

Full Length Research Paper

Compton scattering of 662 keV gamma rays proposed by Klein-Nishina formula

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The free electron Compton scattering cross-section are given by Klein-Nishina, calculated using photon with an energy of 662 keV scattered incoherently through angles from 0-120°. The Compton scattering effect was calculated producing a linear relationship between the reciprocal of the scattered photon energy and $1/E_0$ as a function of $(1-\cos \theta)$, where θ is the scattering angle. It was found that the energy of scattered gamma ray decreases as the scattering angles increase. The differential scattering cross-section as a function of θ was calculated.

Key words: Free electron, cross-section, scattering, photon.

INTRODUCTION

Incoherent scattering is an important process of gamma rays with matter and it is dominant over the energy range of few keV to 2 MeV. It changes the phase and usually the energy of the incident photon. The magnitude of the change depends on the scattering angle and the energy of the incident photon. The incoherent scattering by free electrons is Compton scattering and the bound electron counter-part is often called atomic Compton scattering.

The Klein-Nishina formula gives the differential cross-section of photons scattered from a single free electron in lowest order of quantum electrodynamics (Klein and Nishina, 1929; Krishnaveni and Gowda, 2005; Shahi et al., 2001). At low frequencies (e.g. visible light), this is referred to as Thomson scattering; at higher frequencies (e.g., x-rays and gamma-rays), this is referred to as Compton scattering. However, incoherent scattering cross-sections and the incoherent scattering functions are essential in medical physics, radiography, dosimeter, radiation transport, crystallography, etc.

The discovery of Compton scattering is important because it demonstrates that light cannot be explained

purely as a wave phenomenon. Compton's work convinced the scientific community that light can behave as a stream of particles (photons) whose energy is proportional to the frequency. In the practical cases where the scattering electrons are always bound, the application of this formula implies the restriction that momentum transferred to the struck electron must be large compared to the square root of the binding energy of the electron (Acharya et al., 1981; Erzeneoglu, 1998).

Compton scattering is an interaction in which an incident gamma photon loses enough energy to an atomic electron to cause its ejection, with the remainder of the original photon's energy being emitted as lower energy gamma photon with an emission direction different from that of the incident gamma photon. It is predominant interaction mechanism radioisotopes emitting energy in the range of few hundreds keV. The knowledge of the energy of Compton scattered photons is essential in the determination of cross-section for the elastic scattering of gamma ray. The Compton and elastic peaks overlap, especially for high gamma ray energy and low atomic numbers at small angles (Christillin, 1986). In this study, 662 keV gamma rays were chosen as the source as it commonly used in gamma spectroscopy. At present the Compton scattering of 662 keV gamma rays were calculated in the range of 0-120° using Klein and

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Table 1. Energy $E_{\gamma(\text{theory})}$ and differential scattering with scattering angle.

Angle (θ)	Energy (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{theoretical}}, (\text{cm}^2/\text{sr}) \times E-26$
0	662	7.95
5	659	7.85
10	649	7.53
15	634	6.99
20	614	6.46
25	590	5.81
30	564	5.13
35	536	4.48
40	508	3.89
45	480	3.36
50	452	2.90
55	426	2.52
60	402	2.20
65	379	1.95
70	357	1.75
75	338	1.59
80	320	1.47
85	303	1.38
90	288	1.31
95	275	1.26
100	263	1.22
105	252	1.20
110	242	1.18
115	233	1.17
120	225	1.16

Nishina formula (Klein and Nishina, 1929).

MATERIALS AND METHODS

Compton differential cross section by Klein-Nishina

The values of scattered photon energy, E_{γ} (theory) was calculated

$$E_{\gamma} = \frac{E_{\gamma}}{1 + \frac{E_{\gamma}}{m_0 c^2} (1 - \cos \theta)} \quad (1)$$

The free electron Compton scattering cross-sections are given by the Klein-Nishina (2) formula based on the assumption that the electrons are free and at rest. The formula being:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{theory}} = \frac{r_0^2}{2} \left\{ \frac{1 + \cos^2 \theta}{[1 + \alpha(1 - \cos \theta)]^2} \right\} \times \left\{ 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{[1 + \cos^2 \theta][1 + \alpha(1 - \cos \theta)]} \right\}, \quad (2)$$

$\frac{\text{cm}^2}{\text{sr}}$

Where, $r_0 = 2.82 \times 10^{-13}$ cm (classical electron radius), $\alpha = \frac{E_{\gamma}}{m_0 c^2} = 1.29$, Is the reduced energy of the incoming photon, $d\Omega$ is the

measured solid angle expressed in steradians (for ^{137}Cs), Ω is the solid angle of detector.

This formula is valid for incident energies, several times larger compared to the binding energy of the target atom.

RESULTS

Energy calculation

The values of scattered photon energies, $E_{\gamma(\text{theory})}$ were calculated for various angles ranging from 0-120° using equation (1). It can be seen that, as the scattering angle increases, the energy of photon decreases. This is because, when gamma ray interacts with matter, the energies are decreased. The angular distributions of scattering cross-sections were calculated using equation (2). The theoretical calculation of photon energies and differential cross-sections with scattering angles are given in Table 1.

DISCUSSION

From data in Table 1, one graph of scattering energy, E_{γ} (theory) as a function of angle was plotted in Figure 1. It shows the relationship between scattering angle and scattering energy. Since the relationship between scattered energy E_{γ} and scattering angle θ is non linear

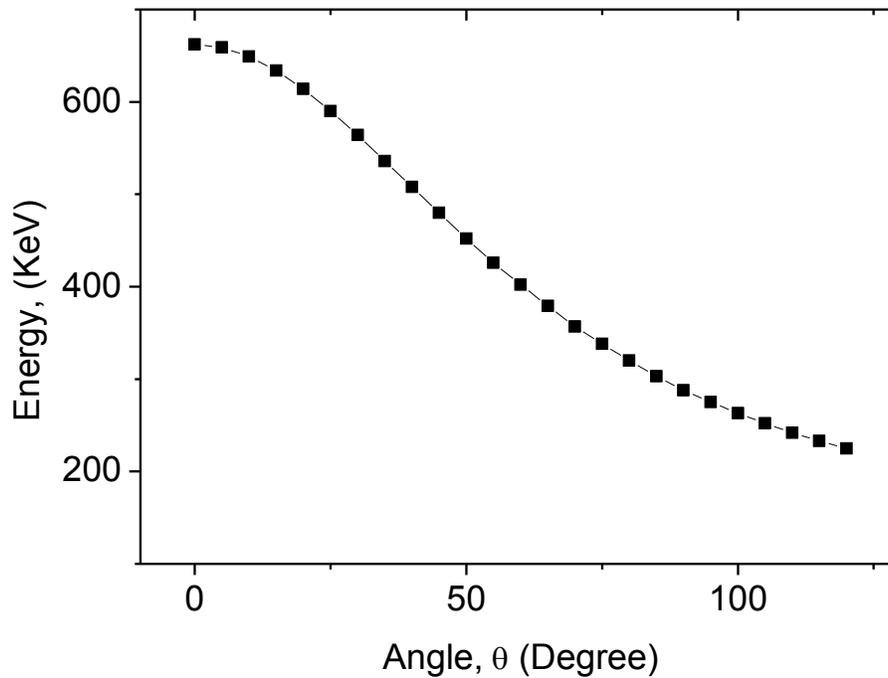


Figure 1. Theoretical Compton scattering energy as function of scattering angle.

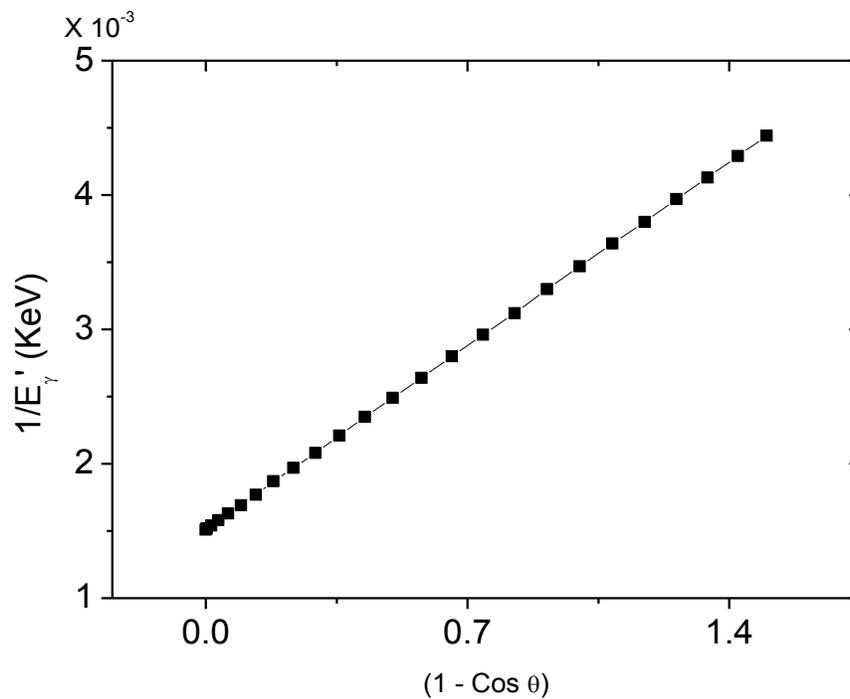


Figure 2. The Compton effects showing a linear relationship between $1/E_{\gamma}'$ and $(1 - \cos \theta)$.

and is only fitted by a curve, a simple manipulation of the Compton formula gives rise to linear relationship between

$1/E_{\gamma}'$ and $(1 - \cos \theta)$ as shown in Figure 2. It is shown that scattered energy decreases when the scattering angle

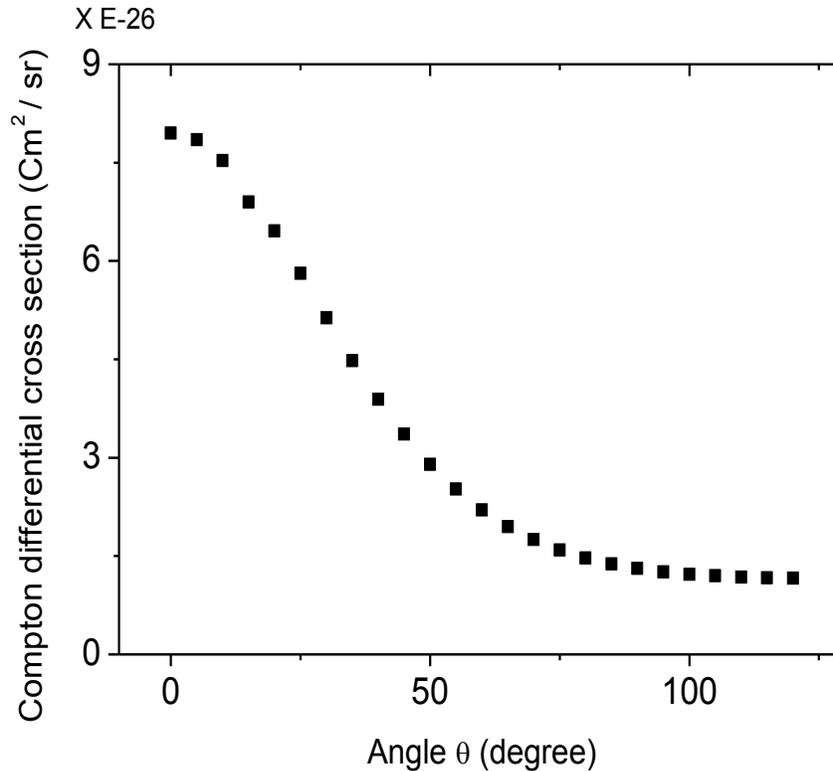


Figure 3. Theoretical Compton scattering cross-sections as function of scattering angle.

increases. In the Compton formula, it is assumed that the electrons in the outer orbital are loosely bound and transfer part of its energy to the electron. Conversion of energy and momentum allows only a partial energy transfer when the electron is not bound tightly enough for the atom to absorb recoil energy. The electron becomes a free electron with kinetic energy equal to the difference of the energy lost by the gamma ray and the electron binding energy. In reality, the electrons were initially in a strongly bound initial state, therefore causing significant deviation from the case of a free initial state. There is no limitation to the change in energy with respect to incidence angle. Compton scattering is strongly affected by the scattering electron's binding.

The theoretical value of differential cross-section area is dependent on angle θ . Figure 3 shows the differential cross-section area as a function of scattering angle. The scattering cross-sections were calculated for the incident 662 keV gamma rays of source ¹³⁷Cs. Theoretically, inelastic scattering cross-sections based on the Klein-Nishina formula are directly applicable in the high-energy region, where the inelastic scattering is a major part of the photon interaction cross-section. It is clearly seen that the curve have an almost similar pattern to scattered energy as a function of scattering angle. The calculated cross-section indicates the probability that photon suffers

deflection though some angles transfer to the electron an amount of momentum commensurate with that predicted for a free electron. The cross-section exponentially decreases as a function of angle θ . The scattering cross-section becomes almost constant at larger angles and rapidly decreases at lower angles. In Klein-Nishina formula, there is only depends on angle in order to evaluate the value of differential cross section.

Conclusions

The effect of Compton scattering of 662 keV gamma rays was calculated by Klein-Nishina formula. The angular distribution of scattered energy and differential cross-sections were calculated theoretically. There is a correlation that if the scattering angle increases; the energy of the scattered gamma ray and scattering cross-sections are decreased. The Compton effects produce a linear relationship between $1/E'_\gamma$ and $(1 - \cos \theta)$.

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