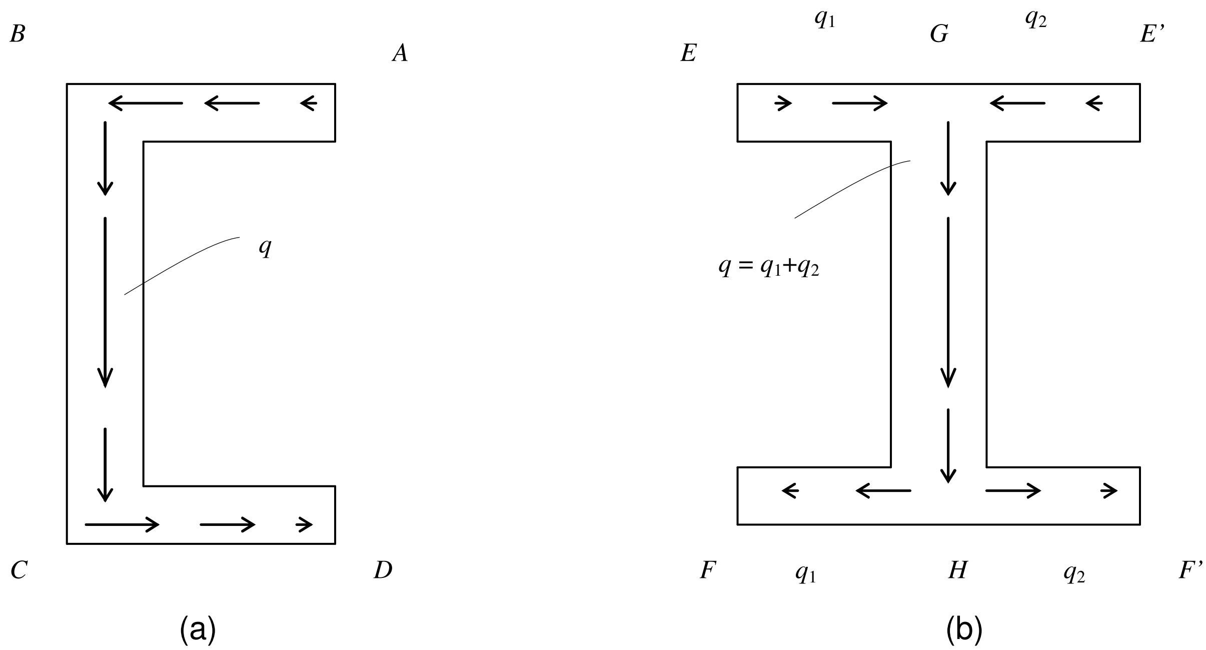

Figure 6. Shear flow variation in (a) a channel (b) an I section.

them change their direction when they come together.

Now, let us investigate the shear flow in quaternion algebra. The shear flow can be obtained:

$$q = \frac{\mathbf{VQ}}{\mathbf{I}} = \frac{V(-\mathbf{e}_2)(-Q)\,\mathbf{e}_3}{(I)\mathbf{e}_1} = \frac{VQ}{I} \tag{46}$$

The shear flow comes out a scalar quantity which is much more consistent.

DISCUSSION AND CONCLUSIONS

In this study, quaternion algebra is used to reevaluate the equations related to find the stress tems in elementary equations of strength of materials. Vector division becomes possible by using quaterion algebra. This becomes a very important aspect in the equations of strength, since vector quantities can be found in the denominators of some Equations like (27), (30), (42) etc.

Using this algebra, a clear distinguish can be made between vectoral scalar quantities. For example, normal stress comes out a scalar quantity, as it is mentioned in Equation (27); or the direction of shearing stresses comes out perpendicular to both the shear forces and the cros sectional area as it is given in Equation (30).

Besides more consistent results can be obtained, especially in sign convention. The unconsistent case occured in shear flow is compensated by using the quaternion algebra. Describing the stress acting at a point by using the quaternion model will be more appropriate than the existing tensor model.

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