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Full Length Research Paper

Research on balance of fuzzy cognitive map

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In order to evaluate better the conflicts in FCM (fuzzy cognitive map), the paper focuses on the research of the balance of FCM. We first further analyze the causal relationship in FCM and the balance of FCM. Based on these, key edge is essential factor to determine whether FCM balanced or not. Moreover, one new balance degree of FCM is used to measure the conflicts. Finally, a method of searching for key edge is given to find the conflicts.

Key words: FCM (fuzzy cognitive maps), balance, balance degree, key edge.

INTRODUCTION

Fuzzy cognitive maps (Kosko, 1986, 1992) have been introduced by Kosko based on Axelord's work on cognitive maps (Axelrod, 1976) and are considered as combination of fuzzy logic and artificial neural networks. FCM is a fuzzy weighted directed graph with feedback and an intelligence tool with cognitive characteristic of the fuzzy causal relationship simulating dynamic behavior of the system. The evaluation of the conflicts that exist in FCM is an important subject, which can give an indication of the dynamical behavior of the system. The balance of FCM can be used to determine whether conflicts exist in FCM or not, how much are the conflicts and where the conflicts are.

The balance was firstly introduced in graph theory by Heider (1946), then was extended by Cartwright and Harary (Cartwright et al., 1956). The balance of the graph is determined by the balance degree. A FCM is imbalanced if we can find two paths between the same two nodes that create causal relations of different sign. In the opposite case it is balanced. Various types of balance degrees have been proposed. The balance degree proposed by Harary is created by the total number of semi-cycles of the digraph (Cartwright et al., 1956), but not taking into account the direction of the arcs. Norman and Roberts still use the total number of semi-cycles and take into consideration the length of different path (Norman and Roberts, 1972). Nozicka and Nakamura based on the idea that as the length of the path increase, the indirect casual relation becomes weaken, proposed that the total effect should have the sign of the shortest path between the two nodes (Nozicka et al., 1976; Nakamura et al., 1982). Eden on the other hand, proposed that the sign of the total effect should be the sign of the most important path that passes through the most important nodes (Eden, 1988). Tsadiras proposed one new balance degree of FCM, which takes into account the imbalanced factor to be the minimum number of positive semi-cycles and negative semi-cycles in graph (Tsadiras et al., 2007).

However, these researches mentioned earlier are insufficient in the analysis of the balance of FCM and are only done in balance degree of FCM from the respective of the number of positive and negative paths of directed graph. But the balance of FCM in essence is for the existence of one or several key edges in the paths, which is the true source. Thus, we studied further balance and the balance degree of FCM in the paper.

MATERIALS AND METHODS

The causality and balance of FCM

The balance of FCM is directly influenced by the causality of FCM.

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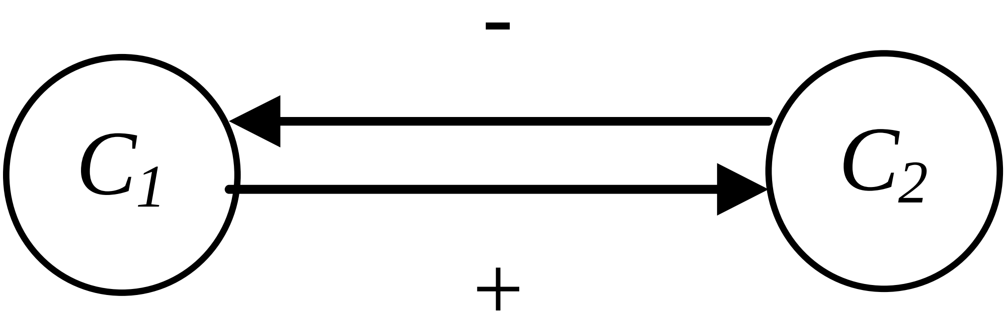

Figure 1. An FCM with feedback loop.

Therefore, first of all, the analysis of causality of FCM is introduced in the following: if there is a route through different nodes from starting node A to destination node B in FCM, there is a causality from A and B. If there is one directed arc directly connecting the two nodes in the route at least, the relationship between the two nodes is direct causality. If there are more than one directed arcs in the route at least, the relationship is indirect causality.

It should be noted that the route that is through different nodes is not a path which may be through same nodes. Otherwise, as shown in Figure 1, in the FCM with feedback loop, there may be a path between C_1 and C_2 accessing these nodes of C_1, C_2, C_1…….C_2, C_1, C_2. The different number of node C_1 or C_2 accessed will cause the different balance of the FCM.

The weight from node A to node B has three cases: positive value that A node has a positive impact on B node, the former increase (or decrease) will cause an increase (or decrease) of the latter; negative value shows the opposite; zero value indicates no causality. The positive and negative of causal degree is discussed as follows:

Theorem 1

If there is an even number of negative direct causal relationship from A to B, the causality is positive from A to B, whereas an odd number of negative direct causal relationship represents that the causality is negative.

Proof: To direct causality, the theorem is the truth. To indirect causality, according to the causality degree, assumed there is a route $C_i \rightarrow C_j \rightarrow C_k$ and $w_{ij}<0, w_{jk}>0$, where one weight is negative. The increase of the state value of C_i will inevitably leads to a decrease of the state value of C_j. The decrease of the state value of C_j will cause a decrease of the state value of C_k. That is to say, the increase of the state value of first node C_i leads to a decrease of the state value of target C_k. When one negative weight is extended to an odd number of negative weights, the truth still holds. The positive weight is the same way.

If in FCM the signs of causality in all routes from node A to node B are same, it is balanced from node A to node B. The balance is complete. If it is (completely) balanced between all nodes in FCM, the FCM is (completely) balanced.

Otherwise, if there are different signs of causality in the routes from node A to node B, it is imbalanced from node A to node B. If the number of positive routes is equal to the number of negative routes from node A to node B, it is completely imbalanced from node A to node B. If there is imbalance between any two nodes of FCM, the FCM is imbalanced. If there is completely imbalance between all nodes of FCM, the FCM is completely imbalanced.

Theorem 2

If the number of routes is odd from node A to node B in FCM, it is impossible that the FCM is completely imbalanced.

Proof: If the number of routes is odd from node A to node B in FCM, the number of positive routes is not equal to the number of negative routes from node A to node B. So it is not completely imbalanced from node A to node B. it is impossible that the FCM is completely imbalanced.

The imbalance from node A to node B in FCM leads to the total influences of concept A to concept B are not estimated rightly, which indicates conflicts exist in the FCM. The conflicts are measured by balance degree.

The key factor of influencing the balance of FCM

The earlier mentioned shows the number or length of routes influence the balance of FCM. In fact, the true key factor is the key edge of the routes.

Definition 1

Key edge is directed edge with causal relationship, which causes FCM to be imbalanced and is limited by the following three conditions: (1) to remove the causality can change the imbalanced FCM to the balanced; (2) the number of the edges to be removed is less; (3) the absolute value weight in causality of the edges is minimum in the same numbers.

One new balance degree of FCM

There are the following three wrong considerations in the calculation of balance degree shown in Equation (1) proposed by Tsadiras and Margaritis (2007):

$$r = \frac{1}{n^2}\left(\sum_i \sum_{j, j\neq i} \frac{\min\{p_{ij}, n_{ij}\}}{\mathrm{int}\left(\frac{p_{ij}+n_{ij}+1}{2}\right)} + \sum_i \frac{\min\{p_{ii}, n_{ii}\}}{\mathrm{int}\left(\frac{p_{ii}+n_{ii}+1}{2}\right)}\right) \quad (1)$$

1. The routes with minimum numbers between two nodes lead to the imbalance between the two nodes.

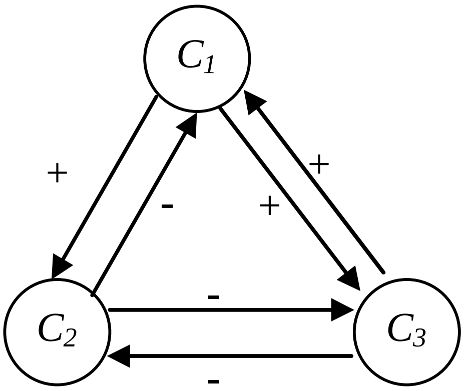

Figure 2. An imbalanced FCM with causality between any two nodes.

2. When the number of routes between two nodes is odd, FCM may be completely imbalanced, such as $p_{ij}=k$ and $n_{ij}=k+1$ or $p_{ij}=k+1$ and $n_{ij}=k$.
3. In the equation of balance degree, the 2 is not achieved, that is to say, when $p_{ij}=k$ and $n_{ij}=k+1$ or $p_{ij}=k+1$ and $n_{ij}=k$, the balance degree from C_i to C_j is not equal to 1.
We propose one new balance degree of FCM shown in Equations (2) and (3).

$$r_{ij}=\begin{cases}1 & n_{ij}=p_{ij}\\ \frac{ke_{ij}}{t_{ij}} & n_{ij}\neq p_{ij}\end{cases} \tag{2}$$

$$r=\frac{1}{n^2}\sum_i\sum_j r_{ij} \tag{3}$$

Where n_{ij} is the number of negative routes from C_i to C_j, p_{ij}, the number of positive routes from C_i to C_j, ke_{ij} is the number of routes with key edge from C_i to C_j, t_{ij} total number of all routes from C_i to C_j and equal to the sum of n_{ij} and p_{ij}, r_{ij} the balance degree from C_i to C_j, r the balance degree of FCM, n the number of all nodes in FCM.

When the number of positive routes n_{ij} is equal to the number of positive routes p_{ij}, which is completely imbalanced from C_i to C_j, the balance degree is 1 from C_i to C_j. When the number of positive routes n_{ij} is not equal to the number of positive routes p_{ij}, the balance degree is ke_{ij}/t_{ij} from C_i to C_j. If it is completely balanced from C_i to C_j, $n_{ij}=0$ or $p_{ij}=0$, that is to say, there is no key edge, $ke_{ij}=0$, the balance degree is 0 from C_i to C_j. Because there are n^2 routes at most in FCM, $1/n^2$ is a factor in Equation (3).

A method of searching for key edge

The key problem is to search for key edge, the root of conflicts, from imbalanced FCM.

Theorem 3

If it is balanced from node A to node B, the map composed of nodes and edges between node A and node B is balanced.

Proof: Assumed that the map composed of nodes and edges between node A and node B is imbalanced, that is to say, there is some route, which is imbalanced, from node C to D and through the edges between node A and node B. Because node C and D are both in the route from node A to node B, there is a route from node A to B through node C and D and some edges, which leads to the route from A to B imbalanced. So the assumption is wrong.

Theorem 4

If it is imbalanced from node A to node B, φ_1 is the set composed of nodes and edges in all routes from A to B, φ_2 is the maximum balanced sub-set of φ_1, key edge must be in the edge set of $\varphi_1-\varphi_2$.

Proof: Assumed that key edge is not in the edge set of φ_1-φ_2, for φ_1 being an imbalanced set, it must be in the set of φ_2, which leads to the map composed of φ_2 is imbalanced. The conclusion is contrary to that φ_2 is balanced sub-set of φ_1. So the theorem is true.

The steps of searching for key edge are as the follows according to the earlier stated theorems:

Step 1: search the longest route in FCM and set φ_1 as the set of nodes and edges in the route.

Step 2: to φ_1:

1. if φ_1 is balanced, the sub-set is also balanced, so key edge is not in φ_1.
2. else:

(a) search maximum sub-set as φ_2 of φ_1.
(b) to each φ_2:

(i) if φ_2 is balanced, key edge is in φ_1-φ_2;
(ii) else, $\varphi_1=\varphi_2$ and go to (a).

RESULTS

As shown in Figure 2, removing the edge from C_1 to C_3, the FCM can be changed into being balanced with no consideration of weights. And to the kind of FCM (causality between any two nodes), the balance only

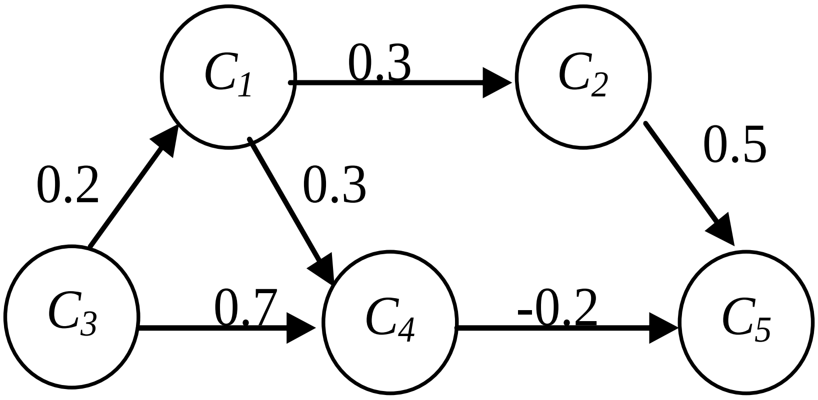

Figure 3. An imbalanced FCM with weights.

exists in two situations. One is all weights of the causal relations in all nodes are positive, the other is to remove some edge.

In Figure 3, it is seen that the two edges from C_4 to C_5 and C_2 to C_5 seem to be viewed as key edge, which leads to opposite signs of the routes from C_1 to C_5 and from C_3 to C_5. Because of $|w_{45}| < |w_{25}|$, the key edge should be the edge from C_4 to C_5. Certainly, to remove the two edges from C_3 to C_1 and from C_1 to C_4 also can transform into a balanced FCM. But it is clear that it does not correspond with minimum number of the edges in the definition of key edge.

The balance degree of FCM in Figure 3 according to Equation (1) is $r = \frac{1}{5^2}(1+\frac{1}{2}) = \frac{3}{50}$.

The new balance degree of FCM in Figure 3 according to Equations (2) and (3) is $r = \frac{1}{5^2}(1+\frac{2}{3}) = \frac{1}{15}$.

The method searching the conflicts in FCM of Figure 3 is: the longest route set φ_1 from C_3 to C_5 is unbalanced. The route set φ_2 included from C_3 to C_2 and from C_3 to C_4 and they are balanced. So the key edge exists in $\varphi_1 - \varphi_2$. Because of the two routes of φ_2, the smallest difference from C_4 to C_5 is key route according to section (3) in Definition 1.

DISCUSSION

For example, in Figure 3, the balance degree proposed by Tsadiras holds that the imbalance from C_3 to C_5 is caused by the route of $C_3 \to C_1 \to C_2 \to C_5$. In fact, the key edge $C_4 \to C_5$ or the routes of $C_3 \to C_1 \to C_4 \to C_5$ and $C_3 \to C_4 \to C_5$ are the root of imbalance of the FCM. Therefore, Equation (1) is not right in computing the balance degree of FCM and Equation (3) are more reasonable, although the two results only differ by $\frac{1}{50} = (\frac{1}{15} - \frac{3}{50})$.

Conclusions

The paper has solved three problems based on traditional balance and balance degree of FCM. Firstly, we can determine whether FCM balanced or not by the studies further on causality and balance of FCM and find the root is key edge if FCM is imbalanced. Secondly, how much conflicts in FCM are measured by one new balance degree. Thirdly, a method is proposed to search for the conflicts in FCM. The methods and results are better than traditional methods.

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