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Full Length Research Paper

Heat transfer and entropy generation in a pipe flow with temperature dependent viscosity and convective cooling

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The second law analysis is performed to study the entropy generation rate in a variable viscosity liquid flowing steadily through a cylindrical pipe with convective cooling at the pipe surface. The system is assumed to exchange heat with the ambient following Newton's cooling law and the fluid viscosity model varies as an inverse linear function of temperature. The analytical expressions for fluid velocity and temperature that were derived essentially expedite the expressions for volumetric entropy generation of numbers, irreversibility distribution ratio and the Bejan number in the flow field.

Key words: Pipe flow, variable viscosity, convective cooling, irreversibility analysis.

INTRODUCTION

Studies related to viscous fluid with temperature dependent properties are of great importance in industries such as food processing, coating and polymer processing (Macosko, 1994; Schlichting, 2000). In industrial system, fluid can be subjected to extreme conditions such as high temperature, pressure and shear rate. External heating such as the ambient temperature and high shear rates can lead to a high temperature being generated with the fluid. This may have a significant effect on the fluid properties. Fluid used in industries such as polymer fluids have a viscosity that varies rapidly with temperature and may give rise to strong feedback effects, which can lead to significant changes in the flow structure of the fluid (Makinde, 2008; Sahin, 1999; Tasnim and Mahmud, 2002). Due to the strong coupling effect between the Navier-Stokes and energy equations, viscous heating also plays an important role in fluid with strong temperature dependence. Elbashbeshy and Bazid (2000) investigated the effect of temperature dependent viscosity on heat transfer over a moving surface. In their investigation, the fluid viscosity model varies as an inverse linear function of temperature. Costa and Macedonio (2003) applied the temperature dependent viscosity model to study magma flows. Makinde (2006) studied the flow of liquid film with variable viscosity along an inclined heated plate. The effects of temperature dependent fluid viscosity on heat transfer and thermal stability of reactive flow in a cylindrical pipe with isothermal wall was reported in Makinde (2007).

Moreover, thermodynamic irreversibility in any fluid flow process can be quantified through entropy analysis. The first law of thermodynamics is simply an expression of the conservation of energy principle. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated with the real processes (Narusawa, 2001). As entropy generation takes place, the quality of energy (that is, exergy) decreases (Ibanez et al., 2003; Makinde, 2008). In order to preserve the quality of energy in a fluid flow process or at least to reduce the entropy generation, it is important to study the distribution of the entropy generation within the fluid volume. The optimal design for any thermal system can be achieved by minimizing entropy generation in the systems. Entropy generation in thermal

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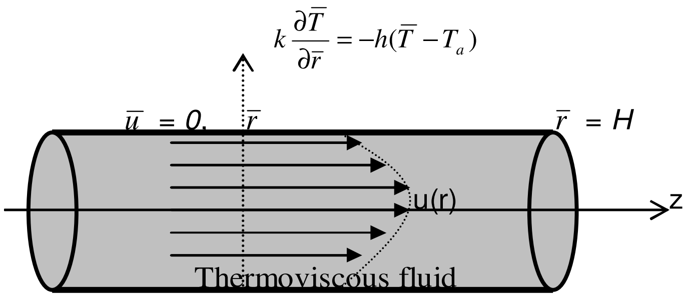

Figure 1. Schematic diagram of the problem.

engineering systems destroys available work and thus reduces its efficiency.

The study of entropy generation in conductive and convective heat transfer processes had assumed considerable importance since the pioneering work of Bejan (1995) and his subsequent book on the subject (1996). Since then, numerous papers have studied entropy generation in heat transfer processes of which references (Reddy et al., 2007; Sahin, 1999; Tasnim and Mahmud, 2002; Taufiq et al., 2007) are a representative sample. Recently, the thermodynamics second law characteristics for variable viscosity channel flow with convective cooling at the walls were discussed by Makinde (2008). It seems to the authors' knowledge that the effect of convective cooling on the entropy generation rate in a variable viscosity flow through a cylindrical pipe had not been investigated.

Motivated by the scarcity of such investigations, the problem of heat transfer and entropy generation in the flow of a variable viscosity fluid through a cylindrical pipe with convective cooling was studied.

MATHEMATICAL MODEL

The configuration of the problem studied in this paper is depicted in Figure 1. The flow is considered to be steady in the $\overline{z}$-direction through a cylindrical pipe of radius *a* and length *L* under the action of a constant pressure gradient, viscous dissipation, convective cooling at the pipe surface. It is assumed that the pipe is long enough to neglect both the entrance and exit effects. The fluid is incompressible and the temperature dependent viscosity ($\overline{\mu}$) can be expressed as (Elbashbeshy and Bazid, 2000; Tasnim and Mahmud, 2002).

$$\overline{\mu}=\frac{\mu_0}{1+m(\overline{T}-T_a)}, \tag{1}$$

where μ_0 is the fluid dynamic viscosity at the ambient temperature T_a

Under these conditions, the continuity, momentum and energy equations governing the problem in dimensionless form may be written as (Makinde, 2007; Schlichting, 2000)

$$\frac{\partial(ru)}{\partial z}+\frac{\partial(rv)}{\partial r}=0, \tag{2}$$

$$\varepsilon Re\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{dr}\right)=-\frac{\partial p}{\partial z}+2\varepsilon^2\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(\frac{\partial u}{\partial r}+\varepsilon^2\frac{\partial v}{\partial z}\right)\right], \tag{3}$$

$$\varepsilon^3 Re\left(u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\frac{2\varepsilon^2}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)+\varepsilon^2\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial r}+\varepsilon^2\frac{\partial v}{\partial z}\right)\right]-2\mu\varepsilon^2\frac{v}{r^2}, \tag{4}$$

$$\varepsilon \operatorname{Re}\operatorname{Pr}\left(u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}\right)=\varepsilon^2\frac{\partial^2 T}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\mu\Phi, \tag{5}$$

where

$$\Phi=\operatorname{Br}\left[2\varepsilon^2\left(\frac{\partial u}{\partial z}\right)^2+2\varepsilon^2\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{\partial u}{\partial r}\right)^2+2\varepsilon^2\left(\frac{v}{r}\right)^2+2\varepsilon^2\frac{\partial v}{\partial z}\frac{\partial u}{\partial r}+\varepsilon^4\left(\frac{\partial v}{\partial z}\right)^2\right] \tag{6}$$

We have employed the following non-dimensional quantities in Equations (2) to (6):

$$r=\frac{\overline{r}}{\varepsilon L}, z=\frac{\overline{z}}{L}, u=\frac{\overline{u}}{U}, v=\frac{\overline{v}}{\varepsilon U}, \varepsilon=\frac{a}{L}, \mu=\frac{\overline{\mu}}{\mu_0}, T=\frac{\overline{T}-T_a}{T_a}, P=\frac{a^2\overline{P}}{\mu_0 UL},$$
$$\alpha=mT_a, Br=\frac{\mu_0 U^2}{kT_a}, \operatorname{Pr}=\frac{\mu_0 c_p}{k}, \operatorname{Re}=\frac{\rho Ua}{\mu_0}, Bi=\frac{ah}{k}. \tag{7}$$

where ρ is the fluid density, *k* is the thermal conductivity, $\overline{T}$ is the fluid temperature, *U* is the velocity scale, α is the viscosity variation parameter, wall temperature, $\overline{u}$ is the axial velocity, $\overline{v}$ is the normal velocity, c_p is the specific heat at constant pressure, $\overline{P}$ is the pressure, *Pr* is the Prandtl number, *Br* is the Brinkman number, *Bi* is the Biot number, *h* is the transfer coefficient, *Re* is the Reynolds number, $\overline{x}$ and $\overline{y}$ are distances measured in streamwise and normal direction, respectively.

Since the pipe is narrow and the aspect ratio $0<\varepsilon<<1$, the lubrication approximation based on an asymptotic simplification of the governing Equations (2) to (6) is invoked and we obtain,

$$0=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+O(\varepsilon), \tag{8}$$

$$0=\frac{\partial p}{\partial r}+O\left(\varepsilon^2\right), \tag{9}$$

$$0=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\mu Br\left(\frac{\partial u}{\partial r}\right)^2+O(\varepsilon), \tag{10}$$

where $\mu = 1/(1+\alpha T)$.

The dimensionless corresponding boundary conditions at the pipe wall is the usual no slip condition for the fluid velocity. The system exchange heat with the ambient, we now follow the Newton's cooling law:

$$u = 0, \ \frac{dT}{dr} = -BiT \quad \text{at} \quad r = 1, \tag{11}$$

and the regularity of the solution along the pipe centreline that is,

$$\frac{du}{dr} = \frac{dT}{dr} = 0 \quad \text{at} \quad r = 0, \tag{12}$$

Solution method

Equations (8) to (10), subject to the boundary conditions can be easily combined to give

$$\frac{du}{dr}=-\frac{rG}{2}(1+\alpha T), \ \frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{r^3G^2Br}{4}(1+\alpha T)=0, \tag{13}$$

where $G = -\partial P/\partial z$ is the constant axial pressure gradient.

Equation (13) with the corresponding boundary conditions is solved exactly and we have the solutions for fluid velocity and temperature profiles as;

$$\begin{aligned}u(r) = \frac{1}{4}\Bigg(&G\,Bi\Bigg(-2r^2\,\text{BesselJ}\left(0, \frac{1}{4}G\sqrt{\alpha Br}\,r^2\right)\\ &- r^2\pi\,\text{StruveH}\left(0, \frac{1}{4}G\sqrt{\alpha Br}\,r^2\right)\text{BesselJ}\Bigg(1,\\ &\frac{1}{4}G\sqrt{\alpha Br}\,r^2\Bigg) + r^2\pi\,\text{StruveH}\Bigg(1,\\ &\frac{1}{4}G\sqrt{\alpha Br}\,r^2\Bigg)\text{BesselJ}\left(0, \frac{1}{4}G\sqrt{\alpha Br}\,r^2\right) + 2\,\text{BesselJ}\Bigg(0,\\ &\frac{1}{4}G\sqrt{\alpha Br}\Bigg) + \pi\,\text{StruveH}\left(0, \frac{1}{4}G\sqrt{\alpha Br}\right)\text{BesselJ}\Bigg(1,\\ &\frac{1}{4}G\sqrt{\alpha Br}\Bigg) - \pi\,\text{StruveH}\left(1, \frac{1}{4}G\sqrt{\alpha Br}\right)\text{BesselJ}\Bigg(0,\\ &\frac{1}{4}G\sqrt{\alpha Br}\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(-\sqrt{\alpha Br}\,G\,\text{BesselJ}\left(1, \frac{1}{4}G\sqrt{\alpha Br}\right)\\ &+ 2\,Bi\,\text{BesselJ}\left(0, \frac{1}{4}G\sqrt{\alpha Br}\right)\Bigg)\end{aligned} \tag{14}$$

$$\begin{aligned}T(r) = &\left(2\,\text{BesselJ}\left(0, \frac{1}{4}G\sqrt{Br\,\alpha}\,r^2\right)Bi\right)\Bigg/\Bigg(\Bigg(\\ &-\sqrt{Br\,\alpha}\,G\,\text{BesselJ}\left(1, \frac{1}{4}G\sqrt{Br\,\alpha}\right) + 2\,Bi\,\text{BesselJ}\Bigg(0,\\ &\frac{1}{4}G\sqrt{Br\,\alpha}\Bigg)\Bigg)\alpha\Bigg) - \frac{1}{\alpha}\end{aligned} \tag{15}$$

Thermal stability criterion

For the temperatures in the flow field to remain finite at any given value of α > 0, the denominator of Equation (15) should not vanish (Makinde, 2007; Makinde and Maserumule, 2008; Squire, 1967). The imposition of this restriction leads to the following thermal stability criterion

$$\frac{\text{BesselJ}\left(0, \frac{1}{4}G\sqrt{Br\,\alpha}\right)}{\text{BesselJ}\left(1, \frac{1}{4}G\sqrt{Br\,\alpha}\right)} \neq \frac{1}{2}\,\frac{G\sqrt{Br\,\alpha}}{Bi} \tag{16}$$

Equation (16) indicates that the thermal stability of the flow system depends not only on the convective cooling parameter but also on the viscous heating parameter and its pressure gradient as well as the parameter characterizing the fluid viscosity variation.

Entropy analysis

The general equation for the entropy generation per unit volume is given by (Bejan, 1995, 1996; Sahin, 1999; Makinde, 2006; Tasnim and Mahmud, 2002):

$$S^m = \frac{k}{T_a^2}\left(\nabla\bar{T}\right)^2 + \frac{\bar{\mu}}{T_a}\Phi. \tag{17}$$

The first term in Equation (17) is the irreversibility due to heat transfer and the second term is the entropy generation due to viscous dissipation. Equation (17) can be easily integrated from $\bar{r}=0$ to $\bar{r}=a$ to give the total entropy generated in the pipe flow as follows:

$$S^T = \int_0^a S^m 2\pi L\bar{r}d\bar{r}. \tag{18}$$

Using Equations (17) and (18), we express the entropy generation number and total entropy generated in dimensionless form as,

$$Ns = \frac{a^2S^m}{k} = \left(\frac{\partial T}{\partial r}\right)^2 + \mu Br\left(\frac{\partial u}{\partial r}\right)^2 + O(\varepsilon^2), \tag{19}$$

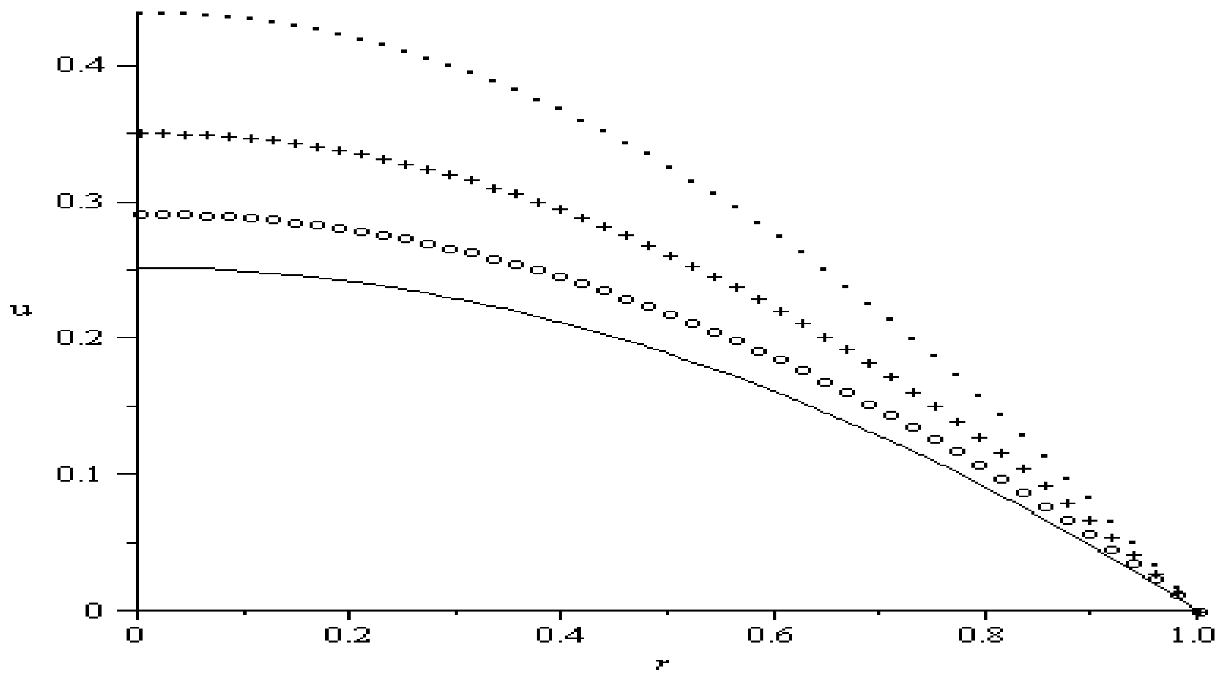

Figure 2. Velocity profile: G = 1; Br = 1; Bi = 1; _______α = 0.1; ooooo α = 2; ++++ α = 4; …….α = 6.

$$N_T = \frac{S^T}{2\pi Lk} = \int_0^1 Nsrdr. \tag{20}$$

In Equation (19), the first term can be assigned as N_1 and the second term due to viscous dissipation as N_2, that is,

$$N_1 = \left(\frac{\partial T}{\partial r}\right)^2, \quad N_2 = \mu Br\left(\frac{\partial u}{\partial r}\right)^2. \tag{21}$$

In order to have an idea whether fluid friction dominates over heat transfer irreversibility or vice-versa, Bejan (1996) define the irreversibility distribution ratio as $\Phi = N_2/N_1$. Heat transfer dominates for $0 \leq \Phi < 1$ and fluid friction dominates when $\Phi > 1$. The contribution of both heat transfer and fluid friction to entropy generation are equal when $\Phi = 1$. In many engineering designs and energy optimisation problems, the contribution of heat transfer entropy N_1 to overall entropy generation rate Ns is needed. As an alternative to irreversibility parameter, the Bejan number (Be) is define mathematically as

$$Be = \frac{N_1}{Ns} = \frac{1}{1+\Phi}. \tag{22}$$

Clearly, the Bejan number ranges from 0 to 1. Be = 0 is the limit where the irreversibility is dominated by fluid friction effects and Be = 1 corresponds to the limit where the irreversibility due to heat transfer by virtue of finite temperature differences dominates. The contribution of both heat transfer and fluid friction to entropy generation are equal when Be = 1/2. It is important to note that using Equations (14) to (15), the explicit expressions for Equations (18) to (22) can be easily obtained using any computer algebra package like MAPLE or MATEMATICA.

RESULTS AND DISCUSSION

For the numerical validation of our results, we have chosen physically meaningful values of the parameters entering into the problem. It is important to note that a positive increase in the parameter value of α indicates a decrease in the fluid viscosity while the convective cooling in the flow system is enhanced by increasing the Biot number (Bi). In Figures 2 to 5, the axial velocity distributions are reported for increasing values of α, Bi, G and Br. Generally a parabolic velocity profile is observed with maximum value along the pipe centerline and minimum at the wall. The velocity increases with increasing values of α, Br and G but decreases with increasing values of Bi. Thus, a decrease in the fluid viscosity coupled with an increase in the viscous heating will enhance the flow velocity, although similar effect is observed by increasing the flow pressure gradient. However, it is noteworthy that an increase in convective cooling slows down the flow process.

Typical variations of the fluid temperature profiles in the normal direction are shown in Figures 6 to 8. Generally, the fluid temperature attained its peak value along the

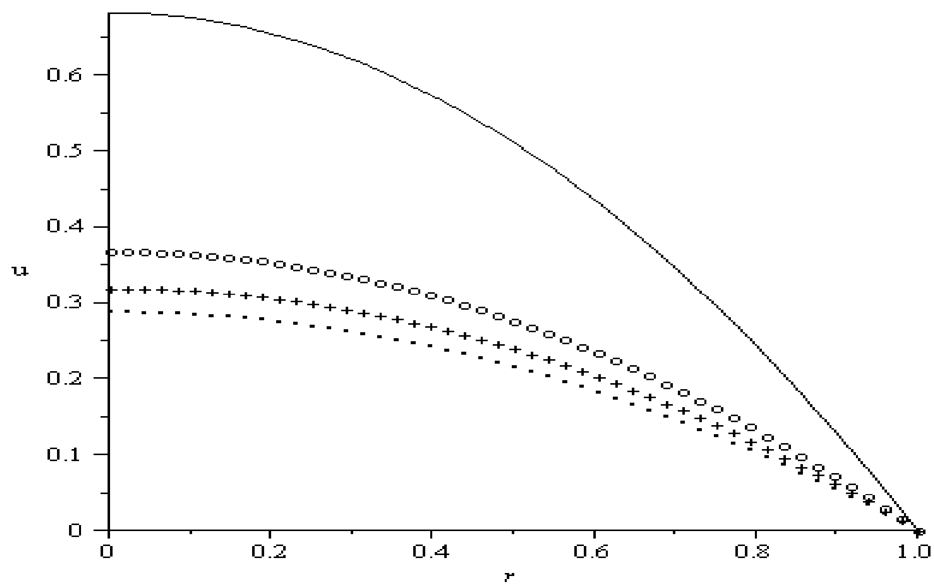

Figure 3. Velocity profile: *G*=1; α = 1; *Br* = 1; ______*Bi* = 0.1; ooooo*Bi* = 0.2; ++++ *Bi* = 0.4; ....*Bi* = 0.5.

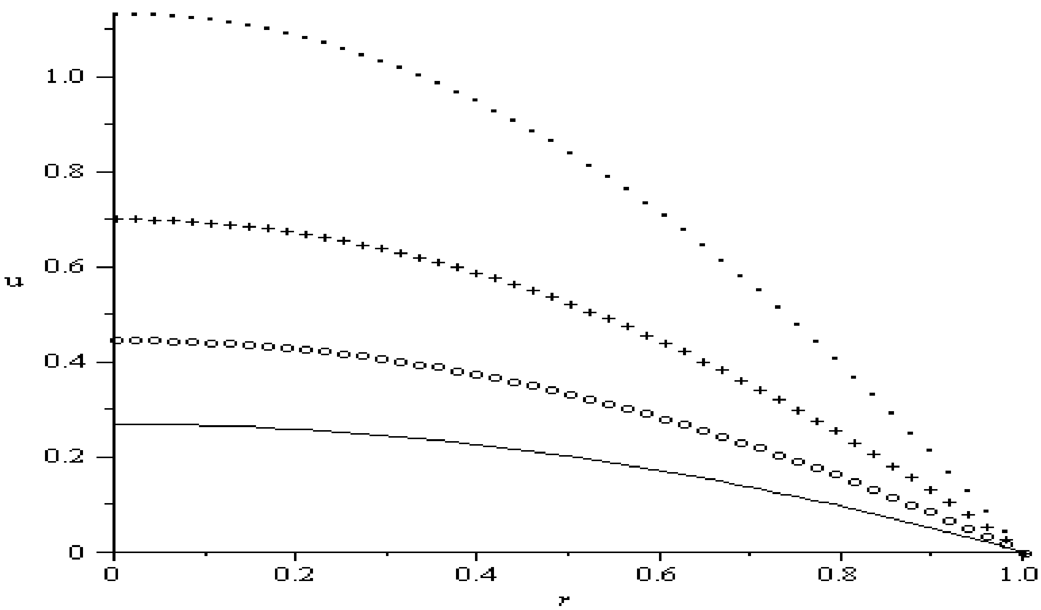

Figure 4. Velocity profile: *Br* = 1; α = 1; *Bi* = 1; ______*G* = 1; ooooo*G* = 1.5; ++++*G* = 2; ....*G* = 2.5.

pipe centreline and decreases gradually towards the wall due to convective heat exchange with the ambient at the wall. However, the fluid temperature increases with increasing values of α, *Br* and decreases with increasing values of Biot number *Bi*.

In Figures 9 to 11, the entropy generation rates in the transverse direction for various parametric values are illustrated. It is noteworthy that entropy generation rate is at the lowest in the region around the pipe centreline and increases quite rapidly near the wall with maximum value at the wall. The zero value of the entropy generation along the pipe centreline can be attributed to the axial – symmetric nature of the pipe flow with zero velocity and temperature gradients along the centreline (Equation 12). We observe that a decrease in the fluid viscosity coupled with an increase in viscous heating (*Br*) results into a further increase in the entropy generation rate at the wall while an increase in *Bi* due to convective cooling results

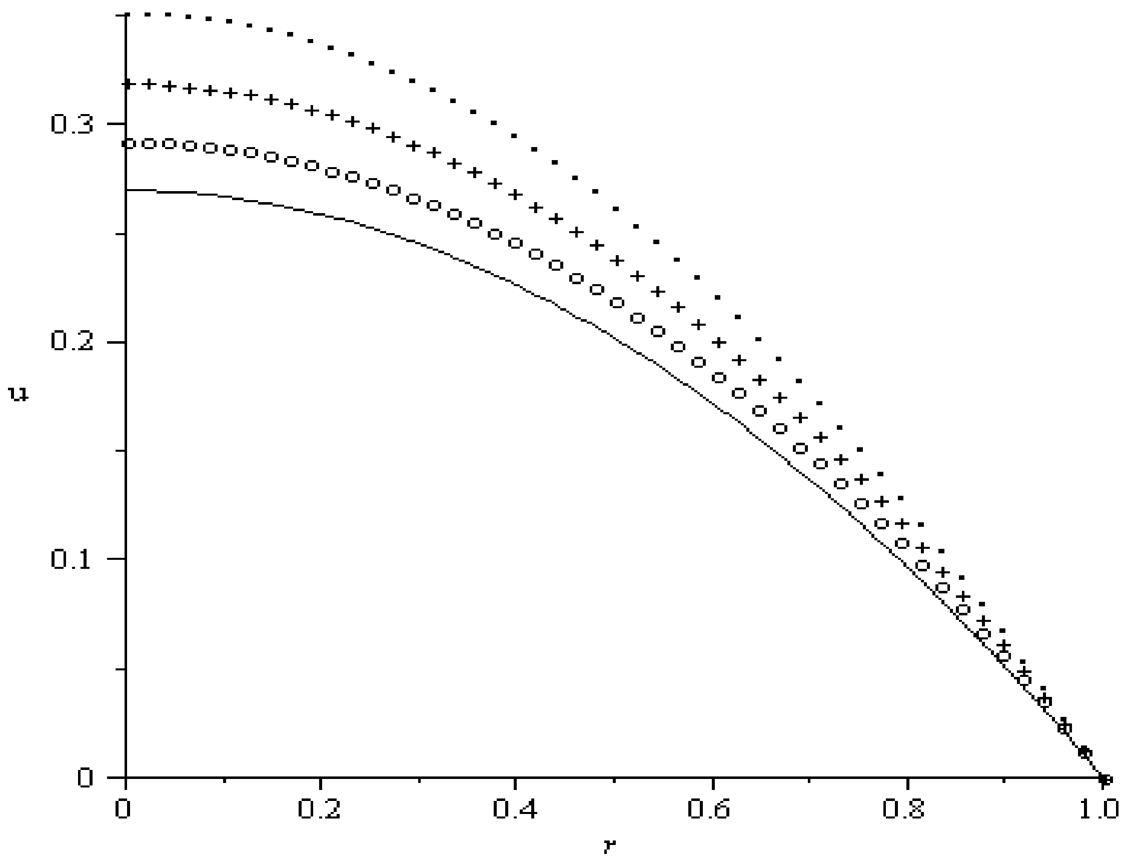

Figure 5. Velocity profile: G = 1; α = 1; Bi = 1; ______Br = 1; oooooBr = 2; ++++Br = 3; …….Br = 4.

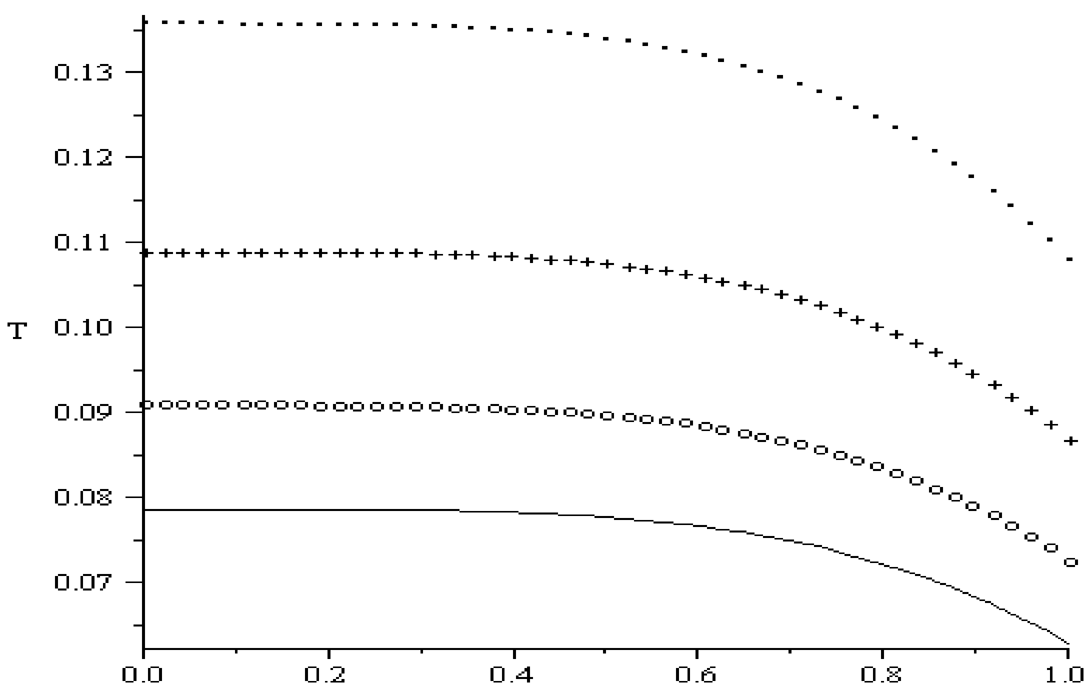

Figure 6. Temperature profile: G = 1; Br = 1;Bi = 1; _______α = 0.1; ooooo α = 2; ++++ α = 4; …….α = 6.

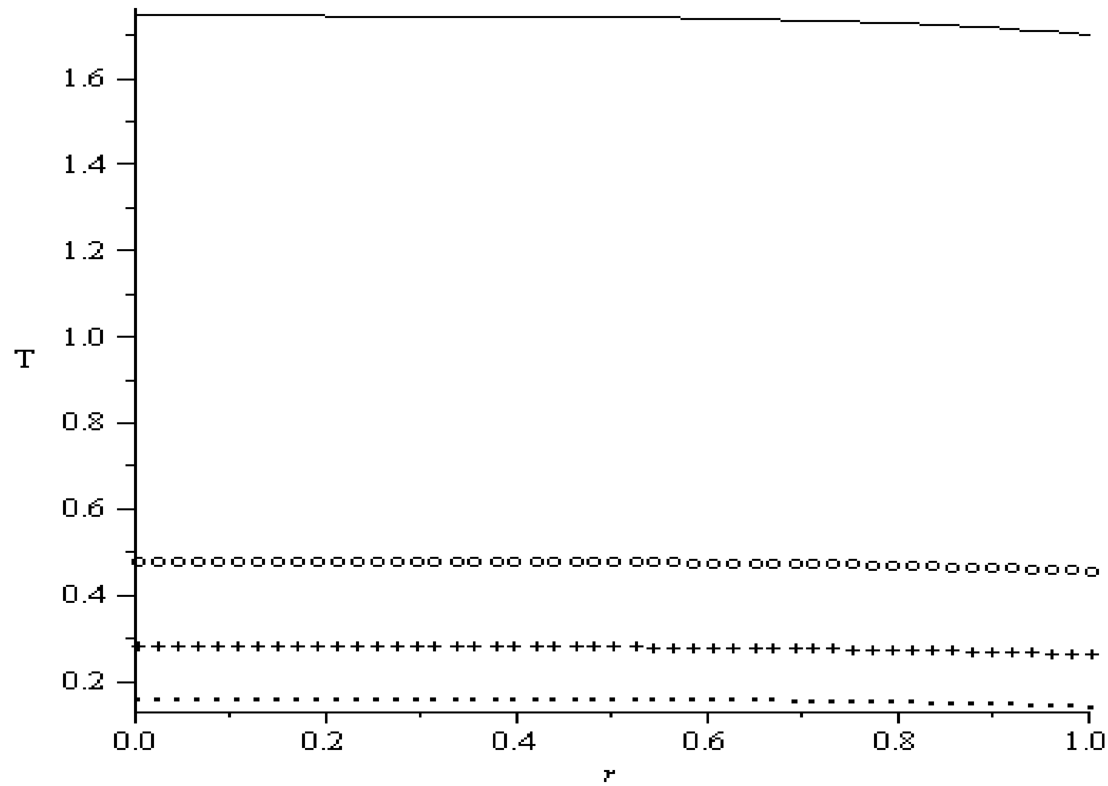

Figure 7. Temperature profile: *G*=1; α = 1; *Br* = 1; ______*Bi* = 0.1; ooooo*Bi* = 0.2; ++++ *Bi* = 0.3; …….*Bi* = 0.5

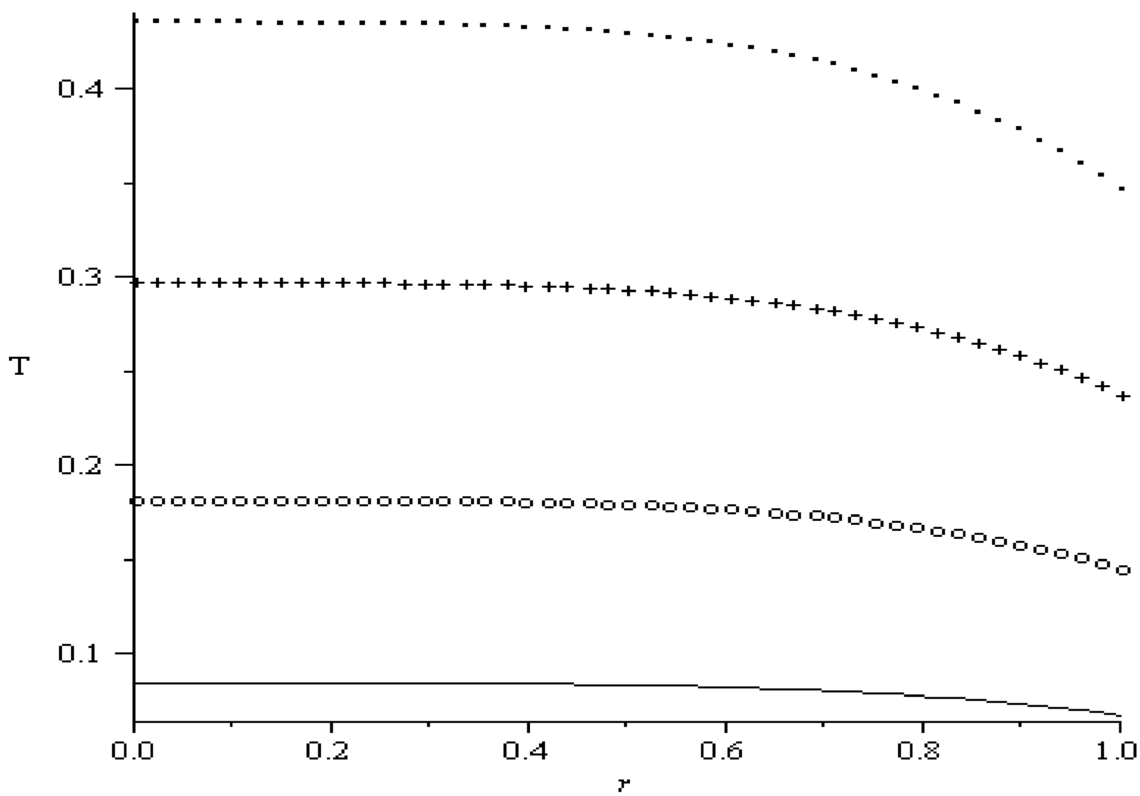

Figure 8. Temperature profile: *G*=1; α = 1; *Bi* = 1; ______*Br* = 1; ooooo*Br* = 2; ++++ *Br* = 3; …….*Br* = 4.

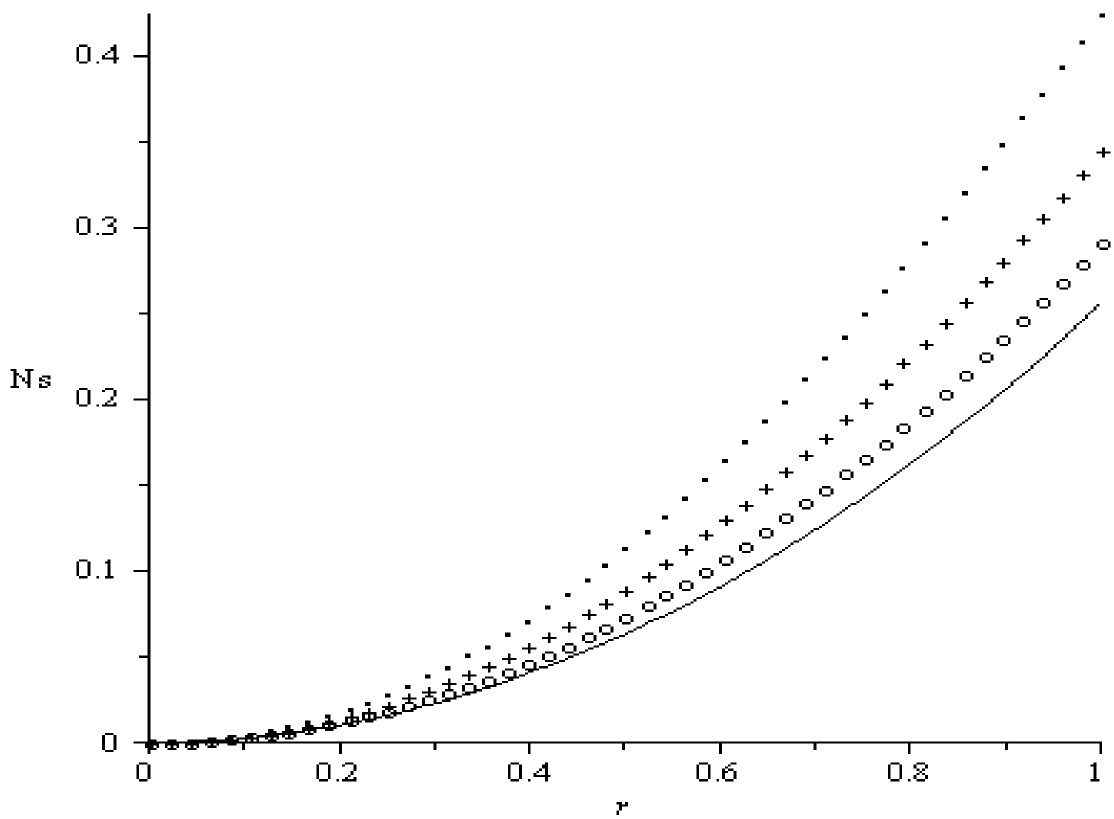

Figure 9. Entropy generation rate: G = 1; Br = 1;Bi = 1; _______α = 0.1; ooooo α = 2; ++++ α = 4; …….α = 6.

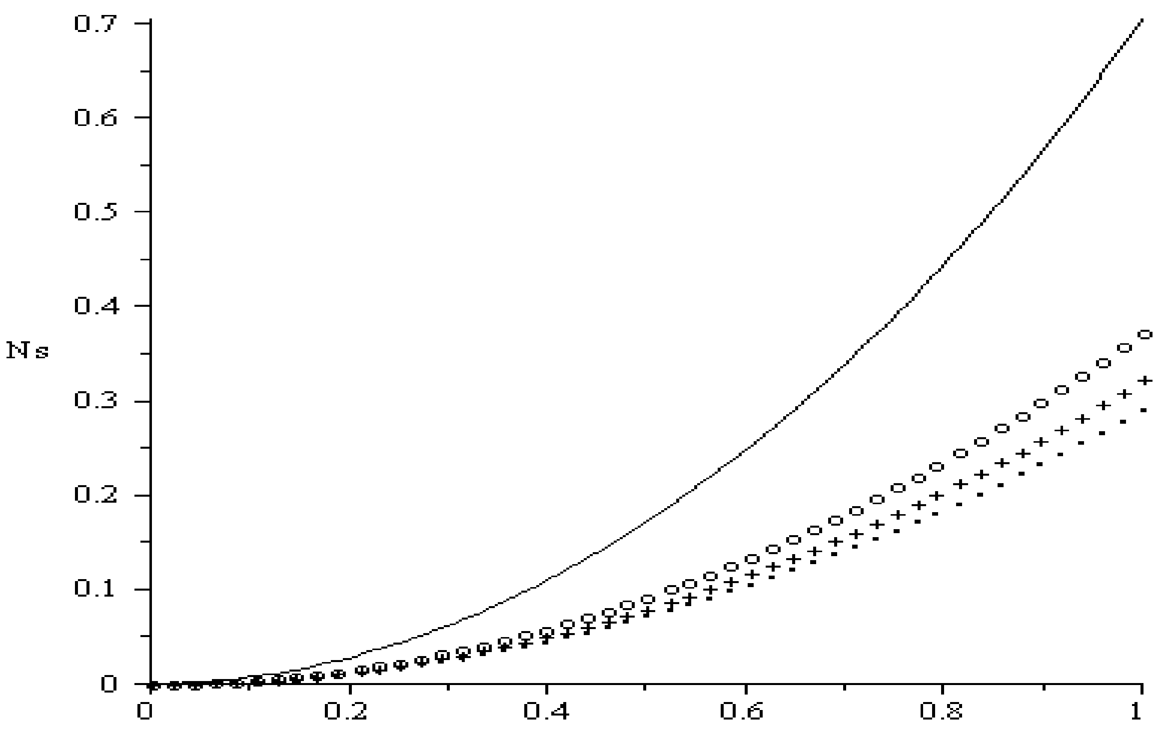

Figure 10. Entropy generation rate: G=1; α = 1; Br = 1; ______Bi = 0.1; oooooBi = 0.2; ++++ Bi = 0.3; …….Bi = 0.5

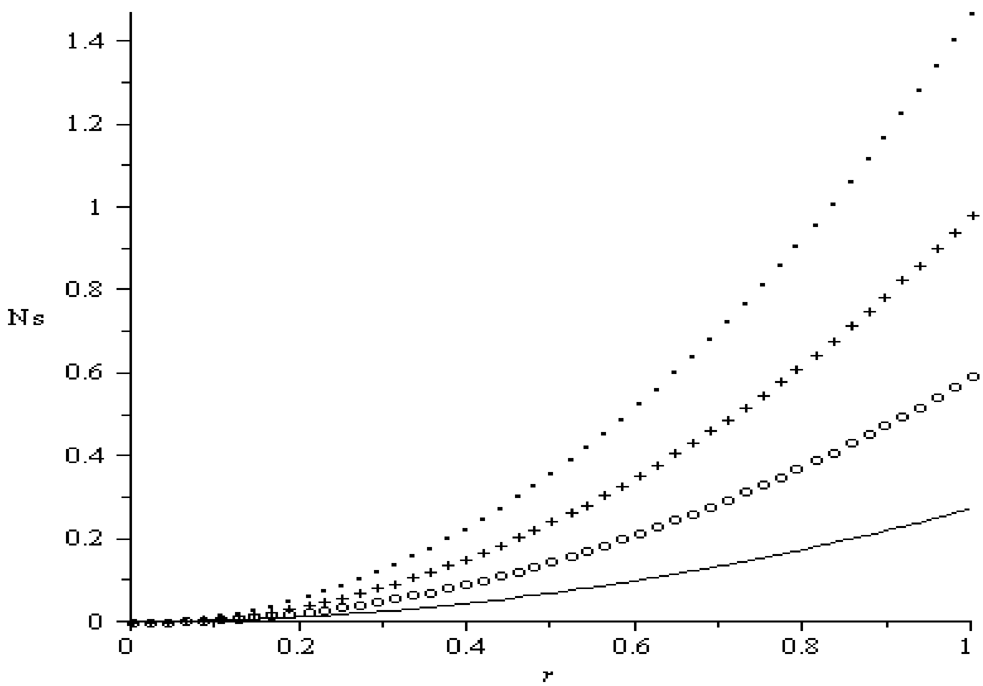

Figure 11. Entropy generation rate: *G*=1; α = 1; *Bi* = 1; ______*Br* = 1; ooooo*Br* = 2; ++++ *Br* = 3; …….*Br* = 4.

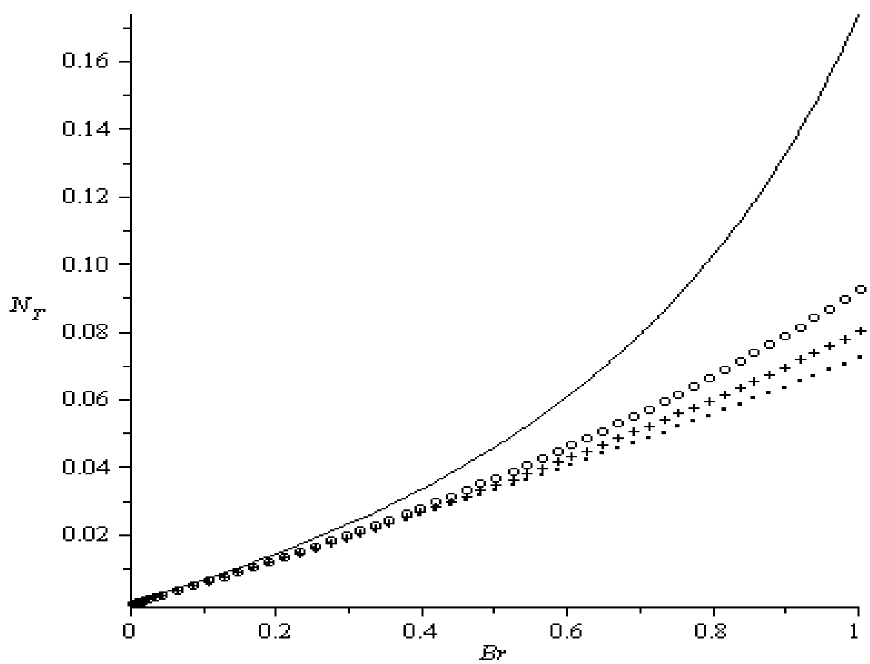

Figure 12. Total entropy generation: *G* = 1; α = 1; _______*Bi* = 0.1; ooooo *Bi* = 0.2; ++++ *Bi* = 0.3; …….*Bi* = 0.5.

into a decrease in entropy generation rate at the wall.

The total entropy generated within the flow field for various parametric values are depicted in Figures 12 to 14. It is interesting to note that the total entropy generation decreases with an increase in convective cooling and increases with a decrease in the fluid viscosity. Moreover, the total entropy generated within the flow field is also augmented by increasing the flow

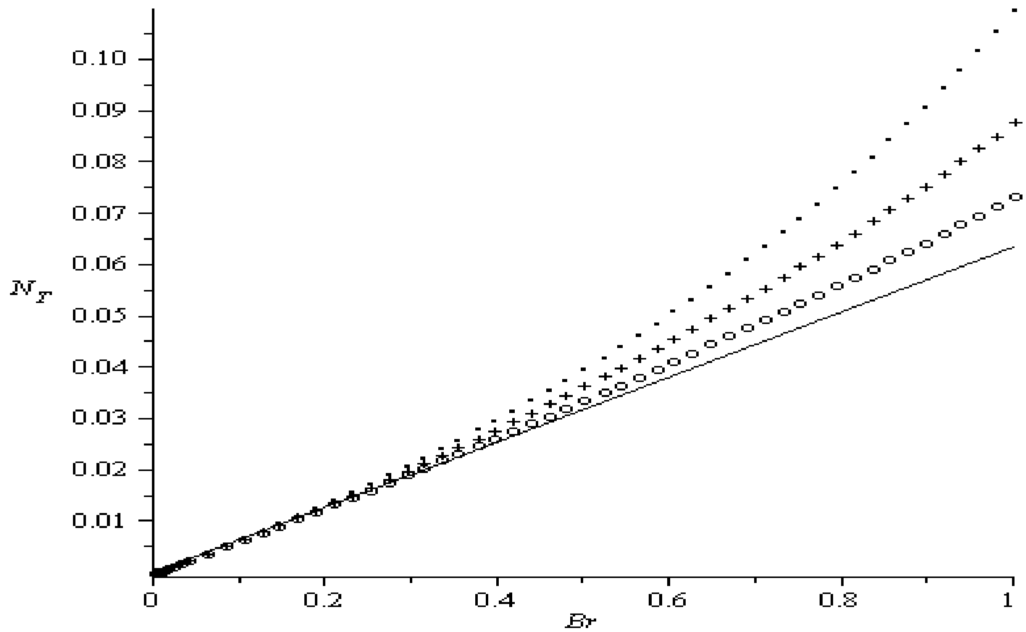

Figure 13. Total entropy generation: G =1; Bi = 1; ______α = 0.1; ooooo α = 2; ++++ α = 4; ....α = 6.

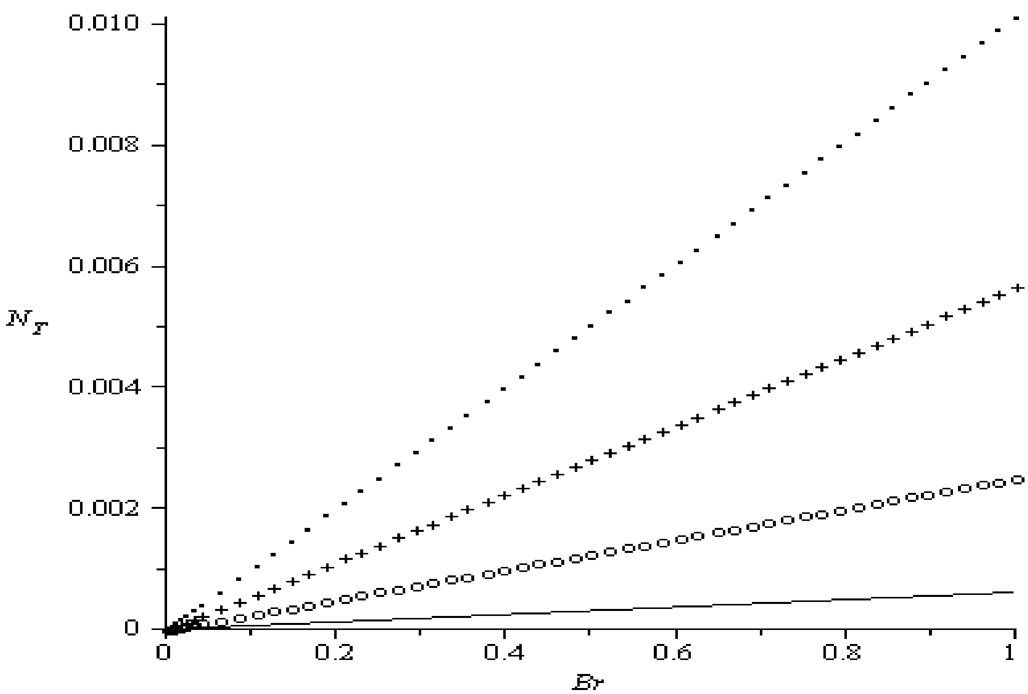

Figure 14. Total entropy generation: α =1; Bi = 1; ______G = 0.1; ooooo G = 0.2; ++++ G = 0.3; ....G = 0.4.

pressure gradient together with viscous heating.

In Figures 15 to 17, the Bejan (Be) number is illustrated for various parametric values. It is observed that the fluid friction irreversibility strongly dominates around the pipe centreline region while near the wall, the effect of fluid friction irreversibility decreases and the heat transfer irreversibility takes over. However, the effect of heat transfer irreversibility near the wall further increases with increasing values of α and Br but decreases with increasing effect of convective cooling at the pipe

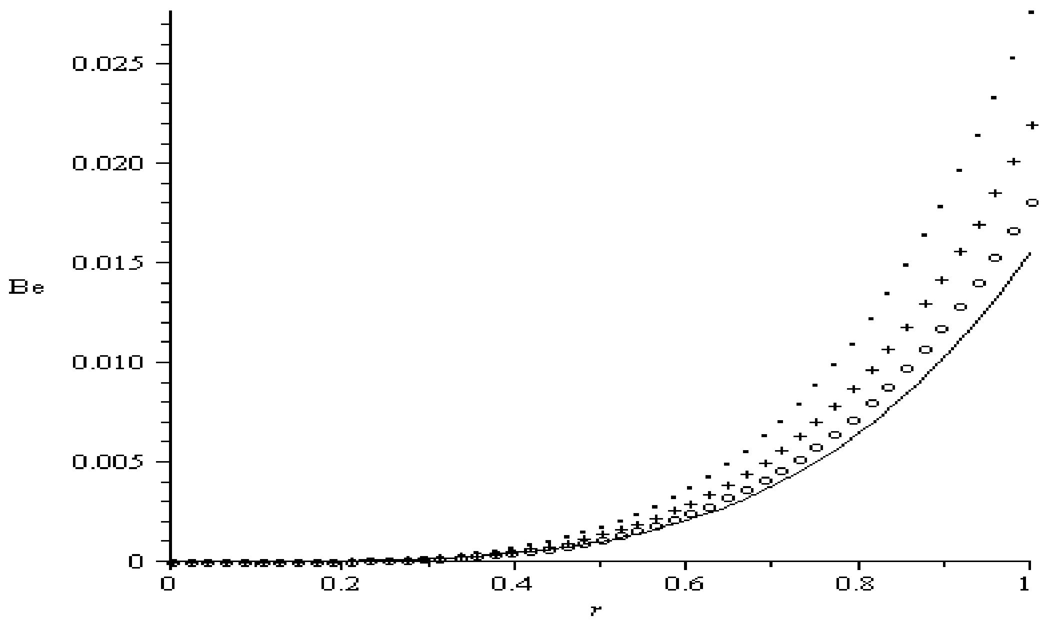

Figure 15. Bejan number: *G* = 1; *Br* = 1;*Bi* = 1; ______α = 0.1; ooooo α = 2; ++++ α = 4; ....α = 6.

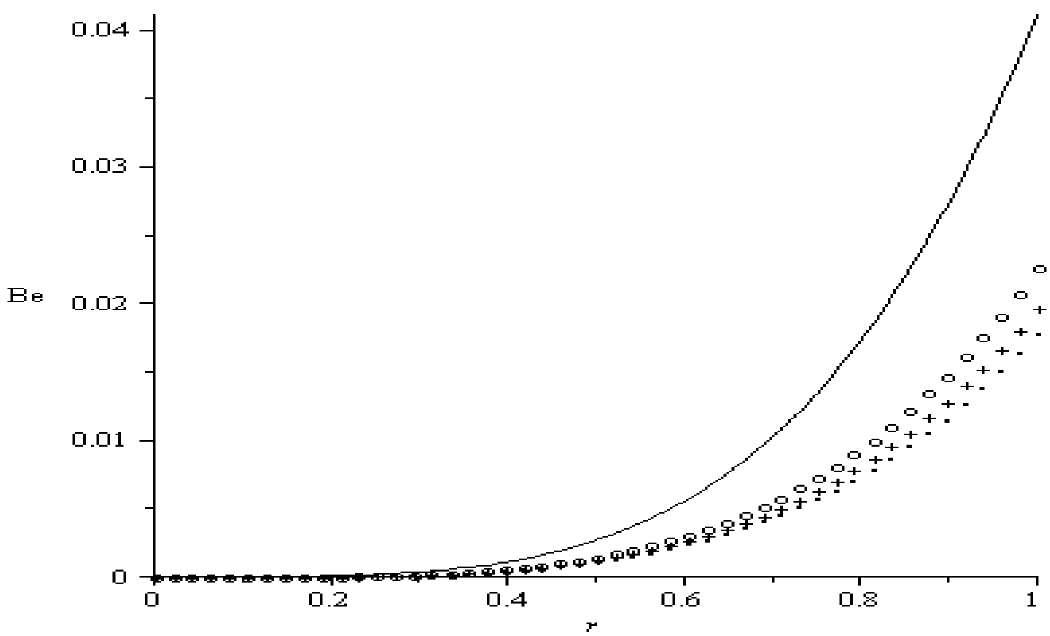

Figure 16. Bejan number: *G*=1; α = 1; *Br* = 1; _____*Bi* = 0.1; ooooo*Bi* = 0.2; ++++ *Bi* = 0.3; ....*Bi* = 0.5.

surface.

Conclusions

In this paper, the heat transfer and entropy generation rate in a temperature dependent viscosity fluid flowing steadily in a cylindrical pipe with convective cooling at the wall was investigated. The velocity and temperature profiles were obtained and used to evaluate the thermal stability criterion and the entropy generation number. Both the fluid velocity and temperature increases with increasing values of α, *G*, *Br* and decreases with increasing value of *Bi.* For all parametric values, the fluid friction irreversibility strongly dominates around the pipe centreline while the effect of heat transfer irreversibility

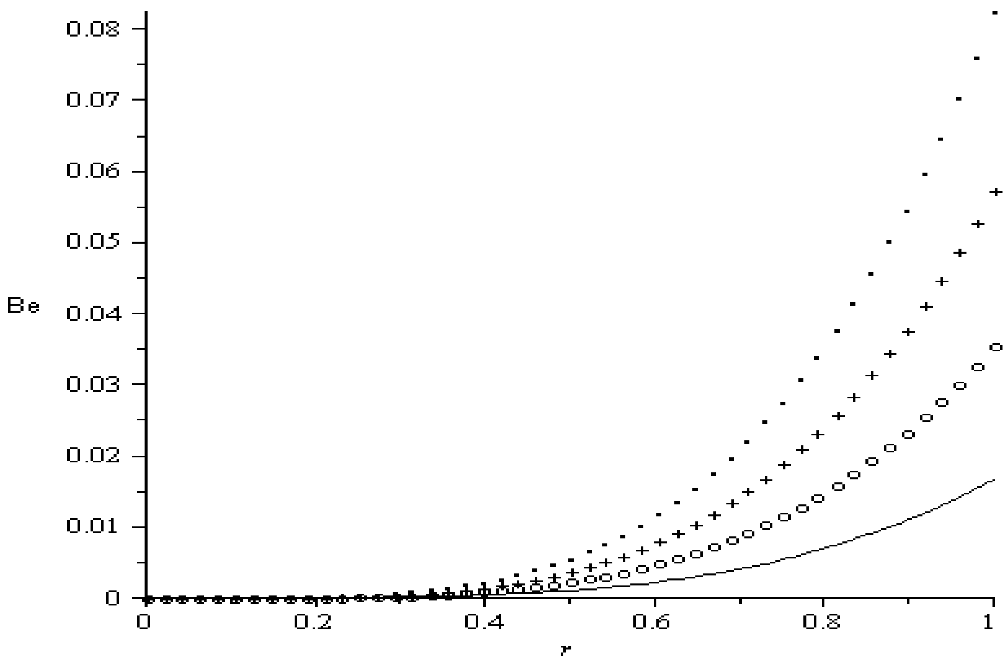

Figure 17. Bejan number: *G*=1; α = 1; *Bi* = 1; _____*Br* = 1; ooooo*Br* = 2; ++++ *Br* = 3; …….*Br* = 4.

increases transversely towards the wall. A decrease in the fluid viscosity and an increase in viscous heating will enhance total entropy generation in the flow field.

REFERENCES

Bejan A (1995). Convective heat transfer, second ed. Wiley. New York.

Bejan A (1996). Entropy generation minimization. CRC Press, New York.

Costa A, Macedonio G (2003). Viscous heating in fluids with temperature dependent viscosity: Implication for magma flows. Non-linear Processing Geophys, 10: 545-555.

Elbashbeshy EMA, Bazid MAA (2000). The effect of temperature dependent viscosity on heat transfer over a continuous moving surface. J. Appl. Phy., 33: 2716-2721.

Ibanez G, Cuevas S, Lopez de Haro M (2003). Minimization of entropy generation by asymmetric convective cooling. Int. J. Heat Mass Transfer., 46: 1321-1328.

Macosko CW (1994). Rheology, Principles, Measurements, and applications. VCH Publishers, Inc.

Makinde OD (2006). Irreversibility analysis for gravity driven non-Newtonian liquid film along an inclined isothermal plate. Physica Scripta., 74: 642-645.

Makinde OD (2007). On steady flow of a reactive variable viscosity fluid in a cylindrical pipe with isothermal wall. Int. J. Num. Methods Heat Fluid Flow, 17(2): 187-194.

Makinde OD (2008). Entropy-generation analysis for variable-viscosity channel flow with non-uniform wall temperature. Appl. Energy, 85: 384-393.

Makinde OD (2008). Irreversibility analysis of variable viscosity channel flow with convective cooling at the walls. Canadian J. Physics, 86(2): 383-389.

Makinde OD, Maserumule RL (2008). Thermal criticality and entropy analysis for variable viscosity Couette flow. Physica Scripta, 6(78): 015402.

Narusawa U (2001). The second law analysis of mixed convection in rectangular ducts. Heat Mass Transfer, 37: 197-203.

Reddy RS, Gorla L, Byrd W, Pratt DM (2007). Second law analysis for microscale flow and heat transfer. Appl. Thermal Eng. 27: 1414-1423.

Sahin AZ (1999). Effect of variable viscosity on the entropy generation and pumping power in a laminar fluid flow through a duct subjected to constant heat flux. Heat Mass Transfer, 35: 499-506.

Schlichting H (2000). Boundary layer theory, Springer-Verlag, New York.

Squire W (1967). A mathematical analysis of self-ignition. Applications of Undergraduate Mathematics in Eng. ed. Noble, B. New York: MacMillan.

Tasnim SH, Mahmud S (2002). Entropy generation in a vertical concentric channel with temperature dependent viscosity. Int. Comm. Heat Mass Transfer, 29(7): 907-918.

Taufiq BN, Masjuki HH, Mahlia TMI, Saidur R, Faizul MS, Niza Mohamad E (2007). Second law analysis for optimal thermal design of radial fin geometry by convection. Appl. Therm. Eng., 27: 1363-1370.