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An extension of fuzzy TOPSIS approach based on centroid-index ranking method

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The last few decades have seen a large number of methods for ranking fuzzy numbers; centroid-index based approaches are the most commonly used among them. However, there are some weaknesses associated with these centroid-indices. Therefore, this paper reviews several fuzzy numbers ranking methods based on centroid-indices and proposes a new centroid-index ranking method that is capable of effectively ranking various types of fuzzy numbers. The proposed centroid-index ranking method uses fuzzy TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) to solve multi-criteria decision making (MCDM) problems, where triangular fuzzy numbers express the ratings of each alternative and importance weight of each criterion. To avoid complicated calculations of fuzzy numbers, the normalized weighted ratings are defuzzified into crisp values to simplify the calculations of distances from each alternative to the ideal and to the negative ideal solutions. A closeness coefficient is defined to determine the ranking order of alternatives. The proposed method is applied to a parting surface evaluation and selection problem in plastic mold design, demonstrating its applicability and computational process.

Key words: Ranking fuzzy numbers, centroid index, fuzzy TOPSIS, parting surface.

INTRODUCTION

Ranking fuzzy numbers plays a very important role in decision making, optimization, and other usages. Following the pioneering work of Jain (1976), who used maximizing sets to order fuzzy numbers, the literature encompasses numerous ranking techniques that have been proposed and investigated (Asady, 2010; Chou et al., 2011; Ezzati et al., 2012; Wang and Lee, 2008; Wang and Luo, 2009; Wang et al., 2006). Among the ranking approaches, the centroid methods are the most commonly used that are highly cited and have wide applications (Abdullah and Jamal, 2010; Chen and Chen, 2003; Cheng, 1998; Chu and Tsao, 2002; Lee and Li, 1988; Mehdizadeh, 2010; Ramli and Mohamad, 2009; Vencheh and Mokhtarian, 2011; Wang and Lee, 2008; Wang et al., 2006; Wang, 2009; Yager, 1980). Yager (1980) was the first researcher to propose a centroid-index for ranking fuzzy numbers. Since then, Cheng (1998) presented a ranking approach for trapezoidal fuzzy numbers based on distance index. The distance index can be defined as $R(A)=\sqrt{\bar{x}_A^2+\bar{y}_A^2}$, with $\bar{x}_A=\dfrac{\int_a^b xf_A^L dx+\int_b^c xdx+\int_c^d xf_A^R dx}{\int_a^b f_A^L dx+\int_b^c dx+\int_c^d f_A^R dx}$, and

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$\bar{y}_A = \varpi . \frac{\int_0^1 y g_A^L dy + \int_0^1 y f_A^R dy}{\int_0^1 g_A^L dy + \int_0^1 g_A^R dy}$; f_A^R and f_A^L are the respective right and left membership functions of A; and g_A^R and g_A^L are the inverse of f_A^R and f_A^L, respectively. The larger the value is of $R(A)$, the better the ranking will be of A. Chu and Tsao (2002) found that the distance method proposed by Cheng (1998) still had some shortcomings. Hence, to overcome the problems, they proposed a new ranking index function $S = \bar{x}_A \bar{y}_A$, where $\bar{x}_A$ is similar to $\bar{x}_A$ in Cheng (1998) and $\bar{y}_A = \frac{\int_0^w y g_A^L dy + \int_0^w y g_A^R dy}{\int_0^w g_A^L dy + \int_0^w g_A^R dy}$. The larger the value is of $S(A)$, the better the ranking will be of A.

In some special cases, the method proposed by Chu and Tsao (2002) also has the same shortcomings as that in Cheng's (1998) method. The shortcomings of Cheng's and Chu and Tsao's centroid-index are as follows. For fuzzy numbers A, B, C and $-A, -B, -C$, according to Cheng's centroid-index $R = \sqrt{(\bar{x})^2 + (\bar{y})^2}$, the same results are obtained - that is, if $A < B < C$, then $-A < -B < -C$. This is clearly inconsistent with mathematical logic. For Chu and Tsao's centroid-index $S = \bar{x}\bar{y}$, if $\bar{x} = 0$, then the value of $S = \bar{x}\bar{y}$ is a constant zero. In other words, the fuzzy numbers with centroids $(0, y_1)$ and $(0, y_1), (y_1 \neq y_2)$ are considered the same. This is also obviously unreasonable.

In a study conducted by Wang et al. (2006), the centroid formulae proposed by Cheng (1998), Chu and Tsao (2002) are shown to be incorrect. Therefore, to avoid any more misapplication, Wang et al. (2006) presented the correct centroid formulae as:

$$\bar{x}_A = \frac{\int_a^b x f_A^L dx + \int_b^c x w dx + \int_c^d x f_A^R dx}{\int_a^b f_A^L dx + \int_b^c w dx + \int_c^d f_A^R dx}, \quad (1)$$

and

$$\bar{y}_A = \frac{\int_0^w y[g_A^R(y) - g_A^L(y)]dy}{\int_0^w [(g_A^R(y) - g_A^L(y)]dy}. \quad (2)$$

To overcome the shortcomings of these existing centroid ranking approaches, this paper proposes a new centroid-index ranking method based upon the centroid formulae of Wang et al. (2006). The proposed method herein uses fuzzy TOPSIS to solve multi-criteria decision making (MCDM) problems, which presents the ratings of each alternative and importance weight of each criterion as triangular fuzzy numbers. To avoid any complicated aggregation of irregular fuzzy numbers, these weighted ratings are defuzzified into crisp values by the proposed centroid-index ranking method. A closeness coefficient determines the ranking order of alternatives by calculating the distances of alternatives to both the ideal and negative-ideal solutions. A parting surface evaluation and selection problem in plastic mold design demonstrates the computational process and applicability of the proposed model.

FUZZY NUMBER

There are various ways of defining fuzzy numbers. This paper defines the concept of fuzzy numbers as follows (Dubois and Prade, 1978; Kaufmann and Gupta, 1991).

Definition 1. A real fuzzy number A is described as any fuzzy subset of the real line *R* with membership function f_A, which has the following properties:

(a) f_A is a continuous mapping from *R* to the closed interval $[0,1]$.

(b) $f_A(x) = 0$, for all $x \in (-\infty, a]$;

(c) f_A is strictly increasing on $[a, b]$;

(d) $f_A(x) = 1$, for all $x \in [b, c]$;

(e) f_A is strictly decreasing on $[c, d]$;

(f) $f_A(x) = 0$, for all $x \in (d, \infty]$,

Where a, b, c and d are real numbers. Unless elsewhere specified, this research assumes that A is convex and bounded (that is, $-\infty < a, d < \infty$).

Definition 2. The fuzzy number $A = [a, b, c, d]$ is a trapezoidal fuzzy number if its membership function is given by:

$$f_A(x) = \begin{cases} f_A^L(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_A^R(x), & c \le x \le d, \\ 0, & \text{otherwise}, \end{cases} \quad (3)$$

Where $f_L^A(x)$ and $f_A^R(x)$ are the left and right membership functions of *A*, respectively (Kaufmann and

Gupta, 1991).

When $b=c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number and can be denoted by $A=(a,b,d)$. Thus, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonical and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. The inverse functions of $f_A^L(x)$ and $f_A^R(x)$ can be denoted by $g_A^L:[0,\omega]\to[a,b]$ and $g_A^R:[0,\omega]\to[c,d]$, respectively. As such, $g_A^L(y)$ and $g_A^R(y)$ are then integrable on the closed interval $[0,\omega]$. In other words, both $\int_0^\omega g_A^L(y)$ and $\int_0^\omega g_A^R(y)$ exist.

Definition 3. **α-cuts:** The α-cuts of fuzzy number *A* can be defined as $A^\alpha=\{x\mid f_A(x)\geq\alpha\},\alpha\in[0,1]$, where A^α is a non-empty bounded closed interval contained in *R* and can be denoted by $A^\alpha=\left[A_l^\alpha, A_u^\alpha\right]$, where A_l^α and A_u^α are its lower and upper bounds, respectively (Kaufmann and Gupta, 1991). For example, if a triangular fuzzy number $A=(a,b,d)$, then the α-cuts of *A* can be expressed as:

$$A^\alpha=[A_l^\alpha,A_u^\alpha]=[(b-a)\alpha+a,(b-d)\alpha+d] \quad (4)$$

Definition 4. Arithmetic Operations on Fuzzy Numbers: Given fuzzy numbers *A* and *B*, where $A,B\in R^+$, the α-cuts of *A* and *B* are $A^\alpha=\left[A_l^\alpha, A_u^\alpha\right]$ and $B^\alpha=\left[B_l^\alpha, B_u^\alpha\right]$, respectively. By the interval arithmetic, some main operations of *A* and *B* can be expressed as follows (Kaufmann and Gupta, 1991):

$$(A\oplus B)^\alpha=\left[A_l^\alpha+B_l^\alpha,\ A_u^\alpha+B_u^\alpha\right] \quad (5)$$

$$(A\ominus B)^\alpha=\left[A_l^\alpha-B_u^\alpha, A_u^\alpha-B_l^\alpha\right] \quad (6)$$

$$(A\otimes B)^\alpha=\left[A_l^\alpha\cdot B_l^\alpha,\ A_u^\alpha\cdot B_u^\alpha\right] \quad (7)$$

$$(A\oslash B)^\alpha=\left[A_l^\alpha/B_u^\alpha, A_u^\alpha/B_l^\alpha\right] \quad (8)$$

$$(A\otimes r)^\alpha=\left[A_l^\alpha\cdot r,\ A_u^\alpha\cdot r\right],\ r\in R^+ \quad (9)$$

CENTROID-INDEX RANKING METHODS FOR FUZZY NUMBERS

This section proposes a new centroid-index ranking method conducted on the basis formulae of Wang et al. (2006). The development of the proposed method is as follows.

For a trapezoidal fuzzy number $A=(a,b,c,d)$, the centroid point $(\bar{x}_A,\bar{y}_A)$ is defined as (Wang et al., 2006):

$$\bar{x}_0(A)=\frac{1}{3}\left[a+b+c+d-\frac{dc-ab}{(d+c)-(a+b)}\right] \quad (10)$$

$$\bar{y}_0(A)=\frac{1}{3}\left[1+\frac{c-b}{(d+c)-(a+b)}\right] \quad (11)$$

Remark. It is clear that $(\varpi/3)\leq\bar{y}_0(A)\prec(\varpi/2)$.

Proof.

$$\bar{y}_0(A)=\frac{\varpi}{3}\left[1+\frac{c-b}{(d+c)-(a+b)}\right]\geq\frac{\varpi}{3}$$
$$\Rightarrow 1+\frac{c-b}{(d+c)-(a+b)}\geq 1$$
$$\Rightarrow \frac{c-b}{(d+c)-(a+b)}\geq 0$$
$$\Rightarrow c\geq b$$

In the case of a triangular fuzzy number, $b=c$ and so $\bar{y}_0(A)=(\varpi/3)$.

$$\bar{y}_0(A)=\frac{\varpi}{3}\left[1+\frac{c-b}{(d+c)-(a+b)}\right]<\frac{\varpi}{2}$$
$$\Rightarrow \frac{2(c-b)}{(d+c)-(a+b)}<1$$
$$\Rightarrow \frac{(c-d)+(a-b)}{(d+c)-(a+b)}<0$$
$$\Rightarrow (c-d)+(a-b)<0$$
$$\Rightarrow c+a<b+d.$$

Because $c+a<c+b\prec b+d$, hence $c+a<b+d$ is satisfied.

This research proposes the new centroid index as follows. Suppose $A_1,A_2,...,A_n$ are fuzzy numbers. First, we calculate the centroid point of all fuzzy numbers $A_i=(\bar{x}_{A_i},\bar{y}_{A_i}),i=1,2,...,n$. We then define $G=(x_{\min},y_{\min})$, such that $x_{\min}=\inf S,\quad S=U_{i=1}^n S_i$,

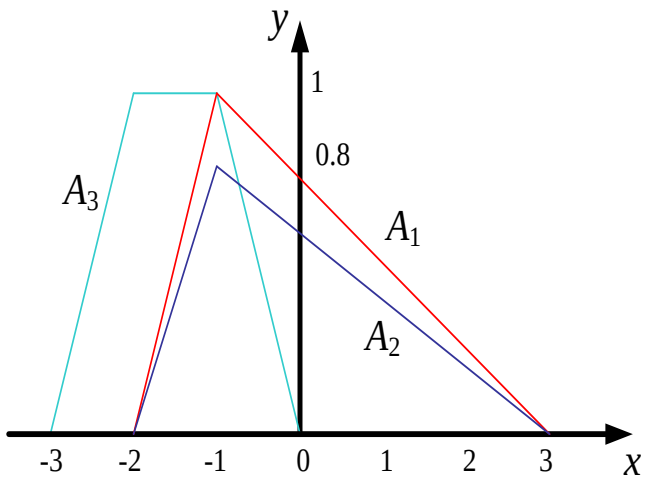

Figure 1. Fuzzy numbers A_1, A_2 and A_3.

Table 1. Comparison between fuzzy numbers A_1, A_2, and A_3.

Fuzzy number	Centroid points		Cheng's ranking index	Chu and Tsao's ranking index $S=\bar{x}_i\bar{y}_i$	Minimum points *G*		Centroid by formulae (12)
	$\bar{x}_{A_i}$	$\bar{y}_{A_i}$	$R=\sqrt{(\bar{x})^2+(\bar{y})^2}$		$x_{\min}$	$y_{\min}$	
A_1	0	1/3	0.3333	0	-3	0.8	3.0091
A_2	0	4/15	0.2667	0	-3	0.8	3.0005
A_3	-3/2	7/18	1.9	1.5496	-3	0.8	1.5049

$S_i=\{x/\ f_{A_i}(x)\succ 0\}$, $y_{\min}=\inf Y$, $Y=U_{i=1}^{n}Y_i$, and $Y_i=\{y/0\prec Y_{A_i}(x)\le\varpi\}$. The proposed distance between the centroid point $A_i=(\bar{x}_{A_i},\bar{y}_{A_i}), i=1,2,...,n$ and the minimum point $G=(x_{\min},y_{\min})$, is as follows:

$$D(A_i,G)=\sqrt{(\bar{x}_{A_i}-x_{\min})^2+(\bar{y}_{A_i}-\frac{\varpi}{3}y_{\min})^2} \quad (12)$$

This paper uses $D(A_i,G)$ to rank fuzzy numbers. The larger $D(A_i,G)$ is, the larger is the fuzzy number A. Therefore, for any two fuzzy numbers A_i and A_j, if $D(A_i,G)<D(A_j,G)$, then $A_i\prec A_j$. If $D(A_i,G)=D(A_j,G)$ then $A_i=A_j$. Finally, if $D(A_i,G)>D(A_j,G)$, then $A_i\succ A_j$. The following example demonstrates that the proposed centroid-index ranking method can overcome the drawbacks of Cheng's (1998), Chu and Tsao's (2002) centroid methods.

Example. Consider a mix of three fuzzy numbers: normal triangular fuzzy number $A_1=(-2,-1,3;1)$, non-normal triangular fuzzy number $A_2=(-2,-1,3;0.8)$, and non-normal trapezoidal fuzzy number $A_3=(-3,-2,-1,0;1)$. Figure 1 shows the pictures of these three fuzzy numbers. Table 1 shows the results obtained by applying Cheng's (1998) centroid-index and Chu and Tsao's (2002) centroid-index and the proposed centroid-index. The final ranking result obtained by using formulae (12) is $A_3<A_2<A_1$.

It is worth mentioning that Chu and Tsao's (2002) centroid-index cannot differentiate A_1 and A_2 - that is, their rankings are always the same. On the other hand,

the ranking order by using Cheng's (1998) centroid-index leads to an incorrect ranking order $A_2 < A_1 < A_3$. Obviously, the proposed centroid-index ranking method can overcome the shortcomings of the inconsistency of Cheng's (1998) and Chu and Tsao's (2002) centroid indices. On the other hand, the proposed centroid-index ranking method is capable of effectively ranking various types of fuzzy numbers.

A TOPSIS METHOD FOR RANKING FUZZY NUMBERS

TOPSIS (technique for order performance by similarity to ideal solution) was first proposed by Hwang and Yoon (1981). The fundamental idea of TOPSIS is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution, in order to solve the MCDM problems (Fouladgar et al., 2011; Jadidi et al., 2008; Jiang et al., 2010; Kutlu and Ekmekçioğlu, 2012; Lashgari et al., 2011; Mohammadi et al., 2011; Wang et al., 2009; Wang and Chang, 2007; Wang and Lee, 2007; Yong, 2006). The fuzzy TOPSIS procedure consists of the following steps:

For a multiple criteria decision making problem, assume that a committee of k decision makers D_t, $t = 1,\ldots,k$ is responsible for evaluating m alternatives A_i, $i = 1,\ldots,m$ under n selection criteria C_i, $j = 1,\ldots,n$ where the suitability ratings of alternatives under each of the criteria, as well as the weights of the criteria, are assessed in linguistic terms (Zadeh, 1976) represented by triangular fuzzy numbers. Criteria are classified into benefit (*B*) and cost (*C*). Benefit criterion has the characteristics of the larger the better. The cost criterion has the characteristic of the smaller the better.

Aggregate ratings of alternative versus criteria

Let $x_{ijt} = (e_{ijt}, f_{ijt}, g_{ijt})$, $i = 1,\ldots,m$, $j = 1,\ldots,n$, $t = 1,\ldots,k$ be the suitability rating assigned to alternative A_i, by decision maker D_t, for criterion C_j. The averaged suitability rating, $x_{ij} = (e_{ij}, f_{ij}, g_{ij})$, can be evaluated as (Yong, 2006):

$$x_{ij} = \frac{1}{k} \otimes (x_{ij1} \oplus x_{ij2} \oplus \ldots \oplus x_{ijt} \oplus \ldots \oplus x_{ijk}), \quad (13)$$

Where $e_{ij} = \frac{1}{k}\sum_{t=1}^{k} e_{ijt}$, $f_{ij} = \frac{1}{k}\sum_{t=1}^{k} f_{ijt}$, and $g_{ij} = \frac{1}{k}\sum_{t=1}^{k} g_{ijt}$.

Aggregate the importance weights

Let $w_{jt} = (o_{jt}, p_{jt}, q_{jt}), w_{jt} \in R^*, j = 1,\ldots,n, t = 1,\ldots,k$ be the weight assigned by decision maker D_t to criterion C_j. The averaged weight, $w_j = (o_j, p_j, q_j)$, of criterion C_j assessed by the committee of *k* decision makers can be evaluated as (Chu and Lin, 2009):

$$w_j = (1/k) \otimes (w_{j1} \oplus w_{j2} \oplus \ldots \oplus w_{jk}) \quad (14)$$

Where
$o_j = (1/k)\sum_{t=1}^{k} o_{jt}$, $p_j = (1/k)\sum_{t=1}^{k} p_{jt}$, $q_j = (1/k)\sum_{t=1}^{k} q_{jt}$.

Normalize performance of alternatives versus objective criteria

To ensure compatibility between average ratings and average weights, the average ratings are normalized into comparable scales. Suppose $r_{ij} = (a_{ij}, b_{ij}, c_{ij})$, is the performance of alternative *i* versus criteria *j*. The normalized value x_{ij} can then be denoted as (Chu and Lin, 2009):

$$x_{ij} = \left(\frac{a_{ij} - a_j^*}{s_j^*}, \frac{b_{ij} - a_j^*}{s_j^*}, \frac{c_{ij} - a_j^*}{s_j^*} \right), j \in B$$
$$x_{ij} = \left(\frac{c_j^* - c_{ij}}{s_j^*}, \frac{c_j^* - b_{ij}}{s_j^*}, \frac{c_j^* - a_{ij}}{s_j^*} \right), j \in C \quad (15)$$

Where
$a_j^* = \min_i a_{ij}, c_j^* = \max_i c_{ij}, s_j^* = c_j^* - a_j^*, i = 1,\ldots,m; j = 1,\ldots,n.$

Develop a membership function of each normalized weighted rating

The membership function of each final fuzzy evaluation value, that is, $R_{ij} = x_{ij} \otimes w_j$, $i = 1,\ldots,m; j = 1,\ldots,n$ can be developed by the interval arithmetic of fuzzy numbers. By Equations (4), (5), and (7), the α-cuts of $R_{ij} = x_{ij} \otimes w_j$, can be presented as follows (Chu and Lin, 2009).

$$(R_{ij})^\alpha = (x_{ij} \otimes w_j)^\alpha = [(f_{ij} - e_{ij})(p_j - o_j)\alpha^2 + [e_{ij}(p_j - o_j) + o_j(f_{ij} - e_{ij})]\alpha + e_{ij}o_j,$$
$$(f_{ij} - g_{ij})(p_j - q_j)\alpha^2 + [g_{ij}(p_j - q_j) + q_j(f_{ij} - g_{ij})]\alpha + g_{ij}q_j. \quad (16)$$

We now have two equations to solve, namely:

$$I_{ij1}\alpha^2 + J_{ij1}\alpha + Q_{ij} - x = 0 \quad (17)$$

$$I_{ij2}\alpha^2 + J_{ij2}\alpha + Z_{ij} - x = 0 \quad (18)$$

where

$$I_{ij1} = (f_{ij} - e_{ij})(p_j - o_j), \quad J_{ij1} = [e_{ij}(p_j - o_j) + o_j(f_{ij} - e_{ij})],$$

$$I_{ij2} = (f_{ij} - g_{ij})(p_j - q_j), \quad J_{ij2} = [g_{ij}(p_j - q_j) + q_j(f_{ij} - g_{ij})],$$

$$Q_{ij} = e_{ij}o_j \quad Y_{ij} = f_{ij}p_j \quad Z_{ij} = g_{ij}q_j$$

Only the roots in [0, 1] will be retained in (17) and (18). The left and right membership functions $f_{R_{ij}}^{L}(x)$ and $f_{R_{ij}}^{R}(x)$ of R_{ij} can be calculated as:

$$f_{R_{ij}}^{L}(x) = \left\{-J_{ij1} + [J_{ij1}^2 + 4I_{ij1}(x - Q_{ij})]^{1/2}\right\} / 2I_{ij1}, \quad Q_{ij} \le x \le Y_{ij}, \quad (19)$$

$$f_{R_{ij}}^{R}(x) = \left\{-J_{ij2} - [J_{ij2}^2 + 4I_{ij2}(x - Z_{ij})]^{1/2}\right\} / 2I_{ij2}, \quad Y_{ij} \le x \le Z_{ij}, \quad (20)$$

For convenience, R_{ij} is expressed as:

$$R_{ij} = (Q_{ij}, Y_{ij}, Z_{ij}; I_{ij1}, J_{ij1}; I_{ij2}, J_{ij2}), i = 1,...,m, j = 1,...,n$$

Defuzzification

This paper applies the proposed centroid-index, which is based on centroid formulae of Wang et al. (2006), to defuzzify all the final fuzzy evaluation values R_{ij}. From Equations (8)-(9), the centroid point of the fuzzy evaluation value, R_{ij}, is produced as:

$$\bar{x}(R_{ij}) = \frac{\int_{Q_{ij}}^{Y_{ij}} x f_{R_{ij}}^{L}(x)dx + \int_{Y_{ij}}^{Z_{ij}} x f_{R_{ij}}^{R}(x)dx}{\int_{Q_{ij}}^{Y_{ij}} f_{R_{ij}}^{L}(x)dx + \int_{Y_{ij}}^{Z_{ij}} f_{R_{ij}}^{R}(x)dx},$$

$$\bar{x}(R_{ij}) = \frac{\int_{Q_{ij}}^{Y_{ij}} x\left\langle\left\{-J_{ij1} + [J_{ij1}^2 + 4I_{ij1}(x - Q_{ij})]^{1/2}\right\} / 2I_{ij1}\right\rangle dx + \int_{Y_{ij}}^{Z_{ij}} x\left\langle\left\{-J_{ij2} + [J_{ij2}^2 + 4I_{ij2}(x - Z_{ij})]^{1/2}\right\} / 2I_{ij2}\right\rangle dx}{\int_{Q_{ij}}^{Y_{ij}} \left\langle\left\{-J_{ij1} + [J_{ij1}^2 + 4I_{ij1}(x - Q_{ij})]^{1/2}\right\} / 2I_{ij1}\right\rangle dx + \int_{Y_{ij}}^{Z_{ij}} \left\langle\left\{-J_{ij2} + [J_{ij2}^2 + 4I_{ij2}(x - Z_{ij})]^{1/2}\right\} / 2I_{ij2}\right\rangle dx}, \quad (21)$$

$$\bar{y}_A = \frac{\int_0^1 y[g_A^R(y) - g_A^L(y)]dy}{\int_0^1 [(g_A^R(y) - g_A^L(y)]dy}.$$

$$\bar{y}_A = \frac{\int_0^1 y\left[(I_{ij2}y^2 + J_{ij2}y + Z_{ij}) - (I_{ij1}y^2 + J_{ij1}y + Q_{ij})\right]dy}{\int_0^1 \left[(I_{ij2}y^2 + J_{ij2}y + Z_{ij}) - (I_{ij1}y^2 + J_{ij1}y + Q_{ij})\right]dy}. \quad (22)$$

Here:

$$g_A^R(y) = I_{ij2}y^2 + J_{ij2}y + Z_{ij}, \quad 0 \le y \le 1$$

$$g_A^L(y) = I_{ij1}y^2 + J_{ij1}y + Q_{ij}, \quad 0 \le y \le 1$$

The distance of fuzzy evaluation value, R_{ij}, is obtained by using Equation (12).

$$D_{ij} = \sqrt{(\bar{x}_{A_i} - x_{\min})^2 + (\bar{y}_{A_i} - \frac{\varpi}{3} y_{\min})^2} \quad (23)$$

Calculation of A^+, A^-, d_i^+ and d_i^-

The fuzzy positive-ideal solution (FPIS, A^+) and fuzzy negative-ideal solution (FNIS, A^-) are obtained as:

$$A^+ = \max_i \{D_{ij}\} \quad (24)$$

$$A^- = \min_i \{D_{ij}\} \quad (25)$$

The distance of each alternative $A_i, i = 1,\ldots,m$ from A^+ and A^- is calculated as:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (D_{ij} - A^+)^2} \quad (26)$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (D_{ij} - A^-)^2}, \quad (27)$$

where D_{ij} is the distance between the centroid points $A_i = (\bar{x}_{A_i}, \bar{y}_{A_i}), i = 1,2,...,n$ and the minimum point $G = (x_{\min}, y_{\min})$, d_i^+ represents the shortest distance of alternative A_i, and d_i^- represents the farthest distance of alternative A_i.

Obtain the closeness coefficient

The closeness coefficient of each alternative, which is usually defined to determine the ranking order of all alternatives, is calculated as (Wang and Lee, 2007; Yong, 2006):

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}. \quad (28)$$

A higher value of the closeness coefficient indicates that an alternative is closer to PIS and farther from NIS simultaneously. The closeness coefficient of each

Table 2. Ratings of alternatives versus criteria.

Criteria	Alternatives	D_1	D_2	D_3	r_{ij}
C_1	A_1	H	H	M	(0.5, 0.7, 0.9)
	A_2	VH	VH	VH	(0.8, 0.9, 1.0)
	A_3	H	M	M	(0.4, 0.6, 0.8)
C_2	A_1	M	H	M	(0.4, 0.6, 0.8)
	A_2	H	VH	H	(0.667, 0.833, 1)
	A_3	H	H	H	(0.6, 0.8, 1)
C_3	A_1	VH	M	H	(0.567, 0.733, 0.9)
	A_2	H	VH	H	(0.667, 0.833, 1)
	A_3	H	VH	M	(0.567, 0.733, 0.9)
C_4	A_1	L	M	M	(0.233, 0.433, 0.633)
	A_2	H	H	VH	(0.667, 0.833, 1)
	A_3	M	M	H	(0.4, 0.6, 0.8)

Table 3. The importance weights of the criteria and the aggregated weights.

Criteria	D_1	D_2	D_3	r_{ij}
C_1	OI	I	I	(0.267, 0.433, 0.6)
C_2	VI	VI	AI	(0.667, 0.833, 0.933)
C_3	VI	VI	VI	(0.6, 0.8, 0.9)
C_4	I	I	I	(0.3, 0.5, 0.7)

alternative is used to determine the ranking order of all alternatives and indicates the best one among a set of given feasible alternatives.

NUMERICAL EXAMPLE

This section implements a computer-aided parting surface selection and evaluation problem to demonstrate the applicability of the proposed method.

Assume that the designer must select a suitable parting surface for an optimal mold design process. After preliminary screening, three parting surfaces, A_1, A_2 and A_3, are chosen for further evaluation. A committee of three decision makers, D_1, D_2 and D_3, conducts the evaluation and selection of the three parting surfaces. Four criteria are considered: projected area (C_1), undercuts (C_2), flatness (C_3), and draw (C_4) (Ravi and Srinivasam, 1990).

This research applies the proposed method to solve this problem and the computational procedure is summarized as follows:

Step 1. Aggregate ratings of alternatives versus criteria: Assume that the decision makers use the linguistic rating set $S = \{\text{VL, L, M, H, VH}\}$, where VL = Very Low = (0.0, 0.0, 0.2), L = Low = (0.1, 0.3, 0.5), M = Medium = (0.3, 0.5, 0.7), H = High = (0.6, 0.8, 1.0), and VH = Very High = (0.8, 0.9, 1.0), to evaluate the suitability of the alternative parting surfaces under each criteria. Table 2 presents the suitability ratings of alternatives versus the four criteria. By Equation (13), the aggregated suitability ratings of three alternatives, A_1, A_2 and A_3, versus four criteria C_1, C_2, C_3 and C_4 from three decision makers can be obtained as shown in Table 2.

Step 2. Aggregate the importance weights: This paper also assumes that the decision makers employ a linguistic weighting set $Q = \{\text{UI, OI, I, VI, AI}\}$, where UI = Unimportant = (0.0, 0.0, 0.3), OI = Ordinary Important = (0.2, 0.3, 0.4), I = Important = (0.3, 0.5, 0.7), VI = Very Important = (0.6, 0.8, 0.9), and AI = Absolutely Important = (0.8, 0.9, 1.0), to assess the importance of all the criteria. Table 3 displays the importance weights of four criteria from the three decision-makers. By Equation (14), the aggregated weights of criteria from the decision

Table 4. Distances between centroid points and minimum point.

Criteria	Alternatives	$\bar{x}_i$	$\bar{y}_i$	x_{min}	y_{min}	D_{ij}
C_1	A_1	0.200	0.333	0.13	0.333	0.080
	A_2	0.089	0.333	0.12	0.333	0.031
	A_3	0.197	0.333	0.12	0.333	0.077
C_2	A_1	0.189	0.335	0.12	0.333	0.069
	A_2	0.122	0.335	0.12	0.333	0.002
	A_3	0.168	0.335	0.12	0.333	0.048
C_3	A_1	0.173	0.335	0.12	0.333	0.053
	A_2	0.164	0.335	0.12	0.333	0.044
	A_3	0.173	0.335	0.12	0.333	0.053
C_4	A_1	0.216	0.333	0.12	0.333	0.096
	A_2	0.201	0.333	0.12	0.333	0.081
	A_3	0.236	0.333	0.12	0.333	0.116

Table 5. The distance measurement.

	d^+	d^-
A_1	0.033	0.148
A_2	0.098	0.093
A_3	0.055	0.152

Table 6. Closeness coefficients of alternatives.

	Closeness coefficient	Ranking
A_1	0.816	1
A_2	0.486	3
A_3	0.735	2

making committee can be obtained as presented in Table 3.

Step 3. Normalize the performance of alternatives versus objective criteria: To make an easier and practical procedure, this paper defines all of the fuzzy numbers in [0,1]. The calculation of Equation (15) is no longer needed, and therefore we have $x_{ij} = r_{ij}$.

Step 4. Develop the membership function of each normalized weighted rating: By Equations (16) ~ (20), the final fuzzy evaluation value of each alternative can be produced.

Step 5. Defuzzification: Equations (21) and (23) produce the centroid point of each alternative and the distance between the centroid point and the minimum point in Table 4.

Step 6. Calculate A^+, A^-, d_i^+ and d_i^-: By Equations (24) and (25), the positive and negative-ideal solutions are obtained. Then, the distance of each alternative from A^+ and A^- is calculated through Equations (26) and (27) as presented in Table 5.

Step 7. Obtain the closeness coefficient: The closeness coefficients of alternatives can be produced by Equation (28) as displayed in Table 6.

Conclusion

This paper reviewed several fuzzy number ranking methods based on the centroid-index and proposed a new centroid-index ranking method that was capable of ranking various types of fuzzy numbers effectively. The proposed method used fuzzy TOPSIS to establish a parting surface evaluation and selection model in plastic mold design. Using the proposed method, the ratings and weights assigned by decision makers were averaged and normalized into a comparable scale. To avoid a complicated calculation of fuzzy numbers, these normalized weighted ratings were defuzzified into crisp values by the proposed centroid-index ranking method to help calculate the distances of each alternative to both the ideal and negative ideal solutions. A closeness coefficient was then defined to determine the ranking order of alternatives.

The applicability of the proposed approach is validated through a numerical example. According to Table 6, among the three parting surface alternatives, A_1 has the

largest closeness coefficient, followed by A_3, and then A_2. Thus, parting surface 1 is the best alternative. Further, it can be seen that the computational procedure is efficient and easy to implement. Thus, for practitioners, the proposed approach is a very effective tool to solve MCDM problems.

Future research may apply the proposed approach to other MCDM problems with similar settings in various industries. This paper employed the new centroid-index to defuzzify the final fuzzy evaluation values to determine the ranking order of the alternatives. Future research may also attempt to use different defuzzification techniques for ranking alternatives, and compare the results with those obtained by the proposed approach.

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