Scientific Research and Essay Vol. 2 (7), pp. 222-231, July 2007
Available online at http://www.academicjournals.org/SRE
ISSN 1992-2248 © 2007 Academic Journals

Full Length Research Paper

A mathematics vision from the usual environment

J. M. Chamoso

Department of Educational Mathematics and Experimental Sciences, Faculty of Education, University of Salamanca, Paseo de Canalejas 169, 37008-Salamanca, Spain. E-mail: jchamoso@usal.es.

Accepted 31 May, 2007

The search of the improvement of teaching and learning processes forces the education community to being kept worried in order to get the best way of presenting contents in the classroom, using the suitable materials and with a prepared organization. In order to obtain it, ideas can arise at any moment or place. One way to do it is using the nearest environment in order to work mathematics contents with activities of different levels. An example is presented, with suggestions for it application in the classroom. Nevertheless, every teacher should only act in this way if he/she thinks that it is the best way to get his/her teaching-learning aims.

Key words: Mathematics Education, mathematics from the environment, teaching-learning mathematics and mathematics practice.

INTRODUCTION

Students are demanded to learn a certain number of concepts with the argument that they will be necessary in their future life, for their later studies, for their working life or for their personal education. In many occasions, this is not easy to accept by them or to be explained by the teacher. It is even more problematic if it is referred to the learning of mathematics, a discipline that seems distant, lacking in perspectives and without direct effect in their personal future. Simultaneously, the mathematics curricula of most countries in the world recommend teaching mathematics in order to form future citizens from students' own contexts.

On the other hand, the educational current trends consider that it is necessary to get an education in which students are capable of reasoning. For this, it seems to be suitable that pupils acquire knowledge by themselves, develop different learning capacities and be active participants in the teaching-learning process. This is possible in a constructivist context in which the development of different types of capacities is looked for and in which it would not be sufficient to raise routine activities. It would also be necessary to modify the work of the teacher. This change implies passing from a teaching model to a learning one, from a magisterial classes model to a diversification of activities one. One of the bases of this reflection is that the aim consists in analysing, describing, explaining and understanding teaching and learning from a conceptual perspective where learning is conceived as a process in which pupils construct knowledge and teaching as the mediation of the teacher in this process.

With these premises a teaching-learning mathematics experience is proposed to develop in the classroom so that it is possible to work in a constructivist environment and based in the daily context. The principal aim is that students consider mathematics close to them. Next, there will appear the theoretical context in which this experience is fitted and, later, it will be showed that this is not only possible to be applied in geometric aspects, which is what is usually done, but in the most part of the mathematics contents. Finally, some reflections and conclusions of this proposal will be formulated.

MATHEMATICS TEACHING AND LEARNING AT PRESENT

In spite of the fact that along the 20th century there have been great and varied theories about how knowledge is acquired, which has repercussions on strong changes in education, these changes have not been reflected in the practice of the classroom. Traditional classes are kept in which mathematics is understood as a set of facts, procedures and solutions known that, basically, are in the manual or textbook. All in all, it seems that students are taught to do, not to think.

Mathematics is not only a set of facts and skills but, rather, a way of thought (Chamoso, 2000; Kehle, 1999).

The directives of the most advanced countries present a vision of students as people who think and reason. Specifically, NCTM (2000) recommends that students should study, principally, the same mathematics that was taught before but with a different approach, so that the purposes that all pupils should obtain with relation to the importance of the mathematical instruction should be: learn to value mathematics, feel confident of their aptitude to do mathematics, manage to solve mathematical problems, learn to communicate by means of the mathematics and learn to reason mathematically.

Many people think that education must help to develop the mental capacities of students. Mathematics teachers work so that students know a lot of mathematics but, besides that they learn a few concrete concepts, the aim must be the personal integral formation. People must be formed in order to be capable of confronting problems, who overcome difficulties and who recognize mistakes that they make. In many occasions it is learned more from mistakes that from successes. All in all, our work consists of preparing students in the today classes to live and work in the world of tomorrow (Burrill, 1998; Chamoso and Rawson, 2003a).

There exist researches that show how a mathematical knowledge is produced in more significant way than in the traditional education (for example, Carpenter et al., 1989; Cobb et al., 1991). It is necessary to pay special attention to the teacher's role so that the negotiation is something more encouraging than the traditional interaction of "step by step" controlled by the teacher, who must allow that students follow diverse ways, in the same way as in the future they will have to face to diverse problems as citizens that will not have an exact way or solution, or that they will make them take a decision that differs from that of the expert when, for example, they have to contract a life insurance, to organize their taxes, etc. That is to say, previously the aim was the community, without thinking about the individual, and now it is the individual but understood inside a group (Burrill, 1998; Voigt, 1994).

On the other hand, it is necessary to find situations that mobilize students both in an emotive and an intellectual way. Probably this one is the major achievement to which the current education can aspire: To manage to imply the pupils, to get them interested by what they are proposed and estimate the utility of what they learn. For it, teachers should approach curricular contents from the point of view of their social relevance. Why not take advantage of the environment potential in order to develop knowledge and forms of mathematical action?

SUGGESTIONS IN ORDER TO WORK GEOMETRIC ASPECTS

With these conditions it is possible to work in many ways. One of them is shown below, organized around geometric aspects. Some examples will be presented according

Figure 1. Tessellation.

to their purpose in the process of teaching, I mean, to introduce a concept, to explain a concept, to study a concept in depth, to explain properties, to demonstrate theorems, to discover geometric forms in the environment, to raise activities and routine or other type of problems.

To introduce a concept

Teacher can try to work the tessellation. His/her aim can be, for example, to see what flat regular forms tessellate the plane or what other types of forms also do it. One way of doing it is to discover tessellation in a nearby context as an introductory element, as those of the figure (Figure 1), from which it is possible to think about different coverings of the plane in soils, walls or artistic elements, and clarifies what is a tessellation and which are the characteristics of the figures that do it. Later, it is possible to continue with the manipulation of objects in the way that seems to be suitable and with the material that the teacher has arranged to get his/her aims of teaching-learning.

To explain concepts

It is possible to explain the concept of parallelism from the images attached (Figure 2). Obviously, parallelism does not exist in all of them, which forces to focus on the concept to decide which are the ones. Even reformulate the concept. It is allowed to be applied in order to discover the different relative positions of two straight lines in the plane, starting, for example, from an image such as this (Figure 3): two straight lines can be cut in 2 points and it is said that they coincide, in 1 point and it is said

Figure 2. Concept of parallelism.

Figure 3. Relative positions of two right lines.

that they cut between them or in 0 points and it is said that they are parallel. The same image allows to introduce the concept of straight angle as the outcome of dividing the plane in 4 equal parts and to observe that the other types of angles are bigger or smaller than it, and therefore they can be considered according to it, as multiples and divisors, or with other type of relations.

Figure 4. Concept of angle.

To study concepts in depth

It is not difficult to introduce the angle concept to students for the quantity of examples that appear in any context in real life. Nevertheless, problems of understanding can arise. For example, the top part of the window of this college is a semi circumference divided in four parts, each of which measures 45º (Figure 4). But in the interior part the reference that marks this opening is missing, which allows to think if the angle is the physical space between two lines, which in this case are not seen, or if it is the opening obtained when a draft that takes one line to another is done.

To establish and to demonstrate properties

There exist multitude of geometric elements in the daily life such as, for example, the polygon of four sides that appears in this wall and the rhombus that is contained in it (Figure 5). Their areas can be compared in order to deduce the general formula of the area of the rhombus connecting the diagonals of the latter with one to the

Figure 5. Area of a rhombus with relation to the rectangle that is contained in it.

Figure 6. Pythagoras's theorem.

sides of the rectangle. Other activities can also be proposed like, for example, to compare the two rectangles that are seen in order to study if they are proportional and to establish general rules for the proportionality of flat figures.

To demonstrate theorems

The mirror of this pub allows to verify Pythagoras's theorem in the particular case of an isosceles triangle (Figure

Figure 7. Golden spiral.

6). For it, it is enough to consider a square of side, the hypotenuse of the triangle and to discover that it is covered by sixteen squares (four by four). The same thing can be done with the other two sides of the triangle. In this way it is possible to verify that the area of the square that has the hypotenuse of the triangle as a side is equal to the sum of the areas of the squares that have its catetos as sides. This particular case allows to enunciate Pythagoras's theorem and to work different proofs and even generalizations of the above mentioned theorem in a similar way, with another type of manipulative materials or with technology (Chamoso and Rodríguez, 2004).

To discover geometric forms in the environment

It is possible to consider curves of usual contexts, discover their names and study how they are constructed like, for example, the spiral of the figure (Figure 7). It may be studied if it is a hyperbolic spiral, of Archimedes, golden or of Fibonacci and look for similar spirals in the environment. Next, it is possible that students are proposed the study of another type of curves that can be discovered anywhere and carry out similar activities.

To raise activities

The spiderweb of a playground (Figure 8) allows developing activities with geometric terms like angles, arches different from the same angle, radiuses, diameters, circular sections, circular sectors and concentric circumferences. Also, it is possible to generate geometric figures of different number of sides, joining successively the points of cut of the radiuses with the exterior circumference. For example, if they are connected in two at a time, a hexagon is formed; if it is done in three at a time, a square is made and, in four at a time, a triangle is formed. If they are joint in five at a time, a figure is not for-med but a star of many tops is created and it is possible to investigate what would happen if the points are connected in six at a time, for example, and with other number of points. There exist other ways of creating a star: for example, from the

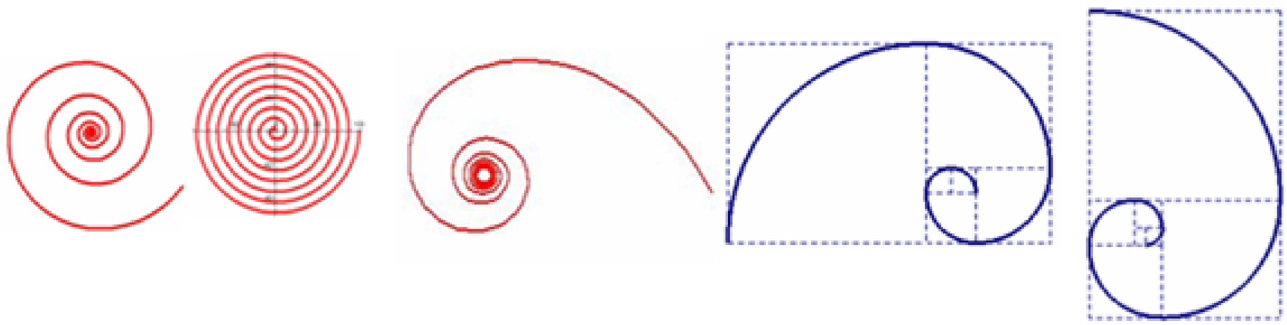

Figure 8a. Schematic drawing of Spiderweb of a playground to build mathematics.

Figure 8 b. Spiderweb of a playground to build mathematics.

Figure 9. Tangents circumferences in a rectangle.

square that was formed previously joining the points in four at a time and connecting the other points that remain free, three equal squares are obtained but turned, that is to say, with the vertexes in different points in the exterior circumference. Something similar could be made with triangles, repeating them in a successive way. The process can be repeated in the different concentric circumferences creating concentric forms that will be proportional if they are regular. And it is possible to study what would happen with the irregular ones. The spiderweb would also allow verifying Thales's theorem. On the other hand, it is possible to work fractions and observe their equivalence just by colouring the zone included between two radiuses in green, the contiguous space in blue, the following one in green again and so on, alternately, therefore 6 out of 12 will be coloured in green, which is the same thing that to say 1 out of 2.

To raise routine problems

Three circumferences of the centre placed inside the rectangle of the grille of this window (Figure 9) allow raising questions: what has major length, any of three circumferences or one of the major sides of this rectangle that contains them? This activity is similar to that which compares the circumference of the top edge of a long glass, in the shape of pipe, with its height. Or, also, what relation is established between the area of the rectangle and the areas of three circles? The three circumferences and the rectangle might be imagined in the space, that is to say, three spheres contained in a cylinder (as tennis balls kept in their case). In this case it is possible to calculate the relation that exists between the volume of three spheres and the space of the cylinder that stays without occupying.

To raise problems of another type

The star of the drugstore (Figure 10) allows wondering if the triangles are equilateral. To know that, it would be necessary to verify that all their angles are equal and measure 60º but it is not possible to do it physically. Probably the triangles could be considered as parts of a hexagon whose side coincide with that of every triangle and check if 6 triangles, as the ones in the star, would fill

Figure 10. Triangles in a star; are they equilateral?

Figure 11. Whole numbers in a thermometer.

the hexagon, even though other personal strategies can be used.

SUGGESTIONS TO WORK OTHER PARTS OF THE MATHEMATICS

Not only can the geometry make use of aspects extracted from the daily environment. It is possible to do the same thing with most of the concepts and mathematical applications. Next, examples in a classification similar to that of the previous section are going to be shown.

Figure 12. Cement sacks to calculate the media.

To introduce a concept

The thermometer of the wall allows discovering that the numbers that are over zero are positive but also those which are below (Figure 11). This fact allows thinking if it is a correct thermometer and if 2 below zero is equivalent to -2. The aim is to bring near the concept of whole numbers and to relate it with the naturals. In addition, it is possible to use the thermometer to establish a procedure that allows calculating the sum and subtraction of whole numbers, rising and going down in the thermometer as it is necessary. On the other hand, the value of the zero like whole number is different from the zero natural. Also it is possible to think other situations of the ordinary life with negative numbers.

To explain concepts

From the cement sacks that are piled up in columns (Figure 12) it is possible to introduce the concept of arithmetical average. To obtain it, it is enough to move the sacks from some columns to others until all of them have the same quantity. In that way it is possible to reach the essence itself of the concept of average instead of doing what is usually done, I mean, to pay attention to the way of calculating it. In this case the obtained result is not exact but it normally happens in the ordinary life. Also, it is possible to observe which is the column that has more sacks, that is to say, the mode. In addition, if the columns are arranged according to the number sacks, it is possible to calculate the median.

To study concepts in depth

The panel that informs about the telephone of Urgencies, 061 (Figure 13), has the digit 0 on its left, something unusual in our system of numeration because it is a digit that can be eliminated when it is on the left of any number. The same thing does not happen when it is on the right

Figure 13. A zero in the left of a number?

Figure 14. Equivalent fractions.

Figure 15. Succession of circles.

side because, for example, 2 designate 2 units whereas 20 symbolizes 2 tens due to the 0 that is on the right of 2. Therefore, 0 has a fundamental importance to represent numbers. It allows thinking of the reasons why, in 061, appears on the left side. On the other hand, it is possible to think about the advantages of using a positional system of this type, which can be compared with the difficulties that Romans had to carry out arithmetical operations with their numerical system. Romans did not either know the zero that, at present, works as any digit that symbolizes a certain value depending of its place in the number provided that it is on the right or in the middle of the digits that compose it, which allows to think over what would happen in our system of current numeration if the zero did not exist. To find other examples in which the zero appears to verify and to strengthen these ideas is not difficult since numbers are everywhere: in the number plates of the cars, in the advertisement of the sale of a flat (price, square meters, number of telephone of contact...) or in a shop window.

To demonstrate properties

This mosaic (Figure 14) allows observing that, since 4 tiles make a figure, 16 tiles complete 4. It allows thinking about the equivalence of fractions because 1/4 is equivalent to 4/16. That is to say, they are different fractions but the same rational number. Later other examples can be thought that make possible to verify the equivalence of fractions. Also symmetries, drafts and adjournments can be discovered.

To demonstrate formulas

The drawing of the van allows discovering a succession of circles of the same colour: 1, 2, 3 and 4 (Figure 15). In order to know how many are of the same colour it is necessary to do the sum of the first 4 natural numbers. It allows thinking about the number of white circles that there would be if the square had, for example, 10 white circles on the side. In addition, it is possible to deduce the formula of the sum of the first natural numbers from these circles. For example, with the first four numbers it is observed that, if it is added as many white circles as there already are and it is placed them the other way round in order that they form a rectangle, there will be as many circles as the product of one side by the other. In this case it will be 4 x 5. This value is, exactly, the double of the intended number. That is to say, 4 x 5/2 will be the sum of the first four natural numbers as it can be verified as 1 + 2 + 3 + 4 = 10. Also, the circles of the same colour can be understood as triangular numbers and another type of numbers can be studied with different placements like, for example, squares, pentagonals, hexagonals, perfect and friends numbers.

Figure 16. Balconies in geometric succession.

Figure 17. Graph in a column.

To raise activities referred to most of the mathematical contents

There appear several examples referred to different parts of mathematics:
i) The balconies of this building (Figure 16) is distributed according to a certain criterion: 1, 2, 4. That is to say, the number of balconies is doubled as you go up floors. It can allow wondering about the number of balconies that would exist if the building had 10 to 100 floors. And even to establish general formulas. In these cases, the sum of balconies is always an odd number. Is it always like that?

Figure 18. Combinations in a door.

ii) From the graph of the column of the figure (Figure 17) where there are bars above and below the central level, it is possible to think of a situation that remains reflected with this graph.
iii) Let's suppose that a family leaves the keys of their house in Paris to some friends in order that they stay there while they are on holidays. On their arrival, the latter discover that, to open the main entrance door, it is necessary to mark a number that makes possible that the door is opened (Figure 18). What are the possibilities of guessing the combination that opens the door depending on the quantity of digits that has the intended number?

To raise routine problems

The pigeonhole (Figure 19) allows observing regularities of the table of multiplying of 9. For example, if it is added to the digits of every number of the same column, the same result is obtained if congruence module 9 are made in all of them, that is to say, if the number obtained has more than one digit, it is necessary to go on adding its digits successively until you get a number with only one digit. For example, in the second column there are the numbers 2, 11, 20, 29, 38, 47, 56, 65... Also it is possible to carry out operations with congruence respect of the same module 9 since if a number from a column and other from a different one is taken, whichever the numbers chosen in each of them, the sum of both will be in the same column. The same thing happens with the subtraction, the multiplication and the division. In addition, other activities might be carried out like, for example, without doing the operation, give the sum of four numbers

Figure 19. Congruence module 9 in a pigeonhole.

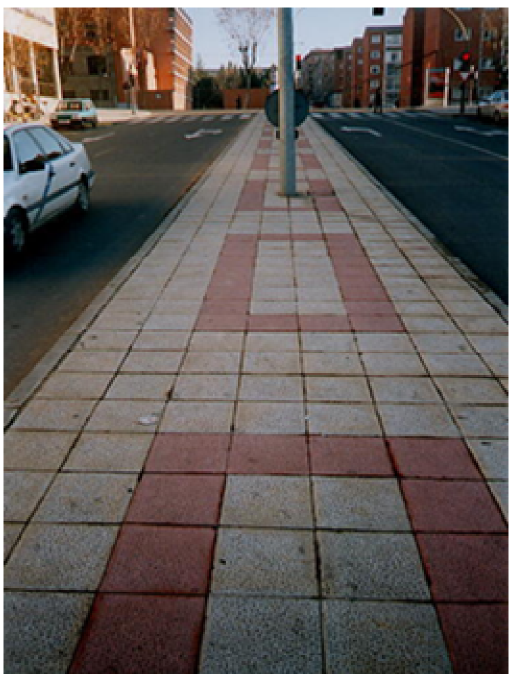

Figure 20. Comparing areas.

that form a square of two numbers of side, ascertain the square chosen. Or four numbers in the shape of an L. Or five in the shape of a cross. And so forth. Does anything slightly similar happen when it is considered a calendar of any month in which the congruence module 7 should be studied?

To raise problems of another type

The rectangle of the soil formed by white colour square tiles surrounded by dark tiles (Figure 20) allows wondering what dimensions should have the interior rectangle in order that its area was equal to the area of the exterior

Figure 21. Building the days of a month with two dices.

band. That is to say, it is a question of calculating the dimensions of the rectangle in order that the number of red squares is equal to the number of white squares. Naturally, it is supposed that the number of squares will have to change both lengthways and widthways. And it is possible to advance more and to wonder what would happen if the exterior band was 2 tiles broad.

To raise problems of "happy idea":

From the calendar composed by two usual dices (Figure 21) that, placed adequately, allow forming all the numbers of the possible days of any month, it is possible to wonder: what numbers will be in the faces of each one of the dices to be able to obtain all the numbers of any month? Though with two dices 36 possibilities can be formed and the maximum days of a month are 31, it is not so simple to obtain it.

CONCLUSIONS

The learning of mathematics can be carried out in very different forms depending on multiple factors like, for example, the aims of teaching, the degree of implication of the teacher, his/her formation and his/her desires to innovate. Traditionally mathematics has been conceived as a definite object that it is necessary to dominate. At present mathematics are understood as an opened form of thought, with margin to the creativity, and respecting the autonomy of every person. It seems that, in this way, the imagination and the own reasoning is stimulated, and the multiplicity of ideas is allowed. That is to say, it is considered that it doesn't exist one unique reality or one only way to solve problems.

For it, it seems to be suitable that learning is carried out from diverse contexts and arranging different didactic resources in order that it is more significant, functional and lasting. A possibility is to use the nearby environment to develop mathematical activities, something which students aren't used to doing. In this way one expects to favour discussions in the classroom that give entry to personal experiences, to take decisions, to the subjectivity and to the connection with other areas that allow to improve the own reasoning and that of the others.

From diverse images taken in the environment it has suggested to work contents and to carry out activities with students in which it is necessary to interpret, to devote and to connect with what one sees. One can work a great quantity of mathematical contents of the curriculum in the way that has been indicated but it is necessary to be able to discover it. Mathematics are neither in the van, nor in the track of the train nor on the walls but they are the result of an own construction. Because of that mathematics can be everywhere if one has a certain interest, puts his/her spirit in it and is specially biased towards it. It must generate rich and opened activities whose offer in the classroom can give place to the creativity, the divergence, the debate, the discussion, the orientation in a personal way, the explanation and the reasoning of the solutions of everyone. Naturally it has disadvantages as, for example, that, in this way, the contents turn out to be interlaced among them and have different levels of mathematical knowledge but it is also interesting. That each teacher uses it or not depend on his/her confidence in this form of work.

A work in this sense would demand a great effort from the teacher but it is also hoped that it would motivate students and would bring the contents over to realities of the daily life. The ideas can arise in any moment and place to be able to use them to work multiple mathematical contents by means of activities of different level in order to, for example, introduce a specific topic, study in depth contents studied before in class or revise and consolidate contents included in one or several didactic units, depending on the possibilities of the educational context, the available time and the pupils themselves. Teachers must decide and organize carefully the methodology that they are going to use in their classes according to the aim of education that is claimed.

But they can also do it in many other ways (for example, Chamoso 2003, 2004, 2006; Chamoso and Rawson, 2003a, 2003b, 2004; Chamoso et al., 2005; Chamoso et al., 2007; Chamoso et al., 2005). Or starting from the own context of the student himself, it is possible to discover what he/she needs to organize his/her room and how he could do it otherwise, how to place his/her clothes in the cupboards depending on their use, study his/her genealogical tree, organize his/her own schedule of class or other activities that carries out, study the expenses of the college understood as books, notebooks and other things and the clothes that it is necessary to buy every year, the calories that he/she should take every day in order that the food is balanced, the weight that can load depending on his/her own one, etc.

It is pretended to look at the world with mathematical eyes. Or, rather, to look at the world and to discover that, in it, there is mathematics.

ACKNOWLEDGEMENT

This work has been partially supported by the Consejería de Educación y Cultura del Gobierno Regional de Castilla y León (Spain) under grant US05/06.

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