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Determination of the most economic thickness and energy source in the design of local hemispherical clay pots

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There is currently great emphasis on research regarding various aspects of energy sources but the question of optimum usage of the energy is equally important. This work seeks to produce an economic computer-aided design of a fire-clay pot based on conditions that give minimum cost per unit of usage time for the consumer. A percentage of energy loss from the fuel to the environment due to inefficiency of the heating system was incorporated as a variable in the calculations. Graphs are presented to indicate the calculated optimum economic pot thicknesses at various values of the inner radii. For a common inner radius of 6.0 cm and 75% energy loss, the optimum pot thicknesses for cooking at temperatures of 60, 80, and 100°C were found to be 9.6, 12.0, and 13.8 mm, respectively. The results show that the economic thickness increases with increase in both inner radius and cooking temperature. It has also been established that the operational cost increases with increase in temperature even at the optimum economic thickness. This work includes a study of the calorimetric value of the most commonly used types of wood and charcoal fuel, in Zambia, for clay-pot cooking. These types of fuel come from the four trees locally known as the Musamba, Mubanga, Mutondo, and Mutiti. Several wood and charcoal samples from all these trees were collected and experimentally analysed using an oxygen bomb calorimeter to determine the amount of heat energy in Joules which can be realizable per kilogram of the fuel materials.

Key words: Calorific value, economic operational cost, computer-aided design.

INTRODUCTION

Clay is a very fine-grained rock matter which is plastic when wet but becomes hard when heated. Plasticity is the property that allows clay to change shape without rupturing. Clay is one of the most affordable and widely used refractory raw materials. The importance of clay pots in the daily lives of people is that clay has some advantages over other cookware materials like steel, iron or aluminium. Clay takes a long time to absorb heat but once it does it spreads that heat evenly throughout the clay pot’s body and releases it slowly to the food cooking within. The somewhat porous nature of clay allows both moisture and heat to circulate through the pot which helps in even, slow and delicate cooking. This slow cooking allows for the flavour of a dish to build slowly.

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for spices to penetrate more deeply and in the case of meat, tough cuts become succulent. A second advantage comes from the unglazed nature of local clay pots. The clay is in contact with the food as it cooks and being alkaline in nature clay helps in neutralizing the pH balance of sour or bitter acids in food with such ingredients as tomatoes, hence, mellowing it and making it to have a little natural sweetness. From a health point of view, clay is a natural material which does not contain any hazardous chemicals to react with the food. It helps the ingredients retain their nutrients and vitamins during cooking. The food retains its natural flavours. Clay is also biodegradable and eco-friendly.

There is a lot of emphasis on research regarding various aspects of energy sources but the question of optimum economic usage of the energy is equally important. An optimum economic design of a product could either be based on conditions giving maximum profit per unit of production or minimum cost per unit of usage time for the consumer (Robert et al., 1997). In this case, the minimum cost per unit time is looked for in terms of energy and material usage of clay pots.

To determine the most economic local energy source for use as cooking fuel, the calorific value of different types of charcoal and wood were determined. The most commonly used types of fire wood and charcoal in Zambia come from the four trees locally known as the Musamba, Mubanga, Mutondo, and Mutiti. Several wood and charcoal samples from all these trees were collected and experimentally analysed using an oxygen bomb calorimeter to determine the amount of heat energy in Joules which can be realizable per kg of the fuel materials.

Wood is mostly used in the rural areas although charcoal is also used there. Charcoal is the most commonly used cooking fuel in our peri-urban areas; it is also widely used in the urban areas where wood is rarely used. There is some research work aimed at improving the properties of charcoal as a fuel (Botrel et al., 2007). Our computational calculations considered charcoal as the source of fuel but the method and computer program used in this work could be used for any other source of fuel. This work uses a computer-aided formulation to solve the problem of finding the thickness of a clay pot that will be most economical in terms of both energy and production material usage. In order to achieve this, the minimum cost per hour was looked for by observing where the total cost of both energy and clay usage goes through a minimum with respect to the pot thickness at various values of the inner pot radius. The cost analysis of heating and material thickness is integrated into the heat transfer phenomenon. It was assumed that heat only enters or leaves the clay pots by the process of conduction, heat transfer due to radiation, and convection was minimal (Lewis et al., 1988) and therefore ignored. The thermal conductivity of ceramics is generally inversely related to the square root of the clay grain size (Chan and Jones, 1962; Bamigbala, 2001). In this work, an average value of the thermal conductivity was used. The heat transfer phenomenon was analyzed using the Fourier law in a hemispherical shell model. Calculations were carried out using the C++ programming language. This aspect of our work is a continuation of previous work carried out by Olatona and Alamu (2013), although the fundamental theoretical formulation of the problem differs in certain ways from that of the previous work.

The present work goes further than the previous work by incorporating a percentage of energy loss from the fuel to the environment as a variable in the calculations, due to inefficiency of the heating system at various temperatures. Graphs are presented to indicate the calculated optimum economic pot thicknesses at various values of the inner radii. For a common inner radius of 6.0 cm and 75% energy loss, the optimum pot thicknesses for cooking at temperatures of 60, 80, and 100°C are found to be 9.6, 12.0 and 13.8 mm, respectively. The 75% energy loss is an assumption based on research which was carried out by Berick (2006). The temperatures of 60, 80 and 100°C were considered as approximate temperatures that could be used for slow, medium speed and fast cooking, respectively. For electric stoves the cooking temperature is controlled by passing the current to the heating element through a bi-metallic strip that thermostatically opens and closes the circuit, this interrupts the current at intervals thereby controlling the average current and temperature. A gas stove control knob adjusts the rate of flow of gas to its burner, hence controlling the temperature. For charcoal and wood fire stoves, the cooking temperature can be controlled by varying the amount of fuel in the stove or the distance of the pot on the stove from where the fire is placed. A heat diffuser could also be placed between the fire and the pot; this also helps to spread out the heat from the burner so that it heats the pot evenly.

**EXPERIMENTAL DETAILS**

Four samples of different wood types (Mubanga, Mutiti, Mutondo, and Musamba) and three samples of charcoal (excluding Mubanga) were collected and ground into powder. The powder was converted into samples which were in the form of pellets, using a pellet press; each pallet was about 1 g. A known length of nichrome fuse wire was passed through the sample with both ends of the wire being attached to the two electrodes of the bomb. The sample was then lowered into bomb and a screw cap was used to seal the bomb.

An oxygen supply line was then connected. The pressure gauge of the oxygen supply line was watched as the bomb pressure rose to the desired filling pressure, usually thirty atmospheres or higher but never more than forty atmospheres. The oxygen supply control valve was closed and the bomb was put inside a calorimeter bucket with 1.4 kg of water. The water-equivalent mass (in terms of heat capacity) of the calorimeter was 0.39 kg. A stirrer was started which ran for 5 min to reach temperature equilibrium after which a timer was started and temperatures were recorded at 1 min intervals for 5 min. At the end of the 5 min, the bomb was fired. The bucket
temperature would start to rise within 30 s after firing. The rise was rapid during the first few seconds; then it became slower as the temperature approached a stable maximum. Temperatures were recorded at 1 min intervals until the difference between successive readings had been constant for 3 min.

The change in temperature was observed before and during the combustion process of the samples, using a precision thermometer. This was done in order to get a corrected temperature (Jessup, 1970), \( T_c \), which was later used to calculate the calorific value of the samples under investigation.

After the last temperature reading, the interior of the bomb was opened and examined for soot or other evidence of incomplete combustion. If such evidence was found, the test was discarded. All unburned pieces of fuse wire were removed from the bomb electrodes and their combined masses were measured. This mass was subtracted from the initial mass of the fuse wire used and the difference was entered on the data sheet as the net amount of wire burned. Calorific values of samples from the four types of trees were obtained after the calorimeter was calibrated (Jessup, 1970) using standard benzoic acid.

The oxygen bomb calorimeter used was the MAHLER BOMB CE-430.

**COMPUTATIONAL DETAILS**

### Inner surface temperature of the local hemispherical clay pot

The inner surface temperature of the hemispherical clay pot was determined as a function of the thickness using Fourier’s law of conduction as the following.

Since the volume of a hemisphere is \( V = \frac{4}{3}\pi r^3 \), then the volume of a hemispherical volume element is,

\[
dV = 2\pi r^2 dr = A dr
\]  \( \text{(1)} \)

where \( A \) is the surface area of a hemispherical shell. By Fourier’s law of conduction,

\[
Q = -kA \left( \frac{dT}{dr} \right).
\]

where \( Q = \) energy transfer per second; \( k = \) is the thermal conductivity; \( dT = \) small change in temperature; and \( dr = \) thickness of hemispherical shell.

Figure 1 is a schematic diagram showing the cross section of a hemispherical clay pot with various parameters used in the calculation.

Fourier’s law for a hemispherical shell is,

\[
Q = -k2\pi r^2 \left( \frac{dT}{dr} \right).
\]

In using Figure 1, we get,

\[
\int_{r_1}^{r_2} \frac{dr}{r^2} = -2\pi k\int_{r_1}^{r_2} dT
\]

or

\[
-Q \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = -2\pi k(T_2 - T_1)
\]

which gives,

\[
Q = 2\pi k r_2 \left( \frac{T_2 - T_1}{r_2 - r_1} \right).
\]

We can write,

\[
Q = U(T_{food} - T_i)
\]

where \( U = \) Heat transfer coefficient.

As a first approximation,

\[ U = JT_i, \]

where \( J \) is a proportionality constant between the inner surface
temperature and heat transfer coefficient. It follows that,

\[ Q = JT(T_{\text{food}} - T_i). \]  

Equating Equations 2 and 3,

\[ 2\pi kr_2 \left( \frac{T_1 - T_2}{r_2 - r_1} \right) = JT(T_{\text{food}} - T_i) \]

which gives

\[ JXT_i^2 + (2\pi kr_2 - JT_{\text{food}})X_1 - 2\pi kr_2 T_2 = 0. \]

where \( X = r_2 - r_1 \), which is the pot thickness. Using the quadratic formula, we get the inner surface temperature, \( T_1 \) of the hemispherical clay pot as a function of the thickness, \( X \),

\[ T_1 = \left( \frac{1}{2JX} \right) \sqrt{\left( 2k \theta r_1 - JT_{\text{food}}X \right)^2 + 8\pi k \theta r_1 T_2 X} \]  

(4)

**Operational cost in kwacha per hour**

The total operational cost of clay and energy in kwacha per hour was arrived at by adding the cost of energy and that of clay as shown subsequently. Here, square brackets are sometimes used next to an expression in an equation. Note that the quantities in the square brackets are not variables but simply units of the associated expression which are included for dimensional clarity.

**Cost of energy in kwacha/hour**

From Equation 2,

\[ Q = 2\pi kr_2 \left( (T_1 - T_2) / (r_2 - r_1) \right) \]  

[J/s].

Let the energy transfer rate per hour be \( Q' \),

\[ Q' = 3600 \times 2\pi kr_2 \left( (T_1 - T_2) / (r_2 - r_1) \right) \]  

[J/hr].

(5)

Also, \( Y \) [J/kg], be the energy equivalent of 1 kg of charcoal or wood in Joules per kg, while \( Z \) [K/kg] is the cost of 1 kg of fuel material in Kwacha (K) per kg.

The cost of heating in Kwacha per hour, \( C'_{\text{energy}} \) is then given by:

\[ C'_{\text{energy}} [K/hr] = \frac{Q'[J/hr] \times Z [K/kg]}{Y [J/kg]}, \]

that is,

\[ C'_{\text{energy}} = Q' \left( \frac{Z}{Y} \right). \]  

(6)

\( P \) was define to be the approximate percentage of energy wasted from the fuel to the surrounding during the heating process. If, \( C'_{\text{energy}} \) is the cost of energy used without loss of heat to the surrounding (that is, \( P = 0 \)), then the total cost including loss to the surrounding is more and can be represented by, \( C_{\text{energy}} \) as,

\[ \left( 1 - \frac{P}{100} \right) C_{\text{energy}} = C'_{\text{energy}}. \]

Therefore, using Equation 6,

\[ C_{\text{energy}} = \left( \frac{100}{100 - P} \right) Q' \left( \frac{Z}{Y} \right). \]

Substituting Equation 5 into the aforementioned expression, we get the operational cost of energy in kwacha per hour as,

\[ C_{\text{energy}} = \left( \frac{100}{100 - P} \right) 3600 \times 2\pi kr_2 ((T_1 - T_2) / (r_2 - r_1)) (Z / Y). \]  

(7)

**Cost of clay in kwacha per hour**

The volume of a hemispherical element as given in Equation 1 is,

\[ dV = 2\pi r^2 dr. \]

The volume of clay in a hemispherical pot is therefore,

\[ V = 2\pi \int_{r_1}^{r_2} r^2 dr \]

giving

\[ V = \frac{2\pi}{3} (r_2^3 - r_1^3) \]  

[m³].

(8)

\( C_{\text{vol}} \left[ \frac{K}{m^3} \right] \) was defined as the cost of clay per unit volume in Kwacha (K) per m³. \( C_{\text{vol}} \) is calculated by obtaining the price of a commercially sold clay pot and dividing it by the volume of the constituent clay, calculated using Equation 8. \( Sl [/hr] \) was also define, as the average service life of a clay pot in hours and \( C_{\text{clay}} \left[ \frac{K}{hr} \right] \) as the effective cost of clay per hour. These quantities are related by the equation,

\[ C_{\text{clay}} \left[ \frac{K}{hr} \right] = \left( C_{\text{vol}} \left[ \frac{K}{m^3} \right] V [m^3] \right) / Sl [hr]. \]  

(9)

Substituting Equation 8 into Equation 9, we get the operational cost of clay in kwacha per hour as,
and 100°C. The ambient temperature, $T_2$ was kept at a constant value of 28°C.

**RESULTS**

**Experimental results**

Several wood and charcoal fuel samples from the Musamba, Mubanga, Mutondo, and Mutiti trees were collected and experimentally analysed using an oxygen bomb calorimeter to determine the amount of heat energy realizable per kg from the materials. Temperature change before and during combustion of the Mutondo charcoal sample is as shown in Figure 2.

It can be seen from Figure 2 that after 5 min there is a rapid rise in temperature, this is the point at which the bomb was fired. It can also be seen in the figure that the rise was rapid during the first few seconds, then it became slower as the temperature approached a stable maximum. Graphs of temperature changes before and during combustion of the other fuel samples were obtained but are not presented here since Figure 2 is representative of their general appearance.

A representative calculation (Jessup, 1970) of the calorific value is given subsequently, for the sample from the Mutondo tree:

**Mass of the sample = 1.0071 g**

**Mass of burnt nichrome fuse wire = 0.0065 g**

**Rate of temperature rise before firing**

$$\frac{24.80708 - 24.62937}{5\text{min}} = 0.035542°C/\text{min}$$

**Rate of temperature decrease after combustion**

$$\frac{29.45374 - 29.49051}{5\text{min}} = -0.007354°C/\text{min}$$

**Cooling correction over 6 minutes**

$$\frac{(-0.007354 + 0.035542)}{2} = 0.084564°C$$

**Corrected temperature rise during firing**, $T_c$ = (maximum temp – initial temp) + cooling correction

$$T_c = (29.49051 - 24.80708) + 0.084564 = 4.76799°C$$

**Calorific value**

$$\frac{[(m \times C_p \times T_c)_{\text{water}} - (Y \times C_V)_{\text{wire}}]}{m_{\text{sample}}}$$

where $m$ = mass of water plus the water-equivalent mass (in terms of heat capacity) of the calorimeter; $C_p$ = specific heat capacity of water at constant pressure; $Y$ = mass of burnt nichrome fuse wire; and $C_V$ = calorific value of nichrome fuse wire.
Table 1. Samples of wood and charcoal with their calorific values in kilojoules per kg.

<table>
<thead>
<tr>
<th>No</th>
<th>Sample type</th>
<th>Calorific value (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Umutiti (wood)</td>
<td>17584.236</td>
</tr>
<tr>
<td>2</td>
<td>Umutiti (charcoal)</td>
<td>29599.067</td>
</tr>
<tr>
<td>3</td>
<td>Umutondo (wood)</td>
<td>16992.138</td>
</tr>
<tr>
<td>4</td>
<td>Umutondo (charcoal)</td>
<td>35395.033</td>
</tr>
<tr>
<td>5</td>
<td>Umubanga (wood)</td>
<td>22509.224</td>
</tr>
<tr>
<td>6</td>
<td>Umusamba (wood)</td>
<td>27640.350</td>
</tr>
<tr>
<td>7</td>
<td>Umusamba (charcoal)</td>
<td>29668.380</td>
</tr>
</tbody>
</table>

Table 2. Average calorific values for charcoal and wood in kilojoules per kg.

<table>
<thead>
<tr>
<th>Sample type</th>
<th>Average calorific value (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>21181.487</td>
</tr>
<tr>
<td>Charcoal</td>
<td>31554.160</td>
</tr>
</tbody>
</table>

\[
= \frac{((1.4+0.39) \times 4.1803 \times 4.76799) - (1600 \times 0.0000065)}{0.0010071}
\]
\[
= 35395.035 \text{ kJ/kg.}
\]

Table 1 shows the calorific values in kilojoules per kg for the samples collected. The average calorific values for charcoal and wood were calculated using the experimental data presented in Table 1. These average values are shown in Table 2.

Computational results

The following data were used as inputs for a C++ computer program which was written according to the computational details.

Energy equivalent of 1 kg of charcoal in joules, \( Y = 31554160.0 \text{ J/kg.} \)

Cost of 1 kg of charcoal, \( Z = 0.63 \text{ K/kg.} \)

Average thermal conductivity value of local clay, \( k = 0.0865 \text{ W/mk.} \)

Average service life of the hemispherical clay pot in hours, \( S_l = 35064.0 \text{ h (4 years).} \)

Cost of clay per cubic meter, \( C_{vol} = 220000.0 \text{ K/m}^3. \)

The approximate percentage loss of energy from the fuel to the surroundings during heating process, \( P = 75.0\%. \)

Inner radius of the clay pot, \( r_1 = 0.06 \text{ m (this was varied from 0.04 m to 0.12 m).} \)

Initial thickness of the clay pot, \( X_0 = 0.005 \text{ m (this was varied with variations in } r_1 \text{).} \)

Shell thickness, \( \Delta X = 0.0002 \text{ m (this was also slightly varied with variations in } r_1 \text{).} \)

Proportionality constant between the inner surface temperature and the heat transfer coefficient, \( J = 0.25. \)

Temperature of the cooking food, \( t_{food} = 373.0 \text{ K (333 K and 353 were also used).} \)

Ambient temperature, \( T_2 = 301.0 \text{ K.} \)

Number of pot thickness values to be plotted per cooking temperature, \( n = 50. \)

Results of calculations carried out using the C++ program are as shown in Figures 3 to 8. Figure 3 gives a graph of the pot thickness against the total operational cost for a typically used inner pot radius of 6 cm at a cooking temperature of 100°C with an energy loss of 75% to the environment. The figure shows that the optimum economic pot thickness with an inner pot radius of 6 cm at a cooking temperature of 100°C with an energy loss of 75% is 13.8 mm.

The results for an inner pot radius of 6 cm at a cooking temperature of 80°C with an energy loss of 75% are shown in the graph of the pot thickness against the total operational cost in Figure 4. Figure 4 shows that the optimum economic pot thickness with an inner pot radius of 6 cm at a cooking temperature of 80°C with an energy loss of 75% is 12.0 mm.

The results for an inner pot radius of 6 cm at a cooking temperature of 60°C with an energy loss of 75% are shown in the graph of the pot thickness against the total operational cost in Figure 5. Figure 5 shows that the optimum economic pot thickness with an inner pot radius of 6 cm at a cooking temperature of 60°C with an energy loss of 75% is 9.6 mm.

The optimum economic pot thickness was calculated for values of inner radii ranging from 4 to 12 cm using an investigation similar to that presented for the inner radius of 6 cm. Figure 6 shows three graphs of the pot thickness against the total operational cost for an inner pot radius of 4 cm at cooking temperatures of 60, 80 and 100°C, respectively, with an energy loss of 75%.

Figure 7 shows three graphs of the pot thickness against the total operational cost for an inner pot radius of 5 cm at cooking temperatures of 60, 80 and 100°C, respectively, with an energy loss of 75%.

Figure 8 shows three graphs of the pot thickness
against the total operational cost for an inner pot radius of 12 cm at cooking temperatures of 60, 80 and 100°C, respectively, with an energy loss of 75%.

Graphs of the pot thickness against the total operational cost were also obtained for inner pot radii of, 8, 9, 10 and 11 cm. The values of the thickness corresponding to the minimum operational cost were determined from all the graphs, the results are summarized in Table 3.

It must be pointed out that the minimum points of the graphs in Figures 3 to 8, which give the optimum economic thicknesses, cannot be determined from the graphs to an error of ±0.01 cm, as presented in Table 3. The results in the table were obtained by an examination of the raw output data from the computer program.

**DISCUSSION**

Table 1 shows the amount of heat energy given out per
Figure 7. Relationship between operational cost and thickness of the clay pot with an inner radius of 5 cm, at cooking temperatures of 60, 80 and 100°C, respectively.

Figure 8. Relationship between operational cost and thickness of the clay pot with an inner radius of 12 cm, at cooking temperatures of 60, 80 and 100°C respectively.

unit mass of the fuel samples collected. Calorific values obtained for the fuel sources ranged from 16992.138 to 27640.224 kJ/kg for wood and from 29599.067 to 35395.033 kJ/kg for charcoal. The average calorific values were 21181.487 kJ/kg for wood and 31554.160 kJ/kg for charcoal. It is observed that the average calorific
value for charcoal is much higher than that for wood. Of the three samples of charcoal studied, the one from the Mutondo had the highest calorific value while the Musamba had the highest value among the four wood samples.

Clay is chemically stable at high temperatures, resistant to chemical corrosion, can withstand considerable, abrasion, impact and fracture due to thermal stresses (Heckroodt, 1991; Kromer and Mörtel, 1988; Borode, 2000). The durability of clay pots generally depends on the type of clay used, the heat treatment (firing) used during production and the design. A material called a temper is normally mixed with the clay to enhance performance. Finely crushed limestone or shells are good temper materials. Some clay does not require any temper but this type of clay is rare. The average clay needs to be enhanced with approximately 10 to 15% temper. Clay products with no temper often crack when fired or have a short duration after firing. If there is too sudden a change in temperature, a clay pot may crack, even if it was properly produced and seasoned. A round flat bottom helps the pot to withstand a little more physical abuse than a curved bottom because there is larger flat surface area to absorb contact pressure or shock from below.

Heat transfer was analyzed using the Fourier law in a hemispherical shell design. Calculations of the optimum economic thicknesses of the clay pots were carried out. Table 3 shows that in order to operate a typical clay pot of inner radius 6 cm in a practical situation where there is a 75% loss of energy to the surroundings and cooking at temperatures of 60, 80 and 100°C, the optimum economic pot thicknesses are 9.6 mm, 12.0 mm and 13.8 mm respectively. The table shows a trend of an increase in economic thickness with temperature at which the pot is cooking, this is observed at all values of inner pot radii from 4 to 12 cm. It is also seen in Table 3 that the graphs of operational cost against pot thickness, at all values of the inner pot radii, shift upwards with increase in temperature. The operational cost therefore increases with increase in temperature even at the optimum economic thickness.

In our preliminary survey, it was found that for commonly used cooking pots the thickness of the walls was from 0.6 to 1.0 cm. The optimum thicknesses obtained in our study were from 0.92 to 1.50 cm, giving a thickness range with greater structural durability while still allowing for heat to be easily and uniformly transferred to the food being cooked.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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