

Full Length Research Paper

Programming of geodetic transformation methods

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Coordinate transformation is widely used in geodetic application. By a coordinate transformation process, position of points with known coordinates in one coordinate system is transformed into a different coordinate system. Mostly, Helmert (similarity), Affine and Projective transformations are used for two dimensional transformations and Bursa-Wolf transformation is used for 3 dimensional transformations. The orthometric heights are used for heights in the mapping and engineering projects. The determined heights with GPS are ellipsoidal heights. Therefore, the ellipsoidal heights to orthometric heights conversion problem has emerged. For this purpose, fair accuracy of geoid undulations must be known. In this study, a program has been developed that can perform one, two and three-dimensional transformations. With this program, a suitable surface is conveyed by utilizing the basic points of which geoid undulations are known and the x, y coordinates that are proved to be compatible with using orthogonal polynomials in one dimension. Geoid undulations that their x, y coordinates are known at certain points can be calculated with this surface. In two and three dimensional transformation, however, outlier test can be conducted by using the both system coordinates of common points, transformation parameters can be determined and according to outlier and the 2nd system coordinates of the points which their coordinates are known in the 1st system, can be calculated in 2 dimensional and 3 dimensional transformations.

Key words: Coordinate transformations, similarity, affine, projective, Bursa-Wolf, geoid, global positioning system, ellipsoid height, orthometric height.

INTRODUCTION

Nowadays, global positioning system (GPS) is widely used in geodetic studies. Latitude, longitude and ellipsoidal height of points at a global geocentric coordinate system are determined with GPS. In map making and engineering works, however, orthometric heights of points have to be used. In order to directly transform Ellipsoid heights obtained by GPS into orthometric height, it is necessary to know geoid undulations with adequate accuracy (Kiliçoğlu, 2002). The process of finding provisions of point coordinates in another coordinate system that were given or calculated in a coordinate system is called "coordinate transformation". As a result of the transformations, there won't be any changes in the physical places of the points but only the coordinates of

the points will be converted from one system into another. The places where the coordinate transformation can be applied are given below:

- 1) The maps that are made in a different coordinate system and re-drawn according to a newly selected system.
- 2) Wrong determination of a selected axis system and related determination of the equivalents in an appropriate system
- 3) In the search of deformation
- 4) In eliminating the differences in Datum
- 5) In calculation of the provisions of AGA points and the coordinates in ED50 Datum in TUTGA system
- 6) In Photogrammetry; Coordinate transformation is applied in transition from tool coordinates to image coordinates, from the image coordinates to terrain coordinates or from the pixel coordinates to photo coordinate system (Başçiftçi and Inal, 2008).

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Figure 1. Introduction screen as a user interface of the program.

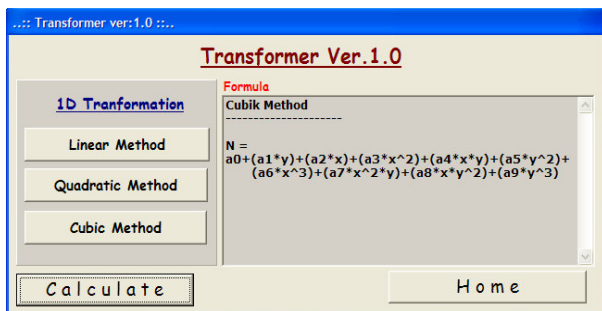


Figure 2. User interface of 1D transformation.

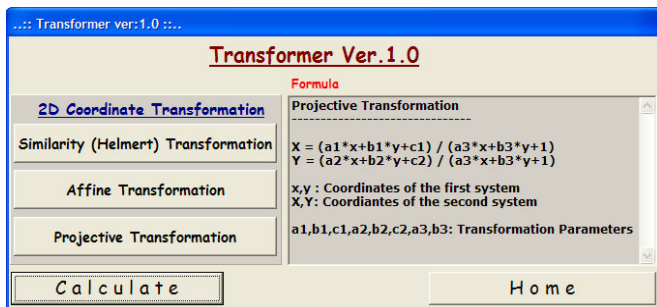


Figure 3. User interface of 2D coordinate transformation.

MATERIALS AND METHODS

In this study, 1, 2 and 3D transformational method programming were made. In 1D transformation the calculations of geoid undulations were made by using orthogonal polynomials in interpolation with polynomial surface. Polynomial interpolation with the surface is explained by many workers (Torg, 1980; Liddle, 1989; Ollikainen, 1997; Zhan-Ji, 1998; Petrie and Kennie, 1987; Inal, 1996; King et al., 1985; Başçiftçi, 2008). In 2D transformation coordinates in x, y system (1st system) are transformed into the XY system (2nd system) by using the transformational parameters calculated by using the known coordinates or enough conjugate point coordinates. Programming of similarity transformation, affine and projective transformations methods were performed in 2D transformation studies which were explained in many studies (Turgut

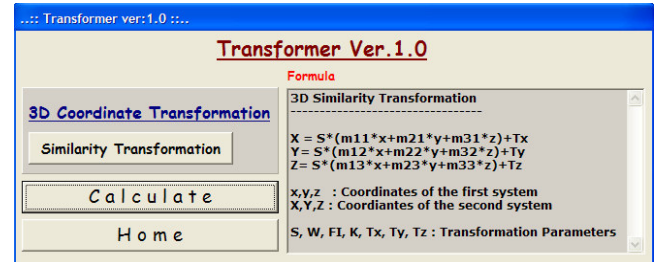


Figure 4. User interface of 3D coordinate transformation.

and Inal, 2003; Inal and Turgut, 2001; Wolf and Ghilani, 1997; Mikhail and Weerawong, 1997; Wolf and Dewitt, 2000; Başçiftçi, 2008). Scale factor is considered as a constant for all directions in 7-parameter similarity transformation in 3D. Similarity transformation is preferred because it needs fewer calculations and its mathematical model appears to be more easily applicable (Ustün, 1996; Bursa, 1962; Wolf, 1963; Başçiftçi, 2008; Hofmann-Wellenhof et al., 1997; Leick 1990).

Programming of transformational methods

A program was written in DELPHI programming language for geodetic transformation method and for the 1, 2 and 3D coordinate transformations to be done and for outliers to be extracted. The program is called "Transformer". With this program, the geoid undulations of control points can be calculated by using the basic points and the orthogonal polynomials. More over, according to 2 and 3D coordinate transformation methods parameters can be calculated, outliers can be extracted and the points whose coordinates are known in the 1st system can be calculated as 2nd system coordinates by using the system coordinates of the common points. (Figure 1) shows the user interface of the program.

In the case of pushing the "One-Dimensional Transformation" button given in (Figure 1), applications of linear, quadratic and cubic methods will appear as in (Figure 2).

In the case of pushing the "Two-Dimensional Coordinate Transformation" button which is shown in (Figure 1), applications of similarity, affine and projective transformation methods will appear as in (Figure 3).

In the case of pushing the "Three-Dimensional Coordinate Transformation" button which is shown in (Figure 1), application of 3D similarity transformation method will appear as in (Figure 4).

The desired method is chosen from 1D transformation methods from the screen at (Figure 2) and by clicking on the "Calculate" button, the selected calculation interface will appear as in (Figure 5).

The desired 2D coordinate transformation method is chosen from the screen when it's at (Figure 3) and by clicking the "Calculate" button, the selected calculation interface will appear as in (Figure 6).

The intended 3D similarity transformation method is chosen from the screen when it's at (Figure 4) and the selected calculation interface by clicking the "Calculate" button will appear as in (Figure 7).

The data can be entered manually to the program as well as it can be entered from a text file created in windows. If the data belonging to the common points are desired to be entered manually, the numbers of points the 1st and the 2nd system coordinates in 2 and 3D coordinate transformation methods with geoid undulation (N) values in 1D transformation method should be written to the appropriate locations and by using of "Write register" button data belonging to other points can be entered by skipping

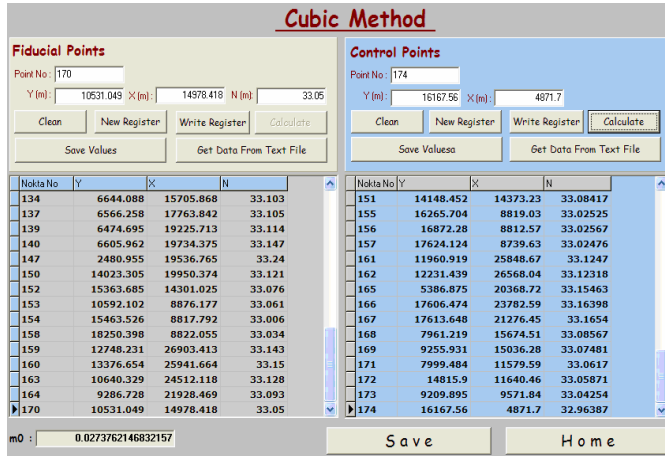


Figure 5. User interface for the computation of 1D transformation.

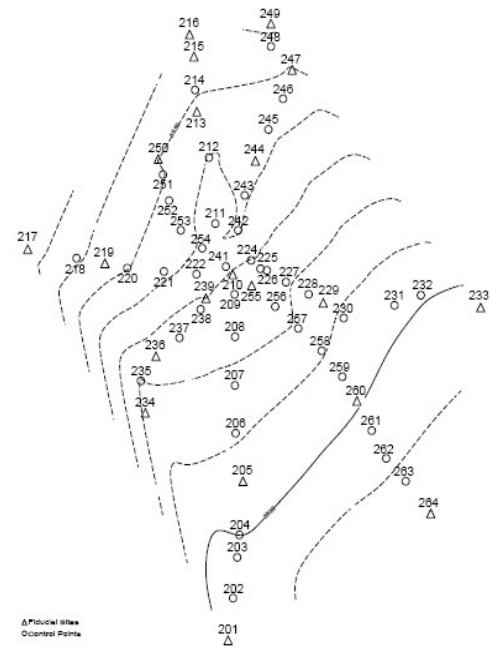


Figure 8. The support sites and the control points.

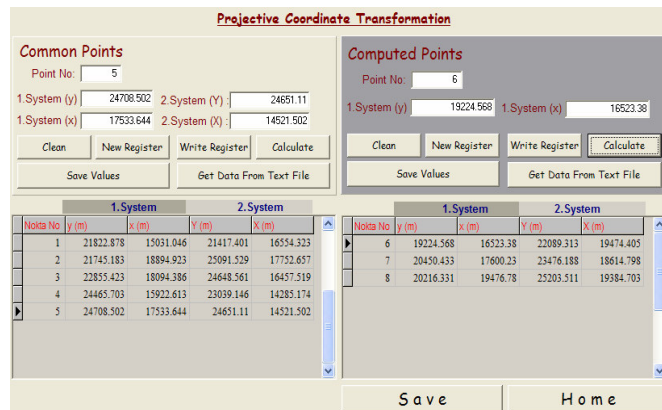


Figure 6. User interface for the computation of 2D coordinate transformation.

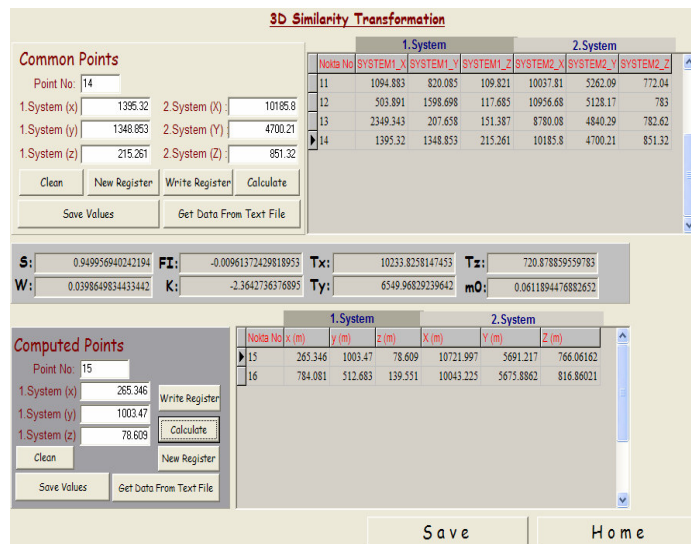


Figure 7. User interface for the computation of 3D coordinate transformation.

the bottom line. All of the entered data can be deleted by using the “Clear” button in the interface, manually entered data values can be stored as text file with “Save” button and transformation parameters can be calculated with “Calculate” button. Manual data entry for the calculated points can also be done as described in common points. The 2nd system coordinates in 2 and 3D coordinate transformation methods and the geoid undulations (N) of points in 1D transformation method are calculated by “Calculate” button.

Application

There are 64 points in the work area that Gauss-Kruger projection coordinates and the geoid undulation are known. In the appropriate distribution, 20 points were the basic points and the remaining 44 points were selected as the control points (Figure 8). Geoid undulations at 44 control points were calculated by means of the “Transformer Programme” which was built in the language of DELPHI.

An incompatibility dimension test had been applied to all orthogonal polynomials surfaces. As a result of this test basic point No. 217 was detected as non-incompatible in linear method and the remaining 19 basic points and the indeterminateness of surface had been re-solved. All of 20 basic points were deemed to be compatible in quadratic and cubic method. Geoid undulations of 44 control points had been determined with the help of GPS leveling and indeterminateness which had been found previously are shown in (Figure 9). Undulations of 44 control points determined with the help of found indeterminateness are shown in (Figure 10).

In order to determine the one that gives the best result among the applied orthogonal polynomial surfaces, the root mean square errors of the unit measurement were calculated by using the deviations from the basic sites of the surface point. The root mean square errors are given in (Table 1). Transformation parameter has been calculated by using the 5 points, whose coordinate accuracy is known in both systems in 2D similarity, affine and projective transformation (Tables 2 and 3). In addition, the 2nd system coordinates

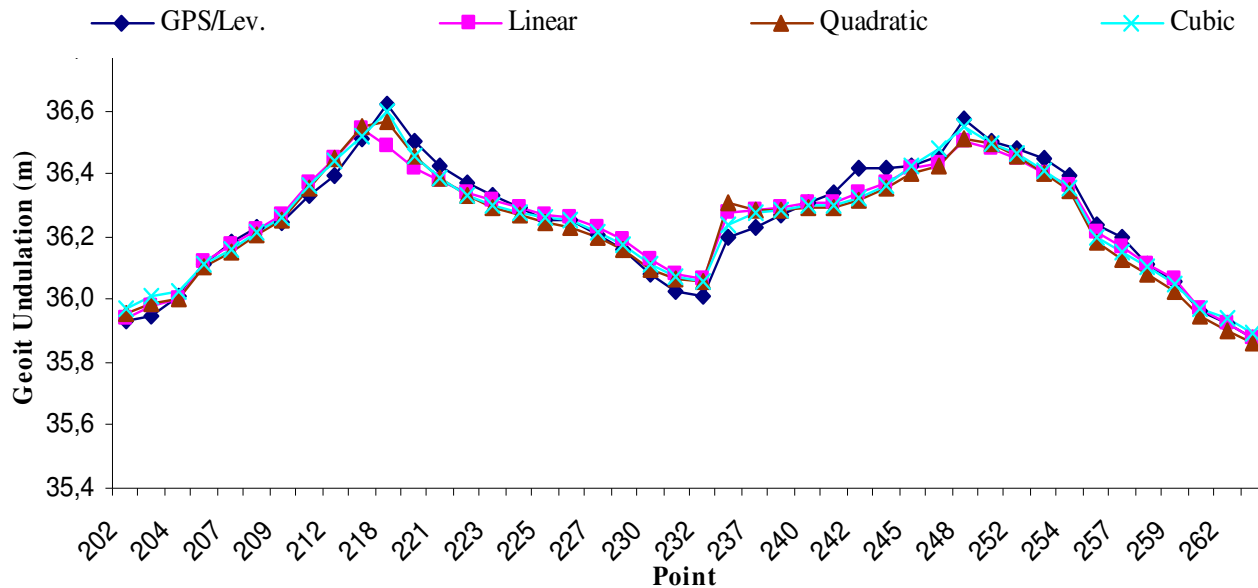


Figure 9. The geoid undulations at the control points.

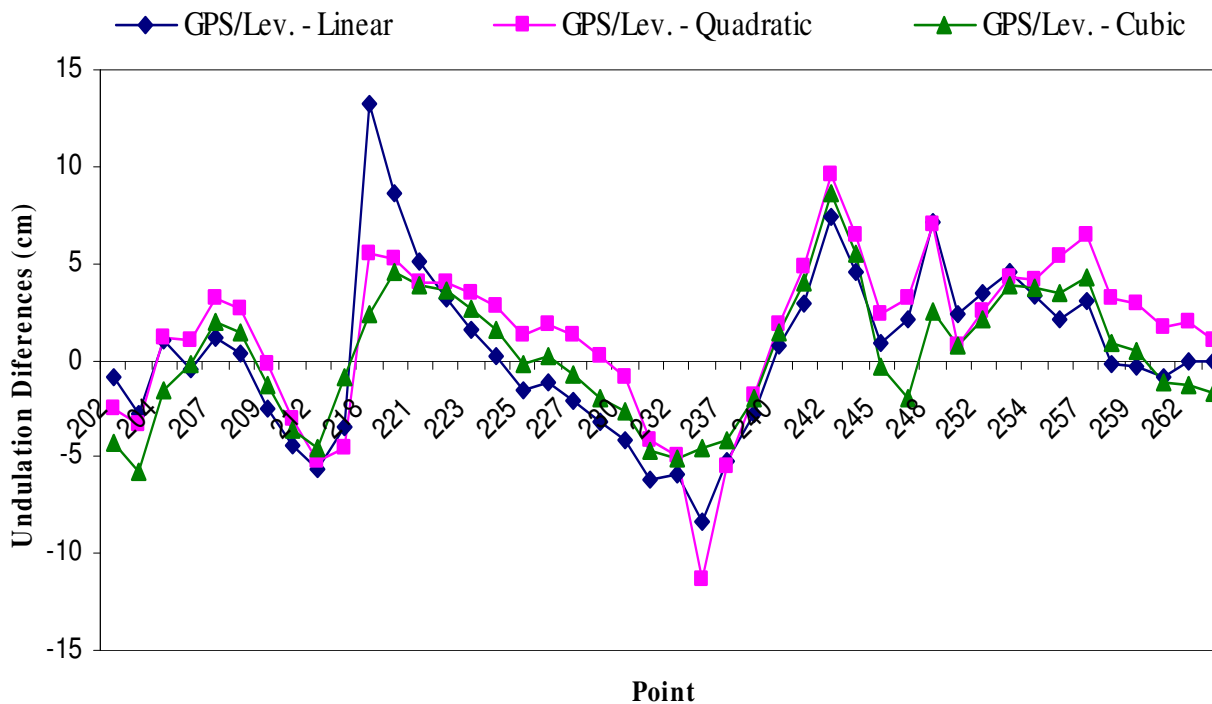


Figure 10. Geoid undulation differences at control points.

Table 1. The root mean square errors determined from the interpolation of the orthogonal polynomials.

	Linear m_0 (cm)	Quadratic m_0 (cm)	Cubic m_0 (cm)
First step	± 6.36	± 6.11	± 3.13
Second step	± 4.86		

Table 2. Coordinates of common points in both systems for Two-dimensional transformation.

NN	First system				Second system			
	y (m)	m _y (m)	x (m)	m _x (m)	Y (m)	m _Y (m)	X (m)	m _X (m)
1	21822.878	0.017	15031.046	0.013	21417.401	0.029	16554.323	0.021
2	21745.183	0.018	18894.923	0.011	25091.529	0.015	17752.657	0.023
3	22855.423	0.016	18094.386	0.011	24648.561	0.012	16457.519	0.009
4	24465.703	0.012	15922.613	0.016	23039.146	0.014	14285.174	0.017
5	24708.502	0.014	17533.644	0.015	24651.110	0.011	14521.502	0.014
6	19224.568		16523.382					
7	20450.433		17600.234					
8	20216.331		19476.782					

Table 3. The transformation coefficients in two-dimensional transformation.

Similarity transformation		
Coefficient	m _x , m _y not considered	m _x , m _y
a	0.29088260	0.29088270
b	0.95673738	0.95673726
c	33060.8160	33060.8114
d	688.74156	688.74093
Affine transformation		
Coefficient	m _x , m _y not considered	m _x , m _y considered
a	0.29089337	0.29088226
b	-0.9567472	-0.95673684
c	33060.8738	33060.8089
d	0.95673969	0.95673774
e	0.29088078	0.29088314
f	688.7465	688.7225
Projective transformation		
Coefficient	m _x , m _y not considered	m _x , m _y considered
a ₁	0.29076861	0.29076860
b ₁	-0.95665885	-0.95665884
c ₁	33060.7185	33060.7185
a ₂	0.95655499	0.95655499
b ₂	0.29099581	0.29099581
c ₂	688.8878	688.8878
a ₃	0.00000000	0.00000000
b ₃	0.00000000	0.00000000

of points No.6, No.7 and No.8 whose coordinates in the 1st system are known and were determined with the help of transformation parameters (Tables 4 and 5). The root mean square errors calculated by considering coordinate accuracy and without considering coordinate accuracy in two-dimensional transformation is given in (Table 6). Transformation parameters were also calculated in 3D similarity transformation by using 4 points whose coordinates and coordinate accuracy are known on both systems (Tables 7 and 8). In addition, the 2nd system coordinates of points No.15 and No.16 whose coordinates in the 1st system are known, were determined

with the help of transformation parameters (Table 9).

RESULTS AND CONCLUSION

By using the developed program called “transformer” 1D, 2D and 3D, transformations can be performed. In this program, linear, quadratic and cubic methods for 1D transformation, similarity, affine and projective

Table 4. Transformed coordinates calculated without considering coordinate accuracy in two-dimensional transformation.

NN	Similarity transformation		Affine transformation		Projective transformation	
	Y (m)	X (m)	Y (m)	X (m)	Y (m)	X (m)
6	22089.369	19474.317	22089.378	19474.363	22089.313	19474.405
7	23476.215	18614.723	23476.223	18614.768	23476.188	18614.798
8	25203.484	19384.553	25203.497	19384.620	25203.511	19384.703

Table 5. Transformed coordinates calculated by considering coordinate accuracy in two-dimensional transformation.

NN	Similarity transformation		Affine transformation		Projective transformation	
	Y (m)	X (m)	Y (m)	X (m)	Y (m)	X (m)
6	22089.369	19474.316	22089.376	19474.351	22089.307	19474.403
7	23476.214	18614.723	23476.222	18614.756	23476.185	18614.793
8	25203.483	19384.553	25203.496	19384.601	25203.510	19384.697

Table 6. The root means errors by the result of two-dimensional transformation.

	Similarity m_0 (cm)	Affine m_0 (cm)	Projective m_0 (cm)
As care of coordinate accuracy	± 1.98	± 1.87	± 1.51
Coordinate accuracy	± 2.08	± 2.06	± 1.62

Table 7. Coordinates of common points on both systems in three-dimensional transformation.

First system						
NN	x (m)	m_x (m)	y (m)	m_y (m)	z(m)	m_z (m)
11	1094.883	0.007	820.085	0.008	109.821	0.005
12	503.891	0.011	1598.698	0.008	117.685	0.009
13	2349.343	0.006	207.658	0.005	151.387	0.007
14	1395.320	0.005	1348.853	0.008	215.261	0.009
15	265.346		1003.470		78.609	
16	784.081		512.683		139.551	
Second system						
NN	X (m)	m_x (m)	Y (m)	m_y (m)	Z(m)	m_z (m)
11	10037.810	0.050	5262.090	0.060	772.040	0.050
12	10956.680	0.040	5128.170	0.060	783.000	0.090
13	8780.080	0.020	4840.290	0.040	782.620	0.020
14	10185.800	0.030	4700.210	0.050	851.320	0.030

transformation methods for 2D and for 3D transformation Bursa-Wolf model which is one of the similarity methods were used. Geoid undulations of control points on test field were calculated by the developed program and the values were compared with the known GPS leveling values of the points in this one-dimensional practice.

As a result of the comparison:

1) The difference between the geoid undulations determined with the linear method and GPS/Leveling is between -8.39 cm and 13.24 cm and the root mean error is (m_0) ± 4.86 cm.

Table 8. Transformation coefficients in three-dimensional transformation.

Coefficient	m_x, m_y not considered	m_x, m_y considered
S	0.94995694	0.94996691
W	2° 17' 02".74	2° 16' 57".62
Φ	-0° 33' 02".97	-0° 33' 00".14
κ	224° 32' 13".5	224° 32' 15".9
T_x	10233.82581	10233.76464
T_y	6549.96829	6550.04551
T_z	720.87886	720.82704

Table 9. Coordinates of new points that transformed to the 2nd system in three-dimensional transformation.

NN	m_x, m_y not considered			m_x, m_y considered		
	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)
15	10721.997	5691.217	766.062	10721.952	5691.292	766.025
16	10043.225	5675.886	816.860	10043.174	5675.954	816.834

2) The difference between the geoid undulations determined with the quadratic method and GPS/Leveling is between -11.35 cm and 9.54 cm and the root mean error is (m0) \pm 6.11 cm.

3) The difference between the geoid undulations determined with the cubic method and GPS/Leveling is between -5.79 cm and 8.64 cm and the root mean error is (m0) \pm 3.13 cm.

4) Cubic method in the work area has a good approximation to the values determined by the GPS/Leveling.

As a result of 2D transformation practices, it is considered that the best way to compare the used methods is comparing the calculated coordinates of test points with the real coordinates. Accordingly, it can be said that projective transformation gives better result in comparison to the other two results when the root mean errors were examined. In addition, the transformation parameters calculated as balanced by taking the accuracy of status into account and it was concluded that the status accuracy of points' impact to the transformed coordinates is not that much. Bursa-Wolf model of similarity transformation practice used in 3D transformation showed that the impact of common points' coordinate accuracy does not have much impact on transformed coordinates.

ACKNOWLEDGEMENT

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