

Full Length Research Paper

Evaluation of recent global geopotential models based on GPS/levelling data over Afyonkarahisar (Turkey)

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This study presents the evaluation of the global geo-potential models EGM96, EIGEN-5C, EGM2008(360) and EGM2008 by comparing model based on geoid heights to the GPS/levelling based on geoid heights over Afyonkarahisar study area in order to find the geopotential model that best fits the study area to be used in a further geoid determination at regional and national scales. The study area consists of 313 control points that belong to the Turkish National Triangulation Network, covering a rough area. The geoid height residuals are investigated by standard deviation value after fitting tilt at discrete points, and height-dependent evaluations have been performed. The evaluation results revealed that EGM2008 fits best to the GPS/levelling based on geoid heights than the other models with significant improvements in the study area.

Key words: Geopotential model, GPS/levelling, geoid height, EGM96, EIGEN-5C, EGM2008(360), EGM2008.

INTRODUCTION

Most geodetic applications like determining the topographic heights or sea depths require the geoid as a corresponding reference surface. The improvements of satellite missions have provided the earth gravity field knowledge with a sufficient accuracy; so global geopotential models (GGMs) that are representation of the earth gravity field have acquired more importance for geosciences.

The release of the Earth Gravitational Model 2008 (EGM2008) by the US National Geospatial Intelligence Agency (Pavlis et al., 2008) and high-resolution GRACE-based gravity field model (EIGEN-5C) by the GFZ-GRGS cooperation (Förste et al., 2008) is a significant achievement in the determination of the earth's mean gravity field.

Among geodesy community, comprehensive efforts have been put to determine EGM2008 and EIGEN-5C's accuracy with several techniques and data sets that were not used for their development and evaluation, after the official release of these models. The external quality

evaluation studies of EGM2008 have been coordinated by the joint working group (JWG) between the International gravity field service (IGFS) and the Commission 2 of the International Association of Geodesy (IAG).

In the selection of a GGM for geoid determination, published error estimates for GGMs cannot be frequently used directly to judge which GGM is best for a certain region as the published quality estimates may be too optimistic and/or presented as global averages; and thus not necessarily representative of the performance of the GGM in a particular region. Hence, the user of a GGM should perform his own accuracy and precision verifications (Kiamehr and Sjöberg, 2005).

The continuous developments in the acquisition, modelling and processing of GPS data have provided geodesists highly reliable and precise external control to evaluate global and regional models for the earth's gravity field (Kotsakis, 2008).

The main objective of this study is comparing EGM2008, EIGEN-5C and the earth geopotential model 1996 (EGM96) (Lemoine et al., 1998) that was used as a reference earth geopotential model for the official regional geoid model Turkish Geoid 2003 (TG-03) (Kılıçoğlu et al., 2005), using GPS/levelling geoid heights

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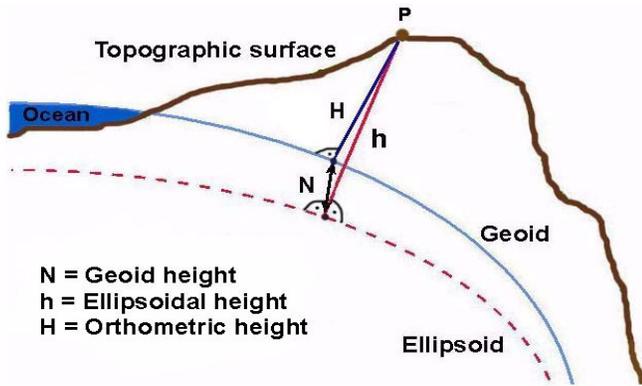


Figure 1. Geoid, ellipsoidal and orthometric heights.

over the Afyonkarahisar study area to quantify the GGMs' accuracy in order to find the geopotential model that best fits the study area to be used in a further geoid determination at regional and national scales.

THEORETICAL BACKGROUND

GPS/Levelling

The points with GPS-derived ellipsoidal heights refer to a reference ellipsoid and orthometric heights refer to an equipotential reference surface which can be incorporated to determine geoid height by geometrical approach. The GPS/Levelling geoid heights are computed by the following equation (Heiskanen and Moritz, 1967):

$$N = h - H \tag{1}$$

Where N is the geoid height, h is the ellipsoidal height computed from GPS and H is the orthometric height computed from levelling (Figure 1).

Geoid heights have been computed based on the known ellipsoidal and orthometric heights (Banarjee et al., 1999).

Global geopotential model

For a better determination of orbits and height systems in science and engineering, it is necessary to significantly improve our knowledge of the gravity field of the earth, both in terms of accuracy and spatial resolution (Rummel et al., 2002).

The GGM is used to determine the long wavelength part of the earth's gravity field and comprises a set of fully-normalized, spherical harmonic coefficients ($\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$) that are obtained from geopotential solutions (Mainville et al., 1992). These coefficients are determined

from the incorporation of satellite observations, land and ship-track gravity data, marine gravity anomalies derived from satellite radar altimetry and airborne gravity data (Rapp, 1997).

The geoid height is represented at a point (φ_p, λ_p) from a set of spherical harmonic coefficients in spherical approximation by the following equation (Heiskanen and Moritz, 1967):

$$N(\varphi_p, \lambda_p) = \frac{GM}{R\gamma} \sum_{\ell=0}^{\infty} \left(\frac{R}{r}\right)^{\ell+1} \sum_{m=0}^{\ell} \bar{P}_{\ell m}(\sin \varphi_p) (\bar{C}_{\ell m} \cos m\lambda_p + \bar{S}_{\ell m} \sin m\lambda_p) \tag{2}$$

Where GM is the geocentric gravitational constant, R is the mean radius of the earth, γ is the mean normal gravity on the surface of the reference ellipsoid, $\bar{P}_{\ell m}$ are the fully-normalized associated Legendre functions, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the fully-normalized harmonic coefficients of the disturbing potential, r is the geocentric radius, φ_p and λ_p are the geodetic latitude and longitude of the point. The infinite series is usually truncated at the maximum degree of the expansion $\ell = L$. The series coefficients allow the determination of the geoid height with:

$$N(\varphi_p, \lambda_p) \cong R \sum_{\ell=2}^L \sum_{m=0}^{\ell} \bar{P}_{\ell m}(\sin \varphi_p) (\bar{C}_{\ell m} \cos m\lambda_p + \bar{S}_{\ell m} \sin m\lambda_p) \tag{3}$$

The long wavelength components of the earth's gravity field are recovered from satellite tracking data and medium and short wavelength components are recovered from satellite altimetry, terrestrial, marine, airborne gravity data. At higher degrees a GGM's accuracy is quite dependent on the geographic coverage of the gravity data that go into the solution (Ellmann and Jürgenson, 2008).

STUDY AREA, DATA SET AND GLOBAL GEOPOTENTIAL MODELS

Afyonkarahisar study area and source data

Afyonkarahisar study area is located in the internal Aegean region of Turkey within the geographical boundaries: $38^{\circ}N \leq \varphi \leq 39.1^{\circ}N$; $30^{\circ}E \leq \lambda \leq 31.1^{\circ}E$, defining a total area of 9000 km² (120 km x 75 km) with a rough topography (Figure 2).

All our GGM evaluation tests based on geoid height

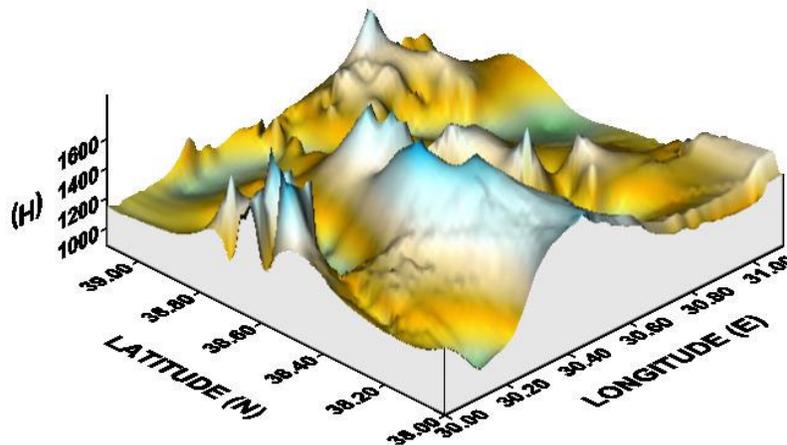


Figure 2. The topography of the Afyonkarahisar study area.

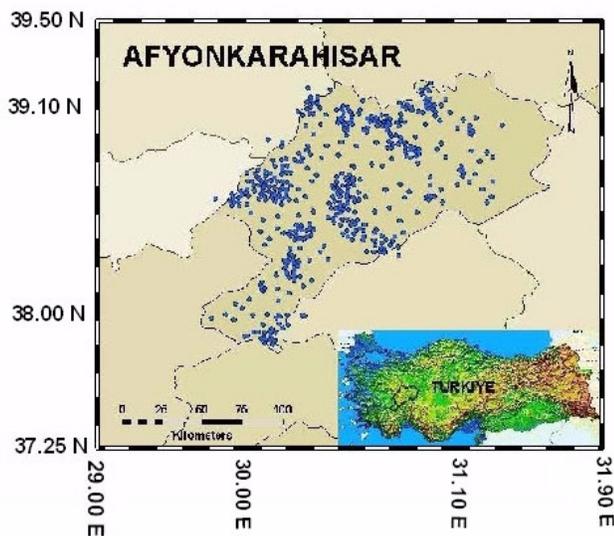


Figure 3. Geographical distribution of the 313 control points over the study area.

refer to the 313 control points that belong to the Turkish National Triangulation Network, in the study area (Figure 3).

Ellipsoidal heights at the 313 control points have been determined using dual-frequency Topcon Hiper GGD GPS receivers and antennas with respect to the well established Turkish National GPS Network (aligned to ITRF96) (reference epoch 2005.00) and orthometric heights at these points have been determined through spirit levelling with respect to the Turkish National Vertical Datum (fixed to mean sea level) as a part of the Digital Cadastre Project by the General Directorate of Land Registry and Cadastre.

Turkish Vertical Control Network (TUDKA99) was re-adjusted due to two Marmara earthquakes that occurred

in 1999, with 243 lines of 25680 points, having a total length of 29316 km. Vertical datum for TUDKA99 was defined with arithmetic mean of instantaneous sea level measurements recorded at Antalya tide gauge between 1936 and 1971. The geopotential value for the datum point was determined by making use of gravity value in Potsdam Datum. In the adjustment, geopotential numbers were used as observations; then, geopotential numbers, Helmert orthometric heights and Molodensky normal heights of all network stations were estimated. Gravity values in Potsdam Datum were used for the calculation of geopotential number differences between network stations. The adjustment resulted in point heights of precisions varying from 0.3 to 9.0 cm, depending on the distance from the datum point. Helmert orthometric height system was selected to be used in Turkey for all geodetic and practical applications, although normal heights of network points were computed as well (Ayhan and Demir, 1992).

Turkish National Fundamental GPS Network (TUTGA) was established in 2001 and a number of the stations have been re-surveyed due to the earthquakes that happened in 1999-2003. The total number of network stations is about 600. For each station 3-D coordinates and their associated velocities were computed in ITRF96. Positional accuracy of the stations is about 1-3 cm while the relative accuracy is better than about 0.01 ppm. Besides, the GPS network has been connected to the Turkish Horizontal and Vertical Control Networks through specified points and time-dependent coordinates of all stations were being computed in the context of the maintenance of the network with periodic GPS observations. Total of 197 GPS network stations have been connected to Turkish vertical network by precise levelling (Kılıçoğlu et al., 2009).

Geoid heights at the 313 control points have been computed according to equation (1) based on the known ellipsoidal and orthometric heights above.

Table 1. Global geopotential models that are presented in this paper (S: Satellite, G: Gravity, A: Altimetry, Tide-free: all tidal effects have been removed).

Model	Degree × Order	Tide System	Data	Reference
EGM96	360 × 360	Tide-free	S (EGM96), G, A	Lemoine et al., 1998
EIGEN-5C	360 × 360	Tide-free	S (Grace), G, A	Förste et al., 2008
EGM2008(360)	360 × 360	Tide-free	S (Grace), G, A	Pavlis et al., 2008
EGM2008	2190 × 2160	Tide-free	S (Grace), G, A	Pavlis et al., 2008

Global geopotential models

Earth geopotential model 1996

EGM96 is a spherical harmonic model of the earth's gravitational potential in degree and order of 360 which corresponds to the spatial resolution of 55 km. EGM96 is developed by combining surface gravity data, ERS-1/GEOSAT altimeter-derived anomalies, extensive satellite tracking data including new data from satellite laser ranging (SLR), the global positioning system (GPS), NASA's tracking and data relay satellite system (TDRSS), the French DORIS system, the US Navy TRANET Doppler tracking system and direct altimeter ranges from TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT (Lemoine et al., 1998).

High-resolution GRACE-based gravity field model

EIGEN-5C is the combined gravity field of the earth complete in degree and order of 360 (corresponding to half-wavelength of 55 km). The model is a combination of GRACE and LAGEOS mission data plus 0.5 × 0.5 degrees gravimetry and altimetry surface data. The combination of the satellite and surface data has been done by the combination of normal equations, which are obtained from observation equations for the spherical harmonic coefficients (Förste et al., 2008).

Earth gravitational model 2008(360)

EGM2008(360) is a version of earth gravitational model 2008 with spherical harmonic coefficients extending up to degree and order of 360.

Earth gravitational model 2008

EGM2008 is a spherical harmonic model of the earth's external gravitational potential in degree and order of 2160, with additional spherical harmonic coefficients extending up to degree of 2190 and order of 2160 that offers a spatial resolution of 9 km. EGM2008 incorporates improved 5x5 min gravity anomalies, altimetry-derived gravity anomalies and has benefited from the latest GRACE based satellite solutions (Pavlis et al., 2008).

GGMs that are compared over the study area are listed in Table 1 with model characteristics.

MODEL EVALUATION

In GGM evaluation, geoid heights based on GPS-derived ellipsoidal heights and spirit levelled orthometric heights at discrete points give an indication of GGMs' accuracy. It is usually acceptable to select a GGM that is a best fit to the geoid heights for a regional geoid model.

EGM96 and EIGEN5C, both are truncated at degree and order of 360, but EGM2008's spherical harmonic coefficients extend up to degree of 2190 and order of 2160. High-resolution model's better performance than the other models over the study area can be considered. Therefore for an actual comparison of GGMs, a version of EGM2008 that truncated at degree and order of 360, EGM2008 (360) is used in the model evaluation.

The geoid heights based on GGMs are interpolated from the closest grid points using software and coefficients obtained from International Centre for Global Earth Models (ICGEM) web page <<http://icgem.gfz-potsdam.de/ICGEM>> by Kriging method (Davis 1986) and refer to the reference ellipsoid GRS80 (Moritz, 1992).

The differences between GPS/Level based on geoid heights and GGM based on geoid heights may be affected by datum inconsistencies. In order to minimize these offsets (that is bias and tilt), a four parameter transformation is used. The geoid heights obtained from GGMs are compared with discrete geoid heights based on GPS/Levelling data after fitting the tilt.

The statistical values of the height data sets that were used for GGM evaluation are given in Table 2.

GPS/Lev-based and GGM-based on geoid heights over the study area are given in Figures 4, 5, 6, 7 and 8.

GGM evaluation was focused on the differences between the GPS/Lev-based and GGM-based on geoid heights and the statistical values of geoid height residuals ($N_{GPS/Lev} - N_{GGM}$) were computed (Table 3).

From the statistical values of $N_{GPS/Lev} - N_{GGM}$, the standard deviations were used to infer the best fit of the GGMs to the GPS/Levelling data for model evaluating because any gravimetric determination of the geoid is deficient in the zero and first-degree terms (Featherstone et al., 1996).

Obviously, EGM2008 gives the best fit to the GPS/levelling data than the other GGMs over the study area

Table 2. Statistics of height datasets over the study area (units in m).

Height	Min.	Max.	Mean	Std. Dev.	Range
h	901.586	1979.749	1348.954	207.216	1078.163
H	866.063	1941.711	1309.418	204.966	1075.648
N _{GPS/Lev}	35.277	38.418	37.628	0.598	2.351
N _{EGM96}	35.206	39.290	38.334	0.676	4.084
N _{EIGEN-5C}	35.771	38.579	38.177	0.428	2.808
N _{EGM2008(360)}	35.625	38.424	37.791	0.480	2.799
N _{EGM2008}	35.270	38.047	37.380	0.403	2.777

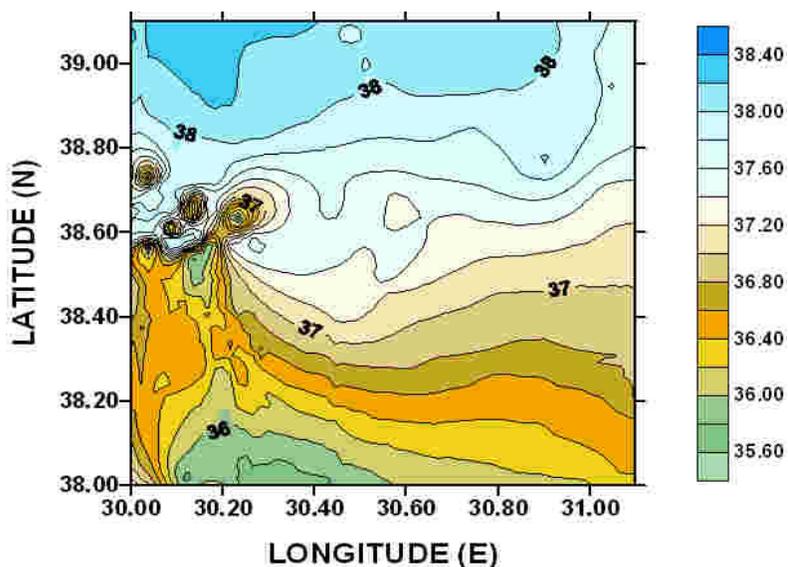


Figure 4. N_{GPS/Lev} in the study area (latitude, longitude in degree, heights in m).

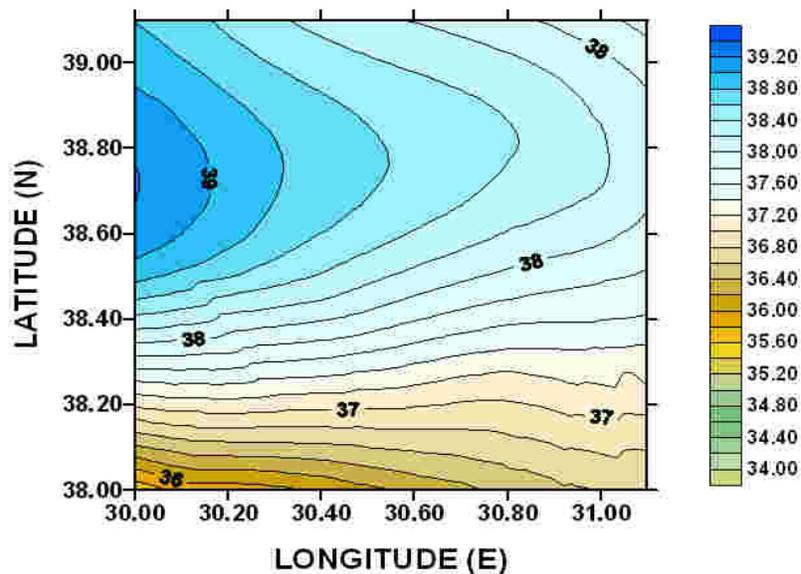


Figure 5. N_{EGM96} in the study area (latitude, longitude in degree, heights in m).

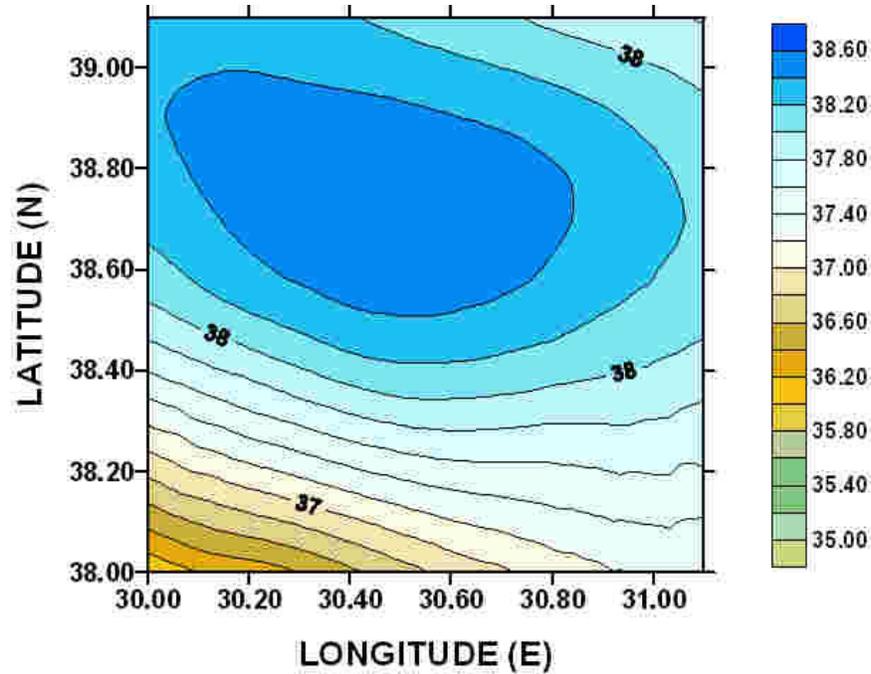


Figure 6. $N_{EIGEN5C}$ in the study area (latitude, longitude in degree, heights in m).

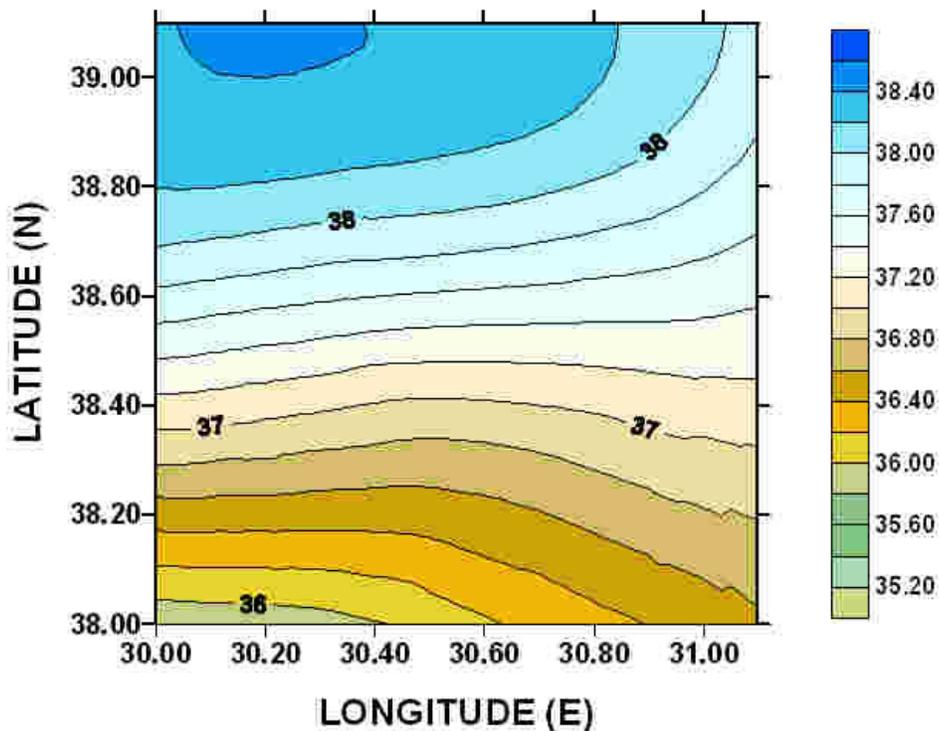


Figure 7. $N_{EGM2008(360)}$ in the study area (latitude, longitude in degree, heights in m).

area with a significant improvement that comes from the higher frequency. This is because the agreements will improve as the maximum degree of the GGMs increases (Rodriguez-Caderot et al., 2006). This is simply due to a

reduction in the omission errors and should not necessarily be interpreted as an improvement in the low frequencies (Amos and Featherstone, 2003).

The standard deviation of the $N_{GPS/Lev} - N_{EGM2008}$ residuals

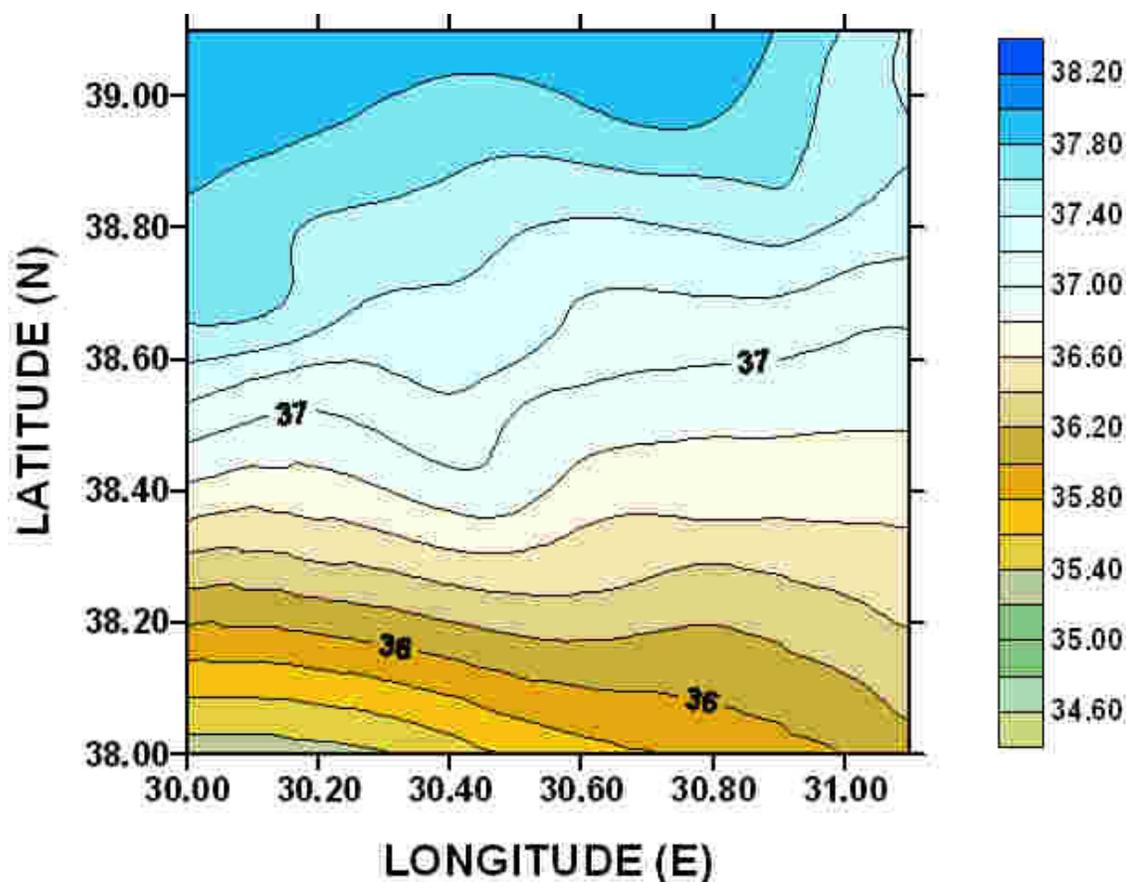


Figure 8. $N_{EGM2008}$ in the study area (latitude, longitude in degree, heights in m).

Table 3. Statistics of the $N_{GPS/Lev} - N_{GGM}$ over the study area after fitting (units in m).

Residual	Min.	Max.	Mean	Std. Dev.	Range
$N_{GPS/Lev} - N_{EGM96}$	-2.188	0.815	-0.806	0.455	3.003
$N_{GPS/Lev} - N_{EIGEN5C}$	-3.326	0.752	-0.866	0.664	4.078
$N_{GPS/Lev} - N_{EGM2008(360)}$	-0.750	0.358	-0.073	0.204	1.108
$N_{GPS/Lev} - N_{EGM2008}$	-0.291	0.707	0.294	0.157	0.998

decreases by a factor of (1.30), compared to the $N_{GPS/Lev} - N_{EGM2008(360)}$; and also decreases by a factor of (2.90), compared to the other residuals.

The geoid height residuals ($N_{GPS/Lev} - N_{GGM}$) over the study area are depicted in Figures 9, 10, 11 and 12.

Figure 13 shows that EGM2008 has a better terrain approximation than the other GGMs over the mountainous ($H > 900$ m.) study area because of its higher frequency content (Kotsakis et al. 2008). It is visible from Figure 13 that height-dependent bias between GPS/Lev based on geoid heights and GGM based on geoid heights is reduced in the case of EGM2008. The remaining height-dependent bias reflects the systematic errors in the orthometric heights (Figure 13).

Conclusion

The results of GGM evaluation in this study have indicated the outstanding of EGM2008 to the other GGMs undoubtedly. EGM2008 has 1.30 times better statistics than the EGM2008 (360) and 2.90 times better statistics than the other GGMs and fits best to the TG-03 at ± 0.157 m. agreement despite the coefficient errors and GPS/Levelling dataset that cannot be considered as an entirely errorless. From our GGM evaluation results we can conclude that EGM2008 can be used as a reference earth geopotential model for a further geoid determination at regional and national scales.

Furthermore, EGM2008 is a major step in earth's mean

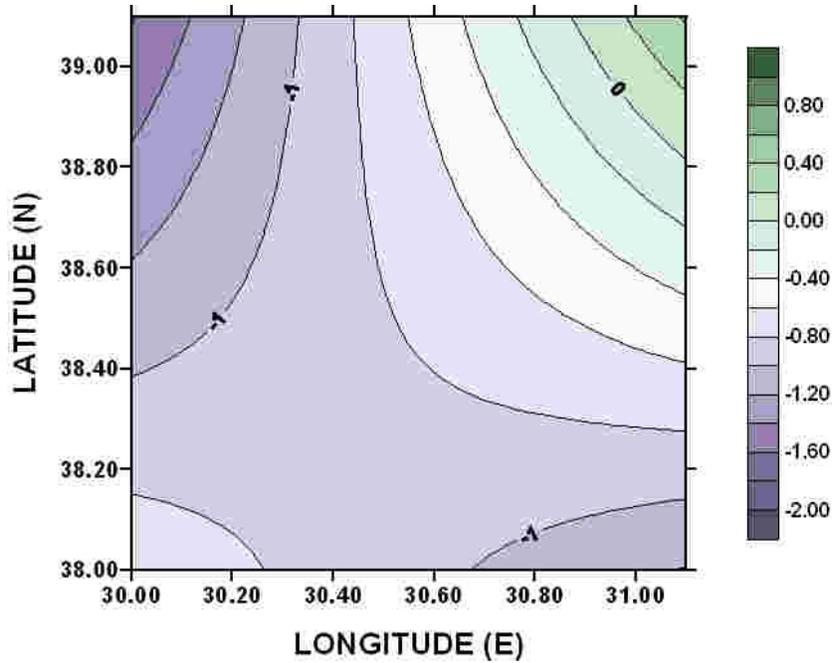


Figure 9. $N_{GPS/Lev} - N_{EGM96}$ residuals (latitude, longitude in degree, heights in m).

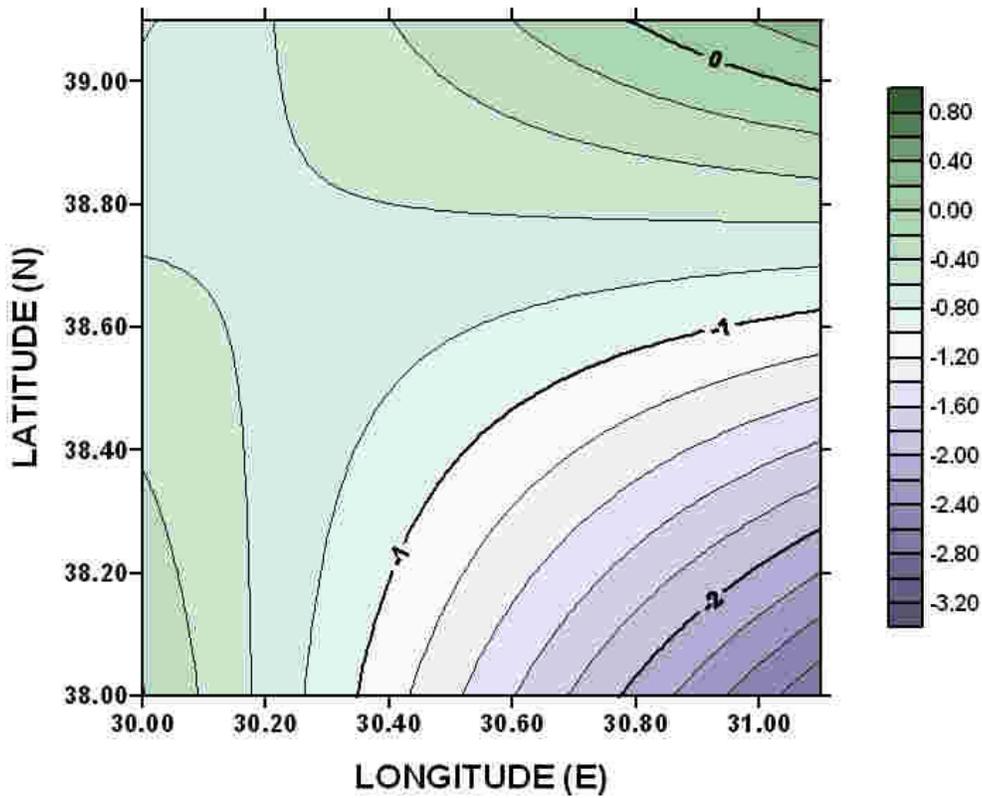


Figure 10. $N_{GPS/Lev} - N_{EIGEN-5C}$ residuals (latitude, longitude in degree, heights in m).

gravity field mapping with a high accuracy. It is very exciting to see the results after the inclusion of the

gravity field and steady-state ocean circulation explorer (GOCE) data that will make for the determination of the

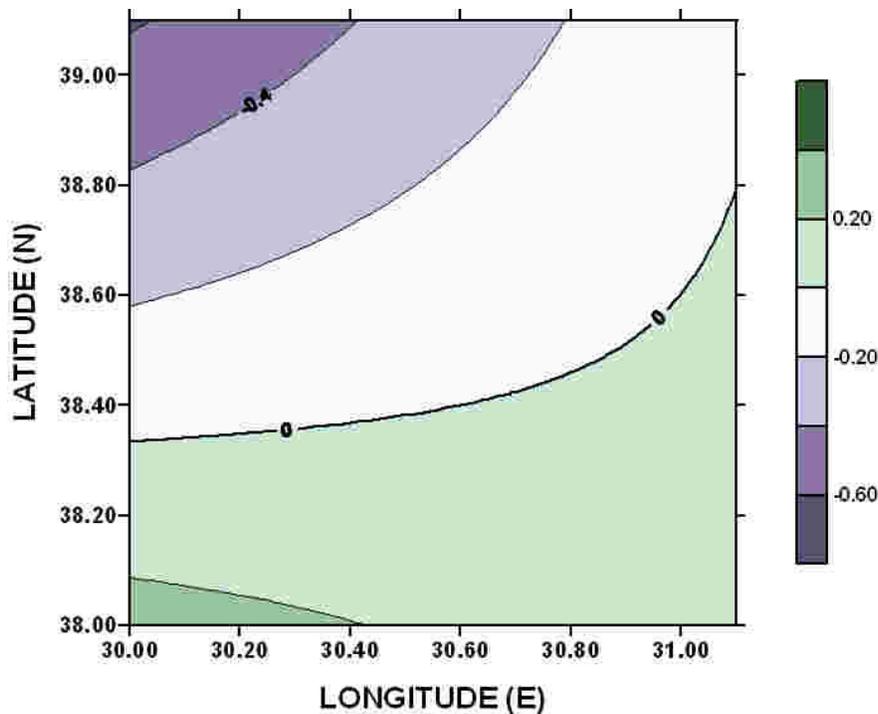


Figure 11. $N_{GPS/Lev} - N_{EGM2008(360)}$ residuals (latitude, longitude in degree, heights in m).

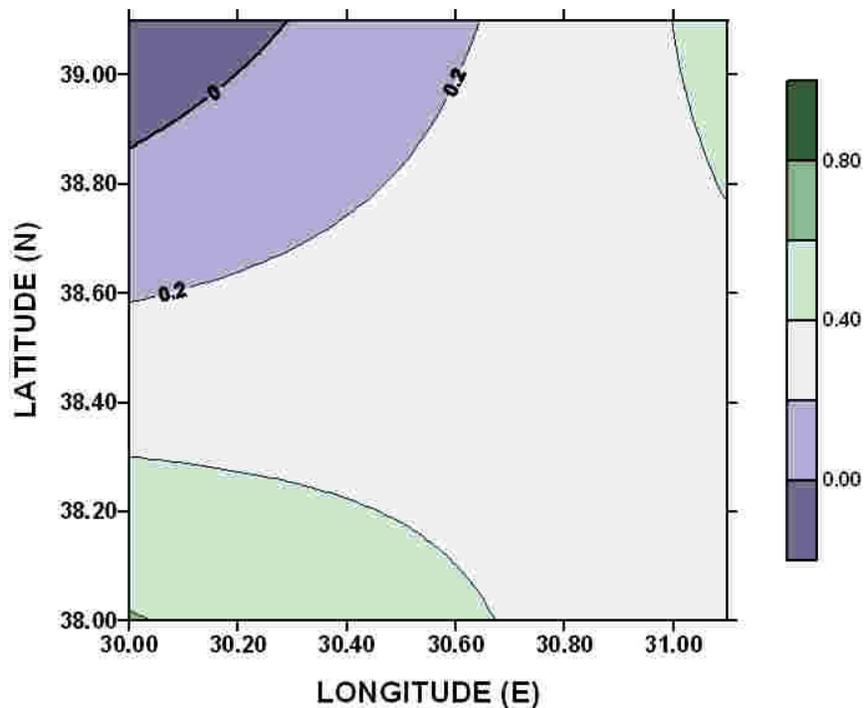


Figure 12. $N_{GPS/Lev} - N_{EGM2008}$ residuals (latitude, longitude in degree, heights in m).

geoid with an accuracy never seen before (mission objective; 1-2 cm) for GGMs in the global scale. In order to

achieve major improvements for the future high-accuracy gravimetric geoid model projects in Turkey, more

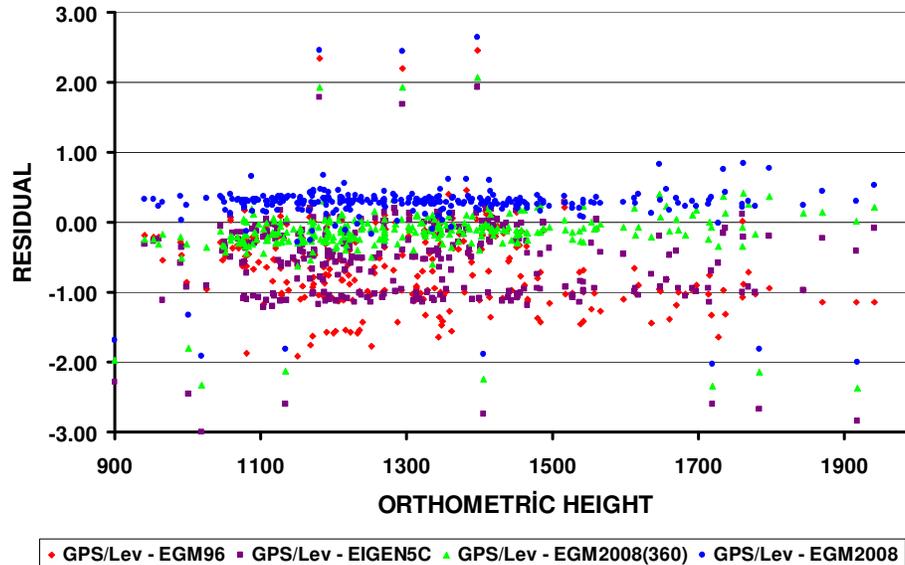


Figure 13. Height dependent N residuals (heights and residuals in m).

more attention must be paid to the EGM2008 and GOCE-based GGMs while high-resolution gravimetric data sets should be constituted

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