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Thermo elastic analysis of a functionally graded cylinder under internal pressure using first order shear deformation theory

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Thermo elastic analysis of a functionally graded cylinder under mechanical and thermal loads is investigated analytically in the present paper. The first order shear deformation theory (FSDT) is employed for analysis of the problem. FSDT recognizes two components for definition of the deformation. Displacement of the mid-plane and rotation about that are two components of deformation. The modulus of elasticity is assumed to vary continuously along the thickness with a power function. Stress strain relation is developed in the cylindrical coordinate system with regarding the thermal strain. Necessary equations for solving the problem have been obtained by taking variation of the energy equation with respect to four assumed functions using Euler equations. The effect of the temperature rising on the distribution of the displacement and stress for different values of non-homogenous index (n) are investigated in the present paper. Evaluation of the axial displacement of cylinder can be presented as an important result of this paper. Furthermore, the present result is compared with results that were obtained using the plane elasticity theory.

Key words: Thermo elastic, shell, temperature, cylinder, energy.

INTRODUCTION

One of the most applicable structures in the mechanical engineering is the shells. Ratio of thickness with respect to radius of curvature for shells is small. In the general state, shells can be classified to two types. The 1st types of them are thin shells. This class is applicable for bearing the membrane and in-plane forces. Membrane theory can be used to utilize this class of shell. The 2nd types of shells are thick shells. In this type, total deformation of shell includes displacement of mid-plane and rotation about outward axis of mid-plane of the shell. Thick shells can be applied to undergo bending and stretching forces, simultaneously.

Lame (Timoshenko, 1976) is studied the exact solution of a thick walled cylinder under inner and outer pressures. The cylinder is supposed to be axisymmetric and isotropic. This solution is applicable for simple and quick solution of pressure vessels. Naghdi and Cooper (1956) considered the effect of lateral shear and conse-quently, constitute the theory of shear deformation. Mirsky and Hermann (1958) employed the first order shear deformation theory for the analysis of an isotropic cylinder. In the beginning of decade 1980, one Japanese group of material scientist, were created new class of materials. Properties of this material vary continuously in terms of components of coordinate system. In the beginning of decade 1990, it started research on the mechanical, thermal and vibration analysis of FG materials (Yamanouchi et al. (1990)).

Tutuncu and Ozturk (2001) presented the exact solution of FG spherical and cylindrical pressure vessels. Jabbari et al. (2002) analyzed the thermo elastic analysis of a FG cylinder under the thermal and mechanical loads. It is supposed that, the material properties are varying as a power function in terms of radial coordinate system. With substituting the derived temperature field into Navier equation, governing differential equation is solved,

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analytically.

Wu et al. (2005) investigated the elastic stability of a FG cylinder. They are employed the shell Donnell's theory to derive the strain-deformation relations. Stress-strain relation can be obtained by regarding the effect of thermal strain in stress-strain relation. Three nonlinear equations of equilibrium according to Donnell's theory are applied. With imposing the condition of prebuckling and a function for the radial displacement, results have been obtained with minimizing of the critical load with respect to the parameter of the problem. They are evaluated in the buckling load of cylinder under uniform temperature rising.

Shao (2005) investigated the thermo elastic analysis of a thick walled cylinder under mechanical and thermal loads. The cylinder is divided into many annular sub cylinders. It is assumed that, every sub cylinders to be isotropic. Based on this assumption, the thermal and the equilibrium equation are employed for every sub cylinder, individually.

After solving the thermal and equilibrium equation in every sub cylinder, the conditions of compatibility for the thermal and mechanical components are imposed within every two layers. This procedure can be followed for the complete cylinder. Finally, the distributions of the temperature and displacement have been obtained, numerically.

Jabbari et al. (2009) investigated the thermo elastic behaviour of a FG cylinder under the thermal and the mechanical loads. At the first, they are employed two-dimensional differential equation of heat transfer for different boundary conditions. By regarding two differential equations of equilibrium in the cylindrical coordinate system and imposing the distribution of temperature, they are obtained two navier equations in terms of two components of displacement. Ghannd et al investigated the elastic analysis of a pressurized thick-walled cylinder using plane elasticity theory and FSDT (Ghannad et al., 2008a, 2008b) Comparison between two theories indicates that, the maximum difference of them is not significant.

Zamani Nejad and Rahimi analyzed the deformations and stresses in rotating FG pressurized thick hollow cylinder under thermal load. They are used the plane elasticity theory to investigates the effect of angular velocity on the distribution of the stresses and displacements (Nejad and Rahimi (2009)). As a main applicable instance of shells, cylindrical shell can be considered in the present paper. Pressure vessels, reactors, heat exchanger and other nuclear and chemical equipments are the instances of cylindrical shells. Present study would improve the manufacturing of chemical and weapon equipments and then increases the strength of them using FGM. The present study considers the effect of pressure and temperature on the behaviour of FG cylinder, simultaneously. Furthermore, the effect of axial distribution of the temperature is investigated on the axial deformation and shear stress for different values of non homogenous index.

FORMULATION

As mentioned in introduction, the first order shear deformation theory (FSDT) is employed to simulate the deformation of every layer of the cylinder (Mirsky and Hermann, 1958; Ghannad et al., 2008b). In order to utilize FSDT, it is necessary to expand Lame's solution for a cylindrical pressure vessel. In the Lame's theory, symmetrical distribution of the radial displacement may be obtained as follows:

$$u = c_1 r + \frac{c_2}{r} \tag{1}$$

Where, r is the radius of every layer of cylinder. In the general case, this distance can be replaced in terms of radius of mid-plane R and distance of every layer with respect to mid-plane z as follows (Ghannad et al., 2008b).

$$r = R + z, \quad R = \frac{r_i + r_o}{2} \quad)$$

With substitution of r from Equation 2 into Lame's solution (Equation 1) and applying Taylor expansion, Equation 1 may be re-written as a function of z as follows:

$$u = c_1 (R+z) + \frac{c_2}{R+z} = c_0' + c_1' z + \ldots \tag{3}$$

This formulation (Equation 3) is known as the first order shear deformation theory (FSDT). Based on this theory, every component of deformation can be stated by two variables that includes the rotation and displacement. For a symmetric cylindrical shell, the radial and axial components of deformation may be regarded as follows (Ghannad et al., 2008b).

$$\begin{Bmatrix} u_x \\ w_z \end{Bmatrix} = \begin{Bmatrix} u \\ w \end{Bmatrix} + z \begin{Bmatrix} \phi_x \\ \phi_z \end{Bmatrix} \tag{4}$$

Where u_x, w_z are the axial and radial components of the deformation, respectively. u, w, ϕ_x, ϕ_z are functions of axial component of coordinate system (x), only. With regarding to the Equation 4, the components of strains are:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u_x}{\partial x} = \dfrac{\partial u}{\partial x} + z\dfrac{\partial \phi_x}{\partial x} \\ \varepsilon_z = \dfrac{\partial w_z}{\partial z} = \phi_z \\ \varepsilon_t = \dfrac{w_z}{r} = \dfrac{w + z\phi_z}{R+z} \\ \gamma_{xz} = 2 \times \varepsilon_{xz} = \dfrac{\partial u_x}{\partial z} + \dfrac{\partial w_z}{\partial x} = \phi_x + \dfrac{\partial w}{\partial x} + z\dfrac{\partial \phi_z}{\partial x} \end{cases} \tag{5}$$

Stress strain relations by regarding the effect of the thermal strain are:

$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x - v(\sigma_t + \sigma_z)}{E} + \alpha T \\ \varepsilon_z = \dfrac{\sigma_z - v(\sigma_t + \sigma_x)}{E} + \alpha T \\ \varepsilon_t = \dfrac{\sigma_t - v(\sigma_x + \sigma_z)}{E} + \alpha T \\ \gamma_{xz} = 2\varepsilon_{xz} = \dfrac{\tau_{xz}}{G} \end{cases} \quad (6)$$

By doing a little mathematic calculation, the components of stress in terms of strains are:

$$\begin{cases} \sigma_x = \dfrac{E}{(1+v)(1-2v)}\{(1-v)\varepsilon_x + v(\varepsilon_t + \varepsilon_z)\} - \dfrac{\alpha T E}{1-2v} \\ \sigma_t = \dfrac{E}{(1+v)(1-2v)}\{(1-v)\varepsilon_t + v(\varepsilon_x + \varepsilon_z)\} - \dfrac{\alpha T E}{1-2v} \\ \sigma_z = \dfrac{E}{(1+v)(1-2v)}\{(1-v)\varepsilon_z + v(\varepsilon_x + \varepsilon_z)\} - \dfrac{\alpha T E}{1-2v} \\ \tau_{xz} = \dfrac{E}{2(1+v)}\{\gamma_{xz}\} \end{cases} \quad (7)$$

Strain energy is equals to the one half of multiplying of the components of the stress tensor in the corresponding components of the strain tensor. Therefore, the strain energy per unit volume ($\bar{u}$) may be obtained as follows:

$$\bar{u} = \frac{1}{2}\{\varepsilon\}^T\{\sigma\} = \frac{1}{2}\{\sigma_x\varepsilon_x + \sigma_z\varepsilon_z + \sigma_t\varepsilon_t + \tau_{xz}\gamma_{xz}\}$$
$$\frac{1}{2}\frac{E}{(1+v)(1-2v)}\left[(1-v)(\varepsilon_x^2 + \varepsilon_z^2 + \varepsilon_t^2) + 2v(\varepsilon_x\varepsilon_z + \varepsilon_x\varepsilon_t + \varepsilon_t\varepsilon_z) + \frac{1-2v}{2}\gamma_{xz}^2\right]$$
$$-\frac{\alpha T E}{2(1-2v)}(\varepsilon_x + \varepsilon_t + \varepsilon_z) \quad (8)$$

Equation 8 includes two different expressions. The first class of them is the mechanical strain energy and second class is the thermal strain energy. The total strain energy must be evaluated by integration of Equation 8 on the volume of the cylinder. The volume element of the cylinder is $2\pi(R+z)dzdx$

$$dV = 2\pi(R+z)dzdx \rightarrow U = \iiint_V \bar{u}\, dV$$
$$U = \pi\int_0^l \int_{-\frac{h(x)}{2}}^{\frac{h(x)}{2}} \frac{E}{(1+v)(1-2v)}\left[(1-v)(\varepsilon_x^2 + \varepsilon_z^2 + \varepsilon_t^2) + 2v(\varepsilon_x\varepsilon_z + \varepsilon_x\varepsilon_t + \varepsilon_t\varepsilon_z) + \frac{1-2v}{2}\gamma_{xz}^2\right](R+z)dzdx$$
$$-\pi\int_0^l \int_{-\frac{h(x)}{2}}^{\frac{h(x)}{2}} \frac{\alpha T E}{1-2v}(\varepsilon_x + \varepsilon_t + \varepsilon_z)(R+z)dzdx = \int_0^1 [U_S(x) - U_T(x)]dx \quad (9)$$

Where, $U_S(x)$ is the mechanical strain energy and $U_T(x)$ is the thermal strain energy. With substitution of the strain component in terms of four displacement and rotation components, equations of mechanical and thermal energy (Equation 9) can be obtained using Equations 10a and 10b, respectively:

$$U_S = \sum_{i=1}^{9} A_i(x) f_i(x) \rightarrow f_i(x) = f_i(u, w, \phi_x, \phi_z) \quad (10a)$$

$$U_T = \sum_{i=1}^{4} B_i(x) g_i(x) \rightarrow g_i(x) = g_i(u, w, \phi_x, \phi_z) \quad (10b)$$

Where, functions $A_i(x), f_i(x), B_i(x), g_i(x)$ are expressed in appendix A.

Calculation of external works

Energy of internal and external pressure is equal to multiplying of the pressure in the radial deformation of the inner and the other surface of cylinder, respectively. Inner pressure applies in the same direction of the deformation; conversely, outer pressure applies in the opposite direction of the deformation. Equation 11 shows work done by the internal and external pressure. Figure 1 shows the schematic figure of the cylindrical pressure vessel. In the present paper, only internal pressure is regarded.

$$W = \int_0^l [C_1 w + C_2\phi_z]dx$$
$$C_1 = 2\pi(P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2}))$$
$$C_2 = 2\pi\frac{h}{2}(-P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2})) \quad (11)$$

Variation of the energy equation

Total energy of the system must be obtained by subtraction of Equation 11 from Equation 9 as follows: (11)

$$U = \int_0^l (U_S - U_T)dx - \int_0^l W dx = \int_0^l F(u, w, x, \phi_x, \phi_z, u_x, w_x)dx$$
$$U_S = \sum_{i=1}^{9} A_i(x) f_i(x), \quad U_T = \sum_{i=1}^{4} B_i(x) g_i(x)$$
$$W = C_1 w + C_2\phi_z, \quad C_1 = 2\pi(P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2})), \quad C_2 = 2\pi\frac{h}{2}(-P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2})) \quad (12)$$

As mentioned above, Equation 12 includes four variables. Using Euler Equation, variation of Equation 12 can be expressed as follows:

$$1)\ \frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial(\frac{\partial u}{\partial x})}\right) = 0$$
$$2)\ \frac{\partial F}{\partial w} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial(\frac{\partial w}{\partial x})}\right) = 0$$
$$1)\ \frac{\partial F}{\partial \phi_x} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial(\frac{\partial \phi_x}{\partial x})}\right) = 0$$
$$2)\ \frac{\partial F}{\partial \phi_z} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial(\frac{\partial \phi_z}{\partial x})}\right) = 0 \quad (13)$$

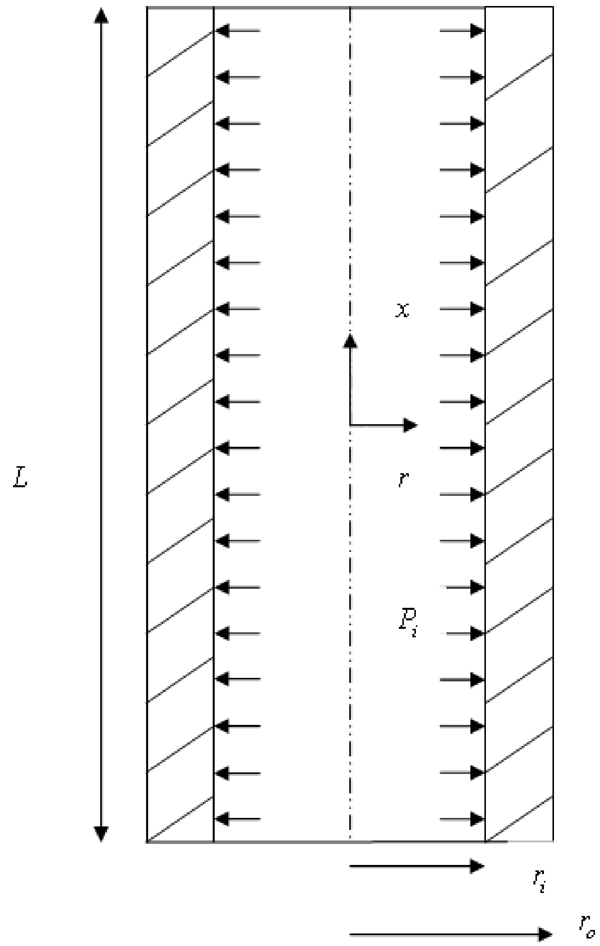

Figure 1. Schematic figure of cylindrical pressure vessel under applied inner and outer pressure.

Where, $u_x = \frac{\partial u}{\partial x}, w_x = \frac{\partial w}{\partial x}$ and functional $F(u, w, x, \phi_x, \phi_z, u_x, w_x)$ in Equation 12 must be regarded as follows:

$$\begin{aligned}
F &= U_S - U_T - W = \sum_{i=1}^{9} A_i(x) f_i(x) - \sum_{i=1}^{4} B_i(x) g_i(x) - C_1 w - C_2 \phi_z = \\
& A_1(x)\times\{(1-\nu)[\phi_z^2 + (\frac{\partial u}{\partial x})^2] + 2\nu[\phi_z \times (\frac{\partial u}{\partial x})] + \frac{1-2\nu}{2}[\phi_x^2 + (\frac{\partial w}{\partial x})^2 + 2\phi_x \frac{\partial w}{\partial x}]\} + A_2(x)\times\{(1-\nu)[2\frac{\partial u}{\partial x}\frac{\partial \phi_x}{\partial x}] \\
& + 2\nu[\phi_z \frac{\partial \phi_x}{\partial x}] + \frac{1-2\nu}{2}[2\phi_x \frac{\partial \phi_z}{\partial x} + 2\frac{\partial \phi_z}{\partial x}\frac{\partial w}{\partial x}]\} + A_3(x)\times\{(1-\nu)[(\frac{\partial \phi_x}{\partial x})^2] + \frac{1-2\nu}{2}[(\frac{\partial \phi_z}{\partial x})^2]\} + A_4(x)\times 2\nu[w\phi_z + w\frac{\partial u}{\partial x}] \\
& + A_5(x)\times(1-\nu)[w^2] + A_6(x)\times 2\nu[\phi_z^2 + w\frac{\partial \phi_x}{\partial x} + \phi_z \frac{\partial u}{\partial x}] + A_7(x)\times(1-\nu)[2\phi_z w] + A_8(x)\times(1-\nu)[\phi_z^2] \\
& + A_9(x)\times 2\nu[\phi_z \frac{\partial \phi_x}{\partial x}]\} - \{B_1(x)\times(\phi_z + \frac{\partial u}{\partial x}) + B_2(x)\times\frac{\partial \phi_x}{\partial x} + B_3(x)\times w + B_4(x)\times\phi_z\} - C_1 w - C_2 \phi_z
\end{aligned} \tag{14}$$

Equilibrium equations (Equation 13) can be represented in terms of resultant of moments and forces. This procedure diminishes the long mathematic equations. Resultant of moments and forces in terms of components of stress are:

$$\begin{aligned}
& N_x = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \sigma_x (1+\frac{z}{R}) dz, \quad N_t = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \sigma_t dz, \quad N_z = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \sigma_z (1+\frac{z}{R}) dz \\
& M_x = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \sigma_x (1+\frac{z}{R}) z dz, \quad M_\theta = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \sigma_\theta z dz, \quad M_{xz} = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \tau_{xz} (1+\frac{z}{R}) z dz \\
& Q_Z = \int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}} \tau_{xz} (1+\frac{z}{R}) dz
\end{aligned} \tag{15}$$

Therefore, the main governing relation on the thermo-elastic behaviour of a functionally graded cylinder can be expressed as follows:

$$\begin{cases}
\frac{\partial N_x}{\partial x} = 0 \\
N_t - \frac{\partial}{\partial x}[RQ_z] + \frac{1}{2\pi} B_3(x) = [P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2})] \\
Q_z - \frac{1}{R}\frac{\partial}{\partial x}[RM_x + \frac{B_2(x)}{2}] = 0 \\
[RN_z + \frac{B_1(x)}{2\pi}] + [M_\theta + \frac{B_4(x)}{2\pi}] - \frac{\partial}{\partial x}[RM_{zx}] = \frac{h}{2}[-P_i(R - \frac{h}{2}) - P_0(R + \frac{h}{2})]
\end{cases} \tag{16}$$

For a special case that the temperature effect is disregarded, $T = 0$ we have $B_i = 0$ and therefore, the equilibrium equations (Equation 16) are in accordance with the literature (Ugural (1981); Ghannad et al. (2008b)). Where, $B_i(x)$ are the functions of the thermal conditions (Appendix A). Equation 16 is the second order system of differential equation with four variables. Equation 17 shows the matrix presentation of Equation 16.

$$[G_1]\frac{d^2}{dx^2}\{X\} + [G_2]\frac{d}{dx}\{X\} + [G_3]\{X\} = \{F\}, \quad \{X\} = [u \quad \phi_x \quad w \quad \phi_z]^T \tag{17}$$

With applying the appropriate matrix operations to Equation 17, G_1, G_2, G_3 and force vector F may be obtained as follows:

$$G_1 = \begin{bmatrix}
(1-\nu)A_1 & (1-\nu)A_2 & 0 & 0 \\
(1-\nu)A_2 & (1-\nu)A_3 & 0 & 0 \\
0 & 0 & \frac{(1-2\nu)}{2}A_1 & \frac{(1-2\nu)}{2}A_2 \\
0 & 0 & \frac{(1-2\nu)}{2}A_2 & \frac{(1-2\nu)}{2}A_3
\end{bmatrix} \tag{18-1}$$

$$G_2 = \begin{bmatrix}
0 & 0 & \nu A_4 & \nu(A_6 + A_1) \\
0 & 0 & -\frac{(1-2\nu)}{2}A_1 + \nu A_6 & -\frac{(1-2\nu)}{2}A_2 + \nu(A_2 + A_9) \\
-\nu A_4 & -\nu A_6 + \frac{(1-2\nu)}{2}A_1 & 0 & 0 \\
-\nu(A_1 + A_6) & -\nu(A_2 + A_9) + \frac{(1-2\nu)}{2}A_2 & 0 & 0
\end{bmatrix} \tag{18-2}$$

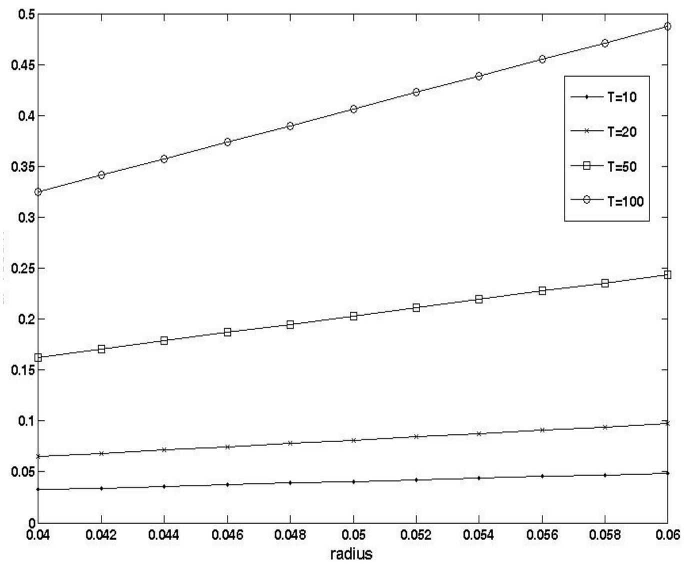

Figure 2. Radial displacement of FG cylinder with n=1 and in terms of different temperature rising.

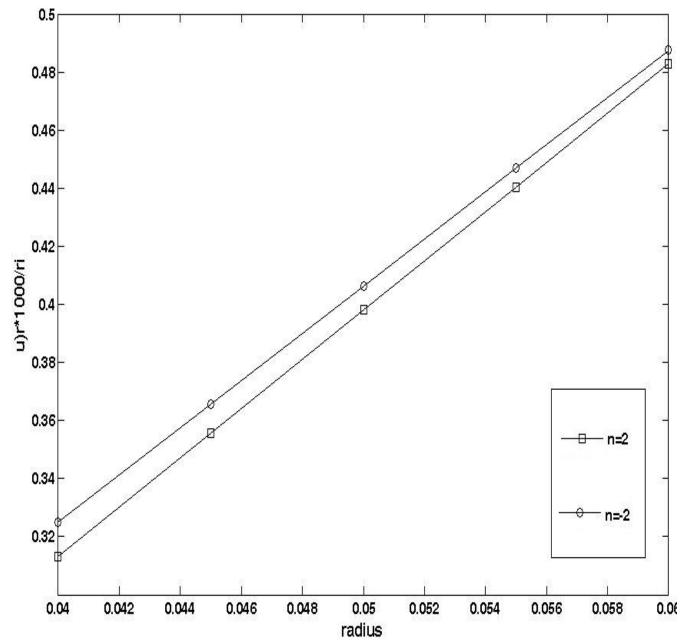

Figure 3. Comparison of radial displacement for 100℃. Temperature rising in terms of two non- homogenous coefficient of elasticity.

$$G_3=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{(1-2v)}{2}A_1 & 0 & 0 \\ 0 & 0 & -(1-v)A_5 & -(1-v)A_7\text{-}vA_4 \\ 0 & 0 & -(1-v)A_7-vA_4 & -(1-v)(A_8+A_1)-2vA_6 \end{bmatrix} \quad (18\text{-}3)$$

$$F=\begin{bmatrix} \dfrac{\partial B_1(x)}{\partial x} \\ -\dfrac{1}{2}\dfrac{\partial B_2(x)}{\partial x} \\ -\dfrac{B_3(x)}{2}-\pi(P_i(R-\dfrac{h}{2})-P_0(R+\dfrac{h}{2})) \\ -\dfrac{B_1(x)+B_4(x)}{2}-\pi\dfrac{h}{2}[-P_i(R-\dfrac{h}{2})-P_0(R+\dfrac{h}{2})] \end{bmatrix} \quad (18\text{-}4)$$

Equations 18 - 1, 18 - 2, 18 - 3 and 18 - 4 are the complete governing differential equations for thermo elastic analysis of a FG cylinder that have been derived yet. At the regions that are adequately far from the both ends of cylinder, following numerical calculation can be investigated.

NUMERICAL RESULTS, COMPARISON AND DISCUSSION

In the present section, numerical results are investigated. It is supposed that, the modulus of elasticity E is graded in the radial direction only, E(z). Before numerical evaluation, non-homogenous modulus of elasticity must be defined as a power function of radial coordinate as follows:

$$E=E_i(\bar{r})^n=E_i(\frac{r}{r_i})^n \xrightarrow{r=R+z}$$
$$E=E_i(\frac{R+z}{r_i})^n=\frac{E_i}{r^n{}_i}(R+z)^n \quad (19)$$

The value of inner and outer radius is

$a=r_i=0.04m$, $b=r_0=0.06m$

Other numerical values are:

$E_i=2\times10^{11}$, $\alpha=5\times10^{-6}$, R=0.05 , v=0.3

Studying of the results in the presence of temperature only

Radial displacement

In this section, inner and outer pressures are assumed zero. Figure 2 shows the radial displacement of a FG cylinder for $n=1$ at the region that is adequately far from the both end of the cylinder. This figure is plotted for four different values of temperature rising. It can be understood that the radial displacement is a linear function of temperature rising. For evaluating of the effect of non-homogenous index n, Figure 3 shows the radial displacement of a FG cylinder for similar temperature rising and two different non-homogenous indexes.

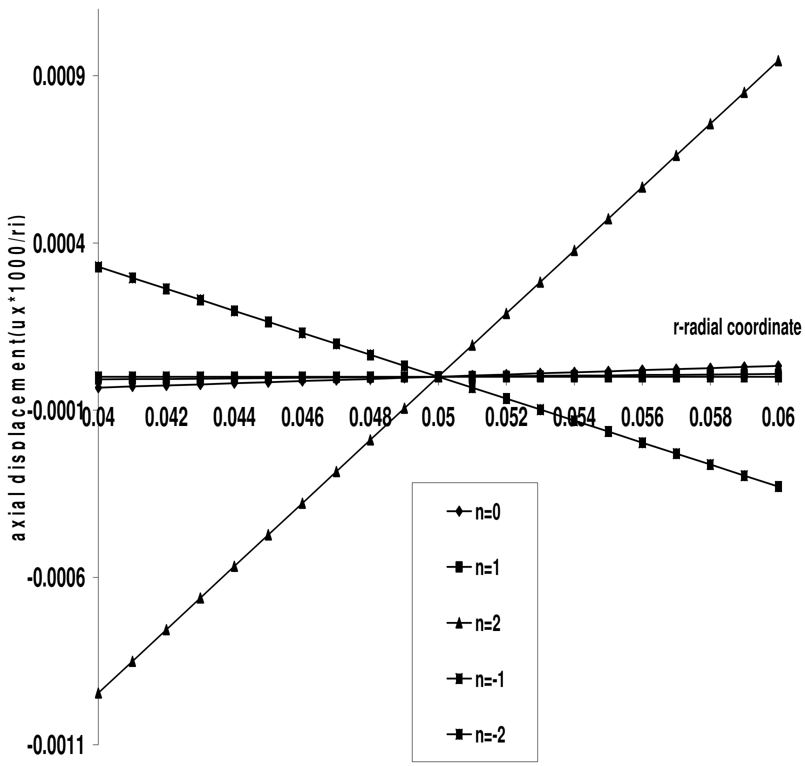

Figure 4. Axial displacement of FG cylinder under 100℃ temperature rising in terms of *n*.

Axial displacement

The axial distribution of the temperature can create the axial displacement and consequently shear stress. Axial displacement can be investigated in the presence of axial distribution of the temperature. Inner and outer pressure does not effect the axial displacement. In this section, it is convenient to investigate the effect of temperature rising on the axial displacement and shear stresses. For example, with regarding a linear distribution of the temperature along the longitudinal coordinate system and by setting $n=2$, rotation function of axial displacement can be calculated in terms of temperature rising and geometric parameters as follows:

$$\phi_x = \frac{\alpha T h^2 (h^2 + 20R^2 + v h^2 + 20 v R^2)}{40R(5\frac{1-2v}{12})(h^2 + 4R^2)} \tag{20-1}$$

For every value of n, it is straightforward to evaluate similar equation and consequently, using Equation 4, axial displacement can be evaluated in the radial direction.

$$u_x = z\phi_x = z\frac{\alpha T h^2 (h^2 + 20R^2 + v h^2 + 20 v R^2)}{40R(5\frac{1-2v}{12})(h^2 + 4R^2)} \tag{20-2}$$

Figure 4 shows axial displacement of a FG cylinder in the radial direction for temperature rising 100˚C. As depicted in the Figure 4, axial displacement for n = -1 is equal to zero. This is due to, B2=0 for n = -1. As stated above, in the presence of axial distribution of temperature rising, it is inevitable that, cylinder bears shear strain and stress. Figure 5 shows the radial distribution of the shear stress in the radial direction.

$$\gamma_{xz} = 2\times\varepsilon_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial w_z}{\partial x} = \phi_x + \cancel{\frac{\partial v}{\partial x}} + z\cancel{\frac{\partial \phi_z}{\partial x}} = \phi_x \tag{21-1}$$

$$\tau_{xz}{}^{n} = \frac{E(z)}{2(1+v)}\{\gamma_{xz}\} = \frac{E_i (R+z)^n}{2r^n{}_i(1+v)}\phi_x \tag{21-2}$$

Figure 5 shows that, value of shear stress for n = 2 are significant among the other values. Shear stress for n = -1 vanishes due to zero shear strain.

Results in the simultaneously presence of temperature and inner pressure

Figures 6 and 7 show the radial displacement of a FG cylinder in terms of n for two values of temperature rising (10 and 100˚C) and inner pressure 80 Mpa, respectively.

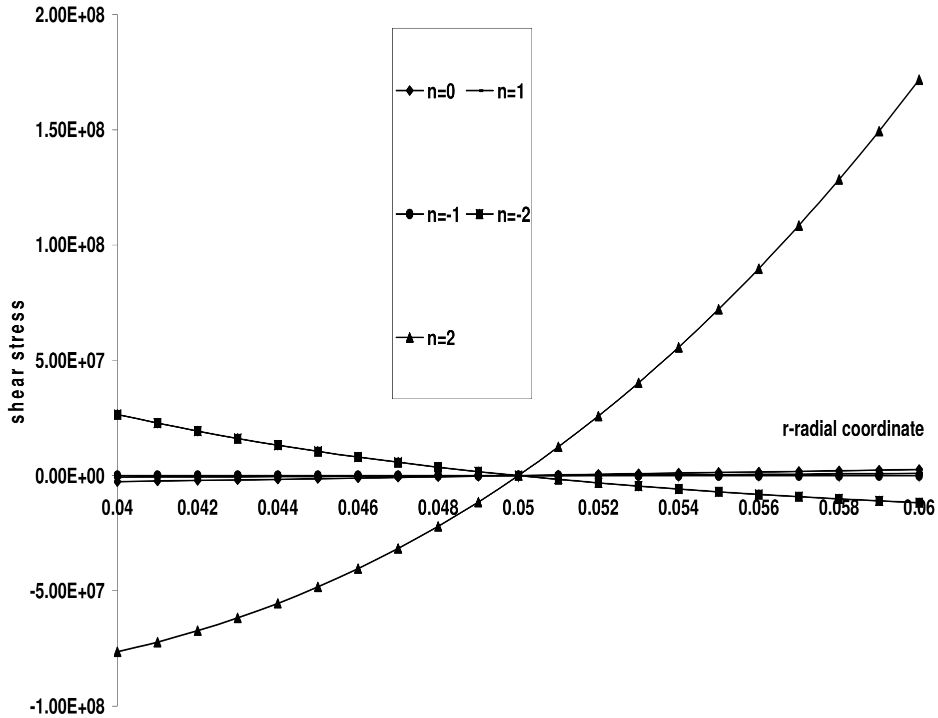

Figure 5. Distribution of shear stress in terms of n for 100℃ temperature rising.

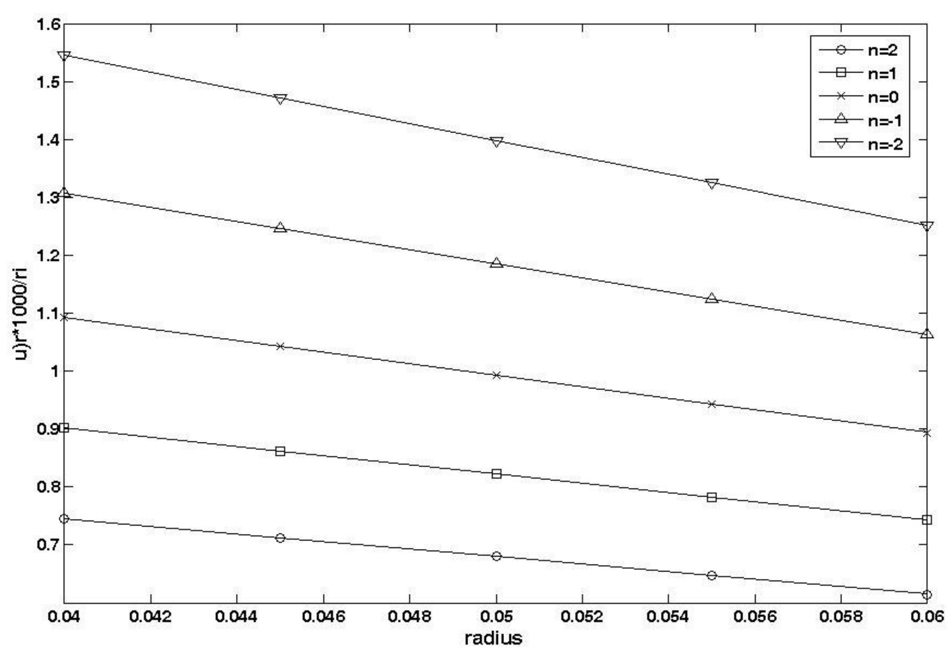

Figure 6. Distribution of radial displacement for 10℃ temperature rising and inner pressure 80 Mpa.

As depicted in Figures 6 and 7, similar temperature rising increases the radial displacement at the outer layer more than inner layer of FG cylinder. As consequence of this observation is that, negative slope of figure decreases with increasing of the temperature rising. Figure 8 shows the radial distribution of the displacement under inner

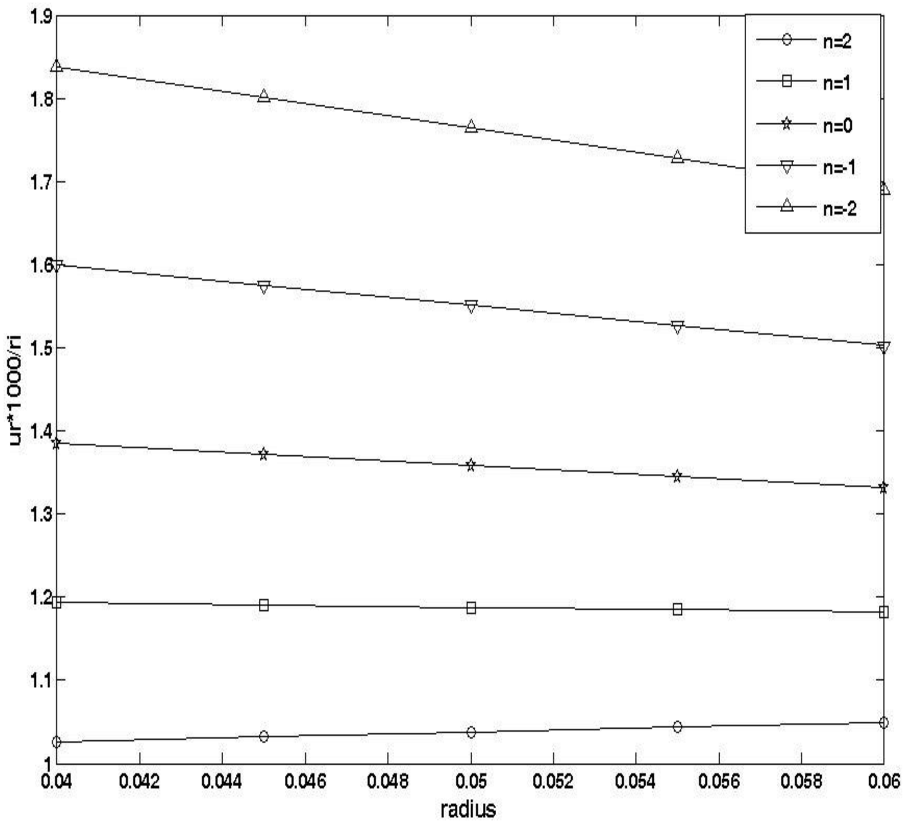

Figure 7. Distribution of radial displacement for 100 centigrade temperature rising and inner pressure 80 Mpa.

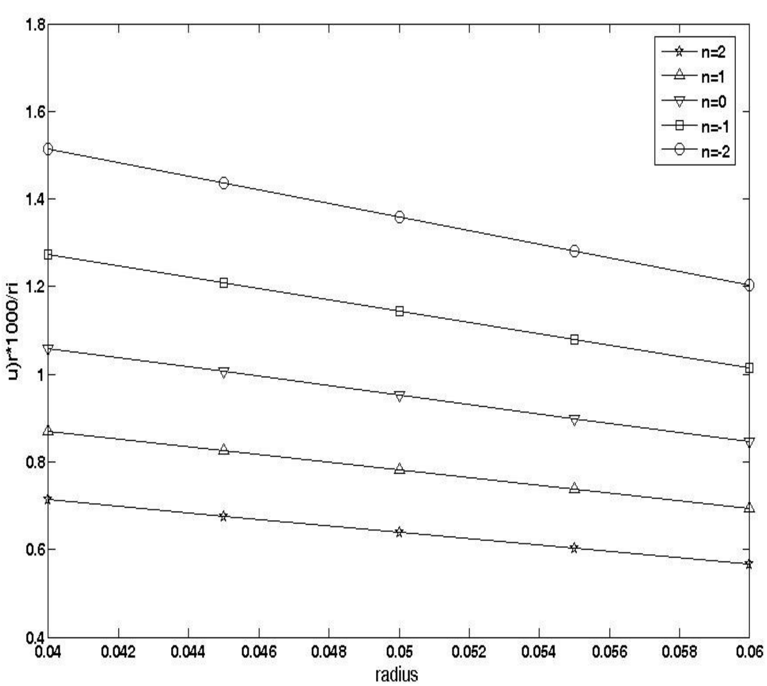

Figure 8. Radial distribution of displacement under only inner pressure (80 Mpa).

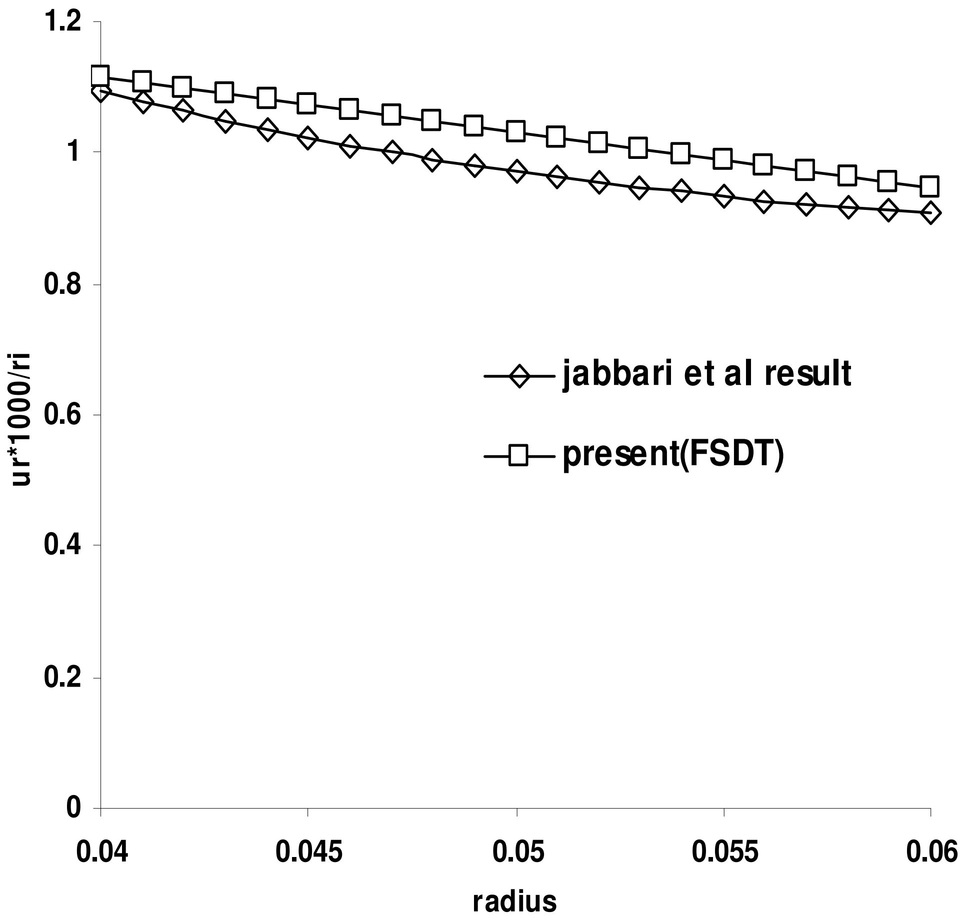

Figure 9. Comparing of the present result (FSDT) with result of plane elasticity theory (Jabbari et al).

pressure only. This figure is exactly identical to previous result (Ghannad, 2008).

Evaluation of results using the plane elasticity theory

In order to compare the present results with the results of plane elasticity theory, in this section, it is evaluated analytical results of a FG cylinder under thermal and mechanical loads Jabbari et al (2002). Differential equation and solution of a FG cylinder using the plane elasticity theory are:

$$u'' + \frac{1+m_1}{r}u' + (\frac{vm_1}{1-v} - 1)u = \frac{1+v}{1-v}\frac{\alpha m_1}{r}T \qquad (22\text{-}1)$$

$$u = u_g + u_p = B'_1 r^{\eta_1} + B'_2 r^{\eta_2} + u_p \qquad (22\text{-}2)$$

Where, B'_1, B'_2 are the constants of integration and η_1, η_2 are the roots of the characteristic equation. With applying the boundary conditions to the displacement, the distribution of the radial displacement can be derived analytically.

$$B.C\begin{cases}\sigma_r = -P_i & \text{r=r}_\text{i}\\ \sigma_r = 0 & \text{r=r}_\text{o}\end{cases} \qquad (22\text{-}3)$$

With evaluating the values of constant, it is convenient to compare the results of two methods (FSDT and plane elasticity theory). Figure 9 shows the results of two methods (FSDT and plane elasticity theory). It is observed that there is an ignorable difference between two theories. Figure 10 shows the percentage of difference between two theories. As depicted in Figure 10, maximum percentage of difference is 5.88%. Therefore, FSDT can be employed as a valid theory for evaluating the solution of thick walled pressure vessel.

Conclusion

The results that were extracted from the present paper are:

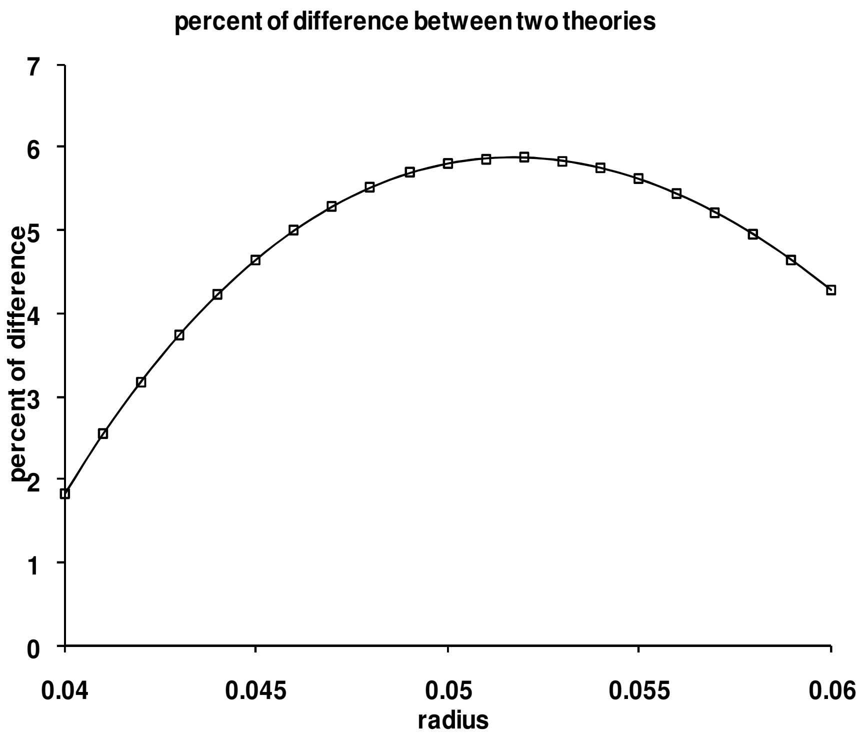

Figure 10. The percentage of difference between of two theories (FSDT and plane elasticity).

(i) In the present paper, thermo elastic formulation of a FG cylindrical shell under pressure based on the first order shear deformation theory (FSDT) is presented. In the previous papers, it is not recognized application of this theory for evaluation of response of a FG cylindrical shell under mechanical and thermal load special for evaluation of the axial deformation.

(ii) In the first section, general formulation of thermo elastic behaviour of pressurized FG cylinder is derived. Derived differential equation (Equation 16) includes a system of four variables and equations. Derived equations in this study (Equation 16) in a special case are compared with literature and the present equations are verified [Ugural (1981)].

(iii) The effect of axial distribution of temperature rising on the deformation of cylinder is studied. FSDT has ability to predict an expression for the axial deformation of cylinder under axial distribution of the temperature. In the plane elasticity theory, axial deformation cannot be evaluated.

(iv) For the better consideration of a FG cylinder, the effect of temperature and pressure on the deformation is investigated, simultaneously. This investigation indicates that the temperature rising diminishes the negative slope of radial displacement along the thickness direction. In the other word, increasing of temperature with constant inner pressure, tend to a uniform distribution of the radial displacement.

(v) As depicted in Figure 9, it is obvious that the present results that derived by employing the first order shear deformation theory (FSDT) is in accordance with the result of plane elasticity formulation. The maximum difference between two theories is not significant.

Therefore, FSDT can be used for the analysis of a FG cylindrical shell as a valid method.

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Appendix1

$$A_1(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi E(R+z)}{(1+v)(1-2v)}dz,\ \mathrm{f}_1=(1-v)[\phi_z{}^2+(\frac{\partial u}{\partial x})^2]+2v[\phi_z\times(\frac{\partial u}{\partial x})]+\frac{1-2v}{2}[\phi_x{}^2+(\frac{\partial w}{\partial x})^2+2\phi_x\frac{\partial w}{\partial x}]$$

$$A_2(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez(R+z)}{(1+v)(1-2v)}dz,\ \mathrm{f}_2=(1-v)[2\frac{\partial u}{\partial x}\frac{\partial \phi_x}{\partial x}]+2v[\phi_z\frac{\partial \phi_x}{\partial x}]+\frac{1-2v}{2}[2\phi_x\frac{\partial \phi_z}{\partial x}+2\frac{\partial \phi_z}{\partial x}\frac{\partial w}{\partial x}]$$

$$A_3(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez^2(R+z)}{(1+v)(1-2v)}dz,\ \mathrm{f}_3=(1-v)[(\frac{\partial \phi_x}{\partial x})^2]+\frac{1-2v}{2}[(\frac{\partial \phi_z}{\partial x})^2]$$

$$A_4(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi E}{(1+v)(1-2v)}dz,\ \mathrm{f}_4=2v[w\phi_z+w\frac{\partial u}{\partial x}]$$

$$A_5(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi E}{(R+z)(1+v)(1-2v)}dz,\ \mathrm{f}_5=(1-v)[w^2]$$

$$A_6(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez}{(1+v)(1-2v)}dz,\ \mathrm{f}_6=2v[\phi_z{}^2+w\frac{\partial \phi_x}{\partial x}+\phi_z\frac{\partial u}{\partial x}]$$

$$A_7(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez}{(R+z)(1+v)(1-2v)}dz,\ \mathrm{f}_7=(1-v)[2\phi_z w]$$

$$A_8(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez^2}{(R+z)(1+v)(1-2v)}dz,\ \mathrm{f}_8=(1-v)[\phi_z{}^2]$$

$$A_9(x)=\int_{-\frac{h(x)}{2}}^{-\frac{h(x)}{2}}\frac{\pi Ez^2}{(1+v)(1-2v)}dz,\ \mathrm{f}_9=2v[\phi_z\frac{\partial \phi_x}{\partial x}]$$

$$B_1(x)=\pi\int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}}\frac{\alpha(R+z)ET}{1-2v}dz,\quad g_1=\phi_z+\frac{\partial u}{\partial x}$$

$$B_2(x)=\pi\int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}}z\frac{\alpha(R+z)ET}{1-2v}dz,\quad g_2=\frac{\partial \phi_x}{\partial x}$$

$$B_3(x)=\pi\int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}}\frac{\alpha ET}{1-2v}dz,\quad g_3=w$$

$$B_4(x)=\pi\int_{-\frac{h(x)}{2}}^{+\frac{h(x)}{2}}z\frac{\alpha ET}{1-2v}dz,\quad g_4=\phi_z$$

Nomenclature

r	Radius of an arbitrary layer of cylinder	G	Shear modulus of elasticity
z	Coordinate of arbitrary layer of cylinder respect to mid-plane	E	Modulus of elasticity
R	Radius of mid-plane of cylinder	$B_i(x)$	General property of material
u_x	Axial component of deformation	P_i	Internal pressure
w_z	Radial component of deformation	P_0	External pressure
u	Displacement component of axial deformation	W	External work(due to pressure)
w	Displacement component of radial deformation	C_1	General force
ϕ_x	Rotational component of axial deformation	C_2	General moment
ϕ_z	Rotationa component of radial deformation	F	general potential function
ε_x	Axial strain	N_x	Resultant of axial force
ε_z	Radial strain	N_t	Resultant of tangential force
ε_t	Circumferential strain	N_z	Resultant of radial force
ε_{xz}	Shear strain in xz plane	M_x	Resultant of axial moment
σ_x	Axial stress	M_θ	Resultant of tangential moment
σ_z	Radial stress	M_{xz}	Resultant of shear force
σ_t	Circumferential stress	Q_Z	Resultant of axial force
τ_{xz}	Shear stess	$\{X\}$	Vector of general deformation
T	Temperature rising	$[G_1]$	4×4 matrice of material property
α	Heat expansion coefficient	$[G_2]$	4×4 matrice of material property
U	Total energy	$[G_3]$	4×4 matrice of material property
$\overline{u}$	Energy per unit volume	$\{F\}$	Vector of general force(thermal and mechanical)
dV	Element of volume	E_i	Modulus of elasticity in the inner raddius
$h(x)$	Local thichness of cylinder	r_i	Inner radius
$U_S(x)$	Mechanical strain energy	r_o	Outer radius
$U_T(x)$	thermal strain energy	a	Inner radius
$f_i(x)$	A function of component of displacement and rotation	b	Outer radius
$A_i(x)$	General property of material	n	Non homogenous index
$g_i(x)$	A function of component of displacement and rotation	m_1	Non homogenous index