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Optimal homotopy asymptotic approach to nonlinear oscillators with discontinuities

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In this paper a version of the optimal homotopy asymptotic method (OHAM) has been proposed and applied to nonlinear oscillators with discontinuities. The main objective was to obtain highly accurate analytical solutions for a particular antisymmetric constant force oscillator using this new procedure. The proposed procedure proved to be very effective and accurate and did not require linearization or small parameters in the equation. It was found that OHAM works very well for whole range of initial amplitudes since an excellent agreement of the approximate frequencies and periodic solutions with the numerical ones has been demonstrated.

Key words: Nonlinear oscillators with discontinuities, minimal sensitivity, optimal homotopy asymptotic method.

INTRODUCTION

In nonlinear science, there appears an ever-increasing interest of scientists and engineers in employing analytical asymptotic techniques for solving nonlinear problems. The most common and widely studied methods for determining analytical approximate solutions of nonlinear oscillatory systems are the perturbation methods. These methods involve the expansion of the solution of an oscillation equation in a series of a small parameter. They include, among others, the Lindstedt-Poincare method (Nayfeh and Mook, 1979; Hagedorn 1988; Mickens, 1996), the Krylov-Bogoliubov-Mitropolski method (Bogoliubov and Mitropolsky, 1963), the multi-time expansion approximate solutions (Nayfeh and Mook, 1979; Hagedorn, 1988; Mickens, 1996), some iteration procedures (Mickens, 1987; Lim and Wu, 2002), the harmonic balance method (Nayfeh and Mook, 1979; Mickens, 1996) etc. In general, the obtained approximate solutions are valid for small values of oscillation amplitude. The so-called small parameter assumption greatly restricts applications of some perturbation techniques and as it is well known, an overwhelming majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all.

There exist some alternative analytical asymptotic approaches, such as some piecewise-linearized methods (Ramos, 2006), the Adomian decomposition method (Abassy, 2010), some modified Lindstedt-Poincare methods (He, 2002; Pakdemirli et al., 2009), some iterative methods (Oziş and Yildirim, 2009; Marinca and Herişanu, 2011), the homotopy analysis method (Liao, 2003), the homotopy perturbation method (He, 2000).

In this paper, a version of an analytical approximate technique, called optimal homotopy asymptotic method (OHAM) is employed to propose an approach to solve nonlinear oscillations. Different from some perturbation methods, the applicability and validity of the OHAM is independent on whether or not there exists a small parameter in the considered nonlinear equations.

In order prove the capabilities of this technique we consider the following antisymmetric constant force oscillators with discontinuities:

$$\ddot{u} + \mathrm{sign}(u) = 0 \qquad (1)$$

with the initial conditions:

$$u(0) = A \;\; , \;\; \dot{u}(0) = 0 \qquad (2)$$

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The function $\mathrm{sign}(u)$ is defined as

$$\mathrm{sign}(u)=\begin{cases}1 & \text{if } u>0\\ -1 & \text{if } u\le 0\end{cases} \tag{3}$$

There exists no small parameter in the equation and therefore the traditional perturbation methods cannot be applied directly. For such problems concerning nonlinear oscillator with discontinuities, the homotopy perturbation method is employed in (Belendez et al., 2008). A modified Lindstedt-Poincare method is used in Liu (2005), an analytical approximate technique which incorporates salient features of both Newton's method and harmonic balance method is applied in Wu et al. (2000), the variational iteration method is employed in Rafei et al. (2007), the parameter-expansion method is successfully used in Zengin et al. (2008).

The procedure proposed in this paper proves to be very effective, simple, and accurate, providing a convenient way to optimally control the convergence of approximate solutions. Finally, this work demonstrates the general validity and great potential of the OHAM.

MATERIALS AND METHODS

In what follows we shortly present the basics of the Optimal Homotopy Asymptotic Method. We consider a nonlinear ODE of the form:

$$\ddot{u}(t)+f(t,u(t))=0 \tag{4}$$

where the dot denotes the derivative with respect to time and f is in general a nonlinear term. Initial conditions are:

$$u(0)=A \quad , \quad \dot{u}(0)=0 \tag{5}$$

The Equation (4) describes a system oscillating with an unknown period T. We switch to a scalar time $\tau=2\pi t/T=\omega t$. Under the transformation:

$$\tau=\omega t \tag{6}$$

the original Equation (4) becomes

$$\omega^2 u''(\tau)+f(\tau,u(\tau))=0 \tag{7}$$

where the prime denotes the derivative with respect to τ.

By the homotopy technique we construct a homotopy in a more general form:

$$\begin{aligned}&H(\phi(\tau,p),\Omega(\lambda,p))=(1-p)L(\phi(\tau,p))-\\&-h(\tau,p)N[\phi(\tau,p),\Omega(\lambda,p)]=0\end{aligned} \tag{8}$$

where L is a linear operator [ω_0 is given by Equation (12)] :

$$L(\phi(\tau,p))=\omega_0^2\left[\frac{\partial^2\phi(\tau,p)}{\partial\tau^2}+\phi(\tau,p)\right] \tag{9}$$

while N is a nonlinear operator:

$$\begin{aligned}&N[\phi(\tau,p),\Omega(\lambda,p)]=\Omega^2(\lambda,p)\frac{\partial^2\phi(\tau,p)}{\partial\tau^2}+\\&+\lambda\phi(\tau,p)+f(\tau,\phi(\tau,p))-p\lambda\phi(\tau,p)\end{aligned} \tag{10}$$

where $p\in[0,1]$ is the embedding parameter, h(τ,p) is an auxiliary function so that h(τ,0)=0, h(τ,p)$\neq$ 0 for p$\neq$ 0 and λ is an arbitrary parameter. From Equation (5) we obtain the initial conditions:

$$\phi(0,p)=A \quad , \quad \left.\frac{\partial\phi(\tau,p)}{\partial\tau}\right|_{\tau=0}=0 \tag{11}$$

Obviously when p=0 and p=1 it holds:

$$\phi(\tau,0)=u_0(\tau) \ , \ \phi(\tau,1)=u(\tau) \ , \ \Omega(\lambda,0)=\omega_0 \ , \ \Omega(\lambda,1)=\omega \tag{12}$$

where $u_0(\tau)$ is an initial approximation of u(τ). Therefore, as the embedding parameter p increases from 0 to 1, ϕ (τ,p) varies from the initial approximation $u_0(\tau)$ to the solution u(τ), so does Ω(λ,p) from the initial approximation ω_0 to the exact frequency ω.

Expanding ϕ (τ,p) and Ω(λ,p) in series with respect to the parameter p, one has respectively:

$$\phi(\tau,p)=u_0(\tau)+pu_1(\tau)+p^2u_2(\tau)+\ldots.. \tag{13}$$

$$\Omega(\lambda,p)=\omega_0+p\omega_1+p^2\omega_2+\ldots... \tag{14}$$

If the initial approximation $u_0(\tau)$ and the auxiliary function h(τ,p) are properly chosen so that the above series converges at p=1, one has:

$$u(\tau)=u_0(\tau)+u_1(\tau)+u_2(\tau)+\ldots. \tag{15}$$

$$\omega=\omega_0+\omega_1+\omega_2+\ldots. \tag{16}$$

We propose the auxiliary function h(τ,p) of the form:

$$h(\tau,p)=pC_1+p^2C_2+\ldots+p^mC_m(\tau)+\ldots. \tag{17}$$

where C_i could be parameters or functions. We emphasize that it is very important to properly choose this function because the convergence of the solution greatly depends on that.

The results of the m th-order approximations are given by:

$$\tilde{u}(\tau)\approx u_0(\tau)+u_1(\tau)+\ldots.+u_m(\tau) \tag{18}$$

$$\tilde{\omega}\approx\omega_0+\omega_1+\ldots.+\omega_m \tag{19}$$

Substituting Equations (13) and (14) into Equation (10) yields:

$$\begin{aligned}&N(\phi,\Omega)=N_0(u_0,\omega_0,\lambda)+pN_1(u_0,u_1,\omega_0,\omega_1,\lambda)+\\&+p^2N_2(u_0,u_1,u_2,\omega_0,\omega_1,\lambda)+\ldots.\end{aligned} \tag{20}$$

If we substitute Equations (20) and (17) into Equation (8) and equate the coefficients of various powers of p, we obtain the following linear equations:

$$L(u_0)=0 \;,\; u_0(0)=A \;,\; u'(0)=0 \tag{21}$$

$$L(u_i)-L(u_{i-1})- -\sum_{j=1}^{i} C_j N_{i-j}(u_0,u_1,\dots,u_{i-j},\omega_0,\omega_1,\dots,\omega_{i-j},\lambda)=0 \tag{22}$$
$$u_i(0)=u_i'(0)=0 \;,\; i=1,2,\dots,m-1$$

$$L(u_m)-L(u_{m-1})-\sum_{j=1}^{m-1} C_j N_{m-j}-C_m(\tau)N_0=0 \;, \tag{23}$$
$$u_m(0)=u_m'(0)=0$$

Note that $\omega_0, \omega_1, \dots \omega_m$ can be determined avoiding the presence of secular terms in the left-hand sides of Equations (22) and (23).

The frequency ω depends upon the arbitrary parameter λ and we can apply the so-called "principle of minimal sensitivity" (Amore and Aranda, 2005) in order to fix the value of λ. We do this imposing that:

$$\frac{d\omega}{d\lambda}=0 \tag{24}$$

We mention that when only one iteration is used, then $\lambda=0$.

At this moment, the m th-order approximation given by Eq.(18) depends on C_1, $C_2,\dots,C_{m-1},C_m(\tau)$. These convergence-control constants C_1, C_2, … C_{m-1} and those constants that appear in the expression of $C_m(\tau)$, can be identified via various methods, for example: the collocation method, the Galerkin method, the least squares method etc.

If $R(\tau,C_1,C_2,\dots,C_q)$ is the residual obtained substituting the m th-order approximation (18) into Equation (7), that is:

$$R(\tau,C_1,C_2,..C_q)=\tilde{\omega}^2\tilde{u}''(\tau)+f(\tau,\tilde{u}(\tau)) \tag{25}$$

and if the functional J is given by the integral:

$$J(C_1,C_2,\dots,C_q)=\int_a^b R^2(\tau,C_1,C_2,\dots,C_q)d\tau \tag{26}$$

where a and b are two values from the domain of the Equation (4), then the coefficients $C_1,C_2,\dots,C_q$ can be determined from the following system of equations:

$$\frac{\partial J}{\partial C_1}=\frac{\partial J}{\partial C_2}=\dots=\frac{\partial J}{\partial C_q}=0 \tag{27}$$

where q is the total number of constants.

Alternatively, another efficient approach to obtain the optimal values of the convergence-control constants C_q is given by solving the system:

$$R(\tau_1,C_i)=R(\tau_2,C_i)=\dots=R(\tau_q,C_i)=0 \tag{27'}$$

where the residual R is given by Equation (25).

We remark that OHAM contains the auxiliary function $h(\tau,p)$, which provides us with a simple way to adjust and optimally control the convergence of the solutions. Note that instead of an infinite series, the OHAM searches for only a few terms (mostly three terms). A similar procedure has been successfully applied by other authors to solve different nonlinear problems (Idrees et al., 2010; Iqbal et al., 2010; Iqbal and Javed, 2011; Babaelahi et al., 2010).

The proposed method will be applied in what follows to the antisymetric constant force oscillator. In this respect, the nonlinear operator (10) corresponding to Equation (1) is given by the equation

$$N(\phi(\tau,p),\Omega(\lambda,p))=\Omega^2(\lambda,p)\phi''(\tau,p)+ +\mathrm{sign}\phi(\tau,p)+\lambda\phi(\tau,p)-p\lambda\phi(\tau,p) \tag{28}$$

Equation (21) can be written as:

$$\omega_0^2(u_0''+u_0)=0 \;,\; u_0(0)=A \;,\; u_0'(0)=0 \tag{29}$$

and has the solution

$$u_0(\tau)=A\cos\tau \tag{30}$$

In our example, $f(u)=\mathrm{sign}(u)$, where u is given by Equation (15), such as:

$$f(u)=f(u_0)+f'(u_0)(pu_1+p^2u_2+\dots)+ +\frac{1}{2}f''(u_0)(pu_1+p^2u_2+\dots)^2+\dots \tag{31}$$

But $f'(u_0)=f''(u_0)=\dots=0$ and therefore we obtain:

$$\mathrm{sign}(u)=\mathrm{sign}(u_0)=\mathrm{sign}(A\cos\tau) \tag{32}$$

The first term in Equation (20) is given by

$$N_0(u_0,\omega_0,\lambda)=\omega_0^2u_0''+\mathrm{sign}(u_0)+\lambda u_0 \tag{33}$$

For i=1 into Equation (22) we obtain the equation in u_1:

$$\omega_0^2(u_1''+u_1)-\omega_0^2(u_0''+u_0)- -C_1\left[\omega_0^2u_0''+\mathrm{sign}(u_0)+\lambda u_0\right]=0 \;,\; u_1(0)=u_1'(0)=0 \tag{34}$$

Substituting Equation (30) into Equation (34) and using the identity

$$\mathrm{sign}(A\cos\tau)=\frac{4}{\pi}\left(\cos\tau-\frac{1}{3}\cos 3\tau+\frac{1}{5}\cos 5\tau-\frac{1}{7}\cos 7\tau+\dots\right) \tag{35}$$

we obtain the following equation:

$$\omega_0^2(u_1''+u_1)=C_1\left[\left(\frac{4}{\pi}+\lambda A-\omega_0^2A\right)\cos\tau+\frac{4}{\pi}\left(-\frac{1}{3}\cos 3\tau+ +\frac{1}{5}\cos 5\tau-\frac{1}{7}\cos 7\tau+\frac{1}{9}\cos 9\tau-\frac{1}{11}\cos 11\tau+\dots\right)\right] \tag{36}$$

Avoiding the presence of a secular term needs:

$$\omega_0^2 = \frac{4}{\pi A} + \lambda \tag{37}$$

With this requirement, the solution of Equation (36) becomes:

$$u_1(\tau) = \frac{C_1}{6\pi\omega_0^2}\Big[(\cos 3\tau - \cos\tau) - \frac{1}{5}(\cos 5\tau - \cos\tau) + \frac{1}{14}(\cos 7\tau - \cos\tau) - \frac{1}{30}(\cos 9\tau - \cos\tau) + \frac{1}{55}(\cos 11\tau - \cos\tau) + \ldots\Big] \tag{38}$$

If we use only one iteration, the first-order approximate solution will be

$$\tilde{u}_1(\tau) = u_0(\tau) + u_1(\tau) \tag{39}$$

From Equations (30), (37) and (38), the first-order approximate solution of Equation (1) becomes

$$\tilde{u}_1(\tau) = A\cos\omega_0 t + \frac{C_1}{6\pi\omega_0^2}\Big[(\cos 3\tau - \cos\tau) - \frac{1}{5}(\cos 5\tau - \cos\tau) + \frac{1}{14}(\cos 7\tau - \cos\tau) - \frac{1}{30}(\cos 9\tau - \cos\tau) + \frac{1}{55}(\cos 11\tau - \cos\tau) + \ldots\Big] \tag{40}$$

where ω_0 is obtained from Equation (37) for λ=0:

$$\omega_0^2 = \frac{4}{\pi A} \tag{41}$$

For m=2 into Equation (23) and choosing

$$C_m(\tau) = C_2(\tau) = C_2 + C_3\cos 2\tau + C_4\cos 4\tau \tag{42}$$

the equation in u_2 has the form:

$$\omega_2^2(u_2'' + u_2) - \omega_0^2(u_1'' + u_1) - C_1(2\omega_0\omega_1 u_0'' + \omega_0^2 u_1'' + \lambda u_1 - \lambda u_0) - (C_2 + C_3\cos 2\tau + C_4\cos 4\tau)(\omega_0^2 u_0'' + \operatorname{sign}(u_0) + \lambda u_0) \tag{43}$$

where the term $2\omega_0\omega_1 u_0'' + \omega_0^2 u_1'' + \lambda u_1 - \lambda u_0$ was obtained from Equation (20) and is $N_1(u_0, u_1, \omega_0, \omega_1, \lambda)$.

Substituting Equations (30), (36) and (38) into Equation (43), the equation in u_2 becomes:

$$\begin{aligned}&\omega_0^2(u_2'' + u_2) - \Big[-2\omega_0\omega_1 A - \lambda A + \frac{C_1}{6\pi}\Big(1 - \frac{\lambda}{\omega_0^2}\Big)\Big(1 - \frac{1}{5} + \frac{1}{14} - \frac{1}{30} + \frac{1}{55}\Big) + \frac{4}{\pi}\Big(\frac{C_3}{6} + \frac{C_4}{15}\Big)\Big]\cos\tau - \Big[\Big(\frac{\lambda}{\omega_0^2} - 9\Big)\frac{C_1^2}{6\pi} - \\ &- \frac{4}{\pi}\Big(\frac{C_1 + C_2}{3} - \frac{C_3}{10} + \frac{C_4}{14}\Big)\Big]\cos 3\tau - \Big[\Big(25 - \frac{\lambda}{\omega_0^2}\Big)\frac{C_1^2}{30\pi} + \frac{4}{\pi}\Big(\frac{C_1 + C_2}{5} - \frac{5C_3}{21} + \frac{C_4}{18}\Big)\Big]\cos 5\tau - \\ &- \Big[\Big(\frac{\lambda}{\omega_0^2} - 49\Big)\frac{C_1^2}{84\pi} - \frac{4}{\pi}\Big(\frac{C_1 + C_2}{7} - \frac{7C_3}{45} + \frac{7C_4}{33}\Big)\Big]\cos 7\tau - \\ &- \Big[\Big(81 - \frac{\lambda}{\omega_0^2}\Big)\frac{C_1^2}{180\pi} + \frac{4}{\pi}\Big(\frac{C_1 + C_2}{9} - \frac{9C_3}{77} + \frac{C_4}{10}\Big)\Big]\cos 9\tau - \\ &- \Big[\Big(\frac{\lambda}{\omega_0^2} - 121\Big)\frac{C_1^2}{330\pi} - \frac{4}{\pi}\Big(\frac{C_1 + C_2}{11} - \frac{C_3}{18} + \frac{C_4}{14}\Big)\Big]\cos 11\tau = 0\end{aligned} \tag{44}$$

No secular term in $u_2(\tau)$ requires that:

$$\omega_1 = -\frac{\lambda}{2\omega_0} + \frac{989C_1}{13860\pi\omega_0 A}\Big(1 - \frac{\lambda}{\omega_0^2}\Big),\ C_4 = -\frac{5}{2}C_3 \tag{45}$$

From Equations (19) and (45) we obtain the frequency in the form:

$$\tilde{\omega} = \omega_0 - \frac{\lambda}{2\omega_0} + \frac{989C_1}{13860\pi\omega_0 A}\Big(1 - \frac{\lambda}{\omega_0^2}\Big) \tag{46}$$

where ω_0 is given by Equation (37).

The parameter λ can be determined applying the “principle of minimal sensitivity”. From Equation (24) we obtain:

$$\lambda = \frac{1}{\pi A}\Big(\sqrt{4 + \frac{1978C_1}{1155}} - 2\Big) \tag{47}$$

This result is substituted into Equation (46) and (37) and we have:

$$\tilde{\omega} = \frac{8 + 2\sqrt{4 + \frac{1978C_1}{1155}}}{3\sqrt{\pi A\Big(2 + \sqrt{4 + \frac{1978C_1}{1155}}\Big)}}\ ,\quad \omega_0^2 = \frac{1}{\pi A}\Big(2 + \sqrt{4 + \frac{1978C_1}{1155}}\Big) \tag{48}$$

Substituting Equations (30), (36), (38) and (45) into Equation (44) and solving Equation (44), we obtain:

$$u_2(\tau)=\left[\frac{C_1^2}{48\pi\omega_0^2}\left(9-\frac{\lambda}{\omega_0^2}\right)+\right.$$
$$+\left.\frac{1}{2\pi\omega_0^2}\left(\frac{C_1+C_2}{3}-\frac{39C_3}{140}\right)\right](\cos 3\tau-\cos\tau)+$$
$$+\left[\frac{C_1^2}{720\pi\omega_0^2}\left(\frac{\lambda}{\omega_0^2}-25\right)-\frac{1}{6\pi\omega_0^2}\left(\frac{C_1+C_2}{5}-\right.\right.$$
$$\left.\left.-\frac{95C_3}{252}\right)\right](\cos 5\tau-\cos\tau)+\left[\frac{C_1^2}{4032\pi\omega_0^2}\left(49-\frac{\lambda}{\omega_0^2}\right)+\right.$$
$$+\left.\frac{1}{12\pi\omega_0^2}\left(\frac{C_1+C_2}{7}-\frac{679C_3}{990}\right)\right](\cos 7\tau-\cos\tau)+$$
$$+\left[\frac{C_1^2}{14400\pi\omega_0^2}\left(\frac{\lambda}{\omega_0^2}-81\right)-\frac{1}{20\pi\omega_0^2}\left(\frac{C_1+C_2}{9}-\right.\right.$$
$$\left.\left.-\frac{113C_3}{308}\right)\right](\cos 9\tau-\cos\tau)+\left[\frac{C_1^2}{39600\pi\omega_0^2}\left(121-\frac{\lambda}{\omega_0^2}\right)+\right.$$
$$+\left.\frac{1}{30\pi\omega_0^2}\left(\frac{C_1+C_2}{11}-\frac{59C_3}{252}\right)\right](\cos 11\tau-\cos\tau) \tag{49}$$

The second order approximate solution is:

$$\tilde{u}_2(\tau)=u_0(\tau)+u_1(\tau)+u_2(\tau) \tag{50}$$

Using Equations (6), (30), (38) and (49), the second order approximate solution of Equation (1) becomes:

$$\tilde{u}_2(t)=A\cos\tilde{\omega}t+\left[\frac{C_1^2}{48\pi\omega_0^2}\left(9-\frac{\lambda}{\omega_0^2}\right)+\frac{1}{2\pi\omega_0^2}\left(\frac{2C_1+C_2}{3}-\right.\right.$$
$$\left.\left.-\frac{39C_3}{140}\right)\right](\cos 3\tilde{\omega}t-\cos\tilde{\omega}t)+\left[\frac{C_1^2}{720\pi\omega_0^2}\left(\frac{\lambda}{\omega_0^2}-25\right)-\right.$$
$$\left.-\frac{1}{6\pi\omega_0^2}\left(\frac{2C_1+C_2}{5}-\frac{95C_3}{252}\right)\right](\cos 5\tilde{\omega}t-\cos\tilde{\omega}t)+$$
$$+\left[\frac{C_1^2}{4032\pi\omega_0^2}\left(49-\frac{\lambda}{\omega_0^2}\right)+\frac{1}{12\pi\omega_0^2}\left(\frac{2C_1+C_2}{7}-\right.\right.$$
$$\left.\left.\frac{679C_3}{990}\right)\right](\cos 7\tilde{\omega}t-\cos\tilde{\omega}t)+\left[\frac{C_1^2}{14400\pi\omega_0^2}\left(\frac{\lambda}{\omega_0^2}-81\right)-\right.$$
$$\left.-\frac{1}{20\pi\omega_0^2}\left(\frac{2C_1+C_2}{9}-\frac{113C_3}{308}\right)\right](\cos 9\tilde{\omega}t-\cos\tilde{\omega}t)+$$
$$+\left[\frac{C_1^2}{39600\pi\omega_0^2}\left(121-\frac{\lambda}{\omega_0^2}\right)+\frac{1}{30\pi\omega_0^2}\left(\frac{2C_1+C_2}{11}-\right.\right.$$
$$\left.\left.-\frac{59C_3}{252}\right)\right](\cos 11\tilde{\omega}t-\cos\tilde{\omega}t) \tag{51}$$

where $\lambda, \tilde{\omega}$ and ω_0 are given by Equations (47) and (48).

Substituting Equations (40) and (51), respectively, into Equation (25) we obtain two corresponding residuals:

$$\begin{aligned}R_1(t,C_1^*)&=\ddot{\tilde{u}}_1+\mathrm{sign}(\tilde{u}_1),\\R_2(t,C_2,C_3C_4)&=\ddot{\tilde{u}}_2+\mathrm{sign}(\tilde{u}_2)\end{aligned} \tag{52}$$

RESULTS

Case 1

In the case of the first-order approximate solution (40), for A=1, following the proposed procedure we obtain

$$C_1^*=-1.3553$$

and the explicit first-order approximate solution becomes

$$\begin{aligned}\tilde{u}_1(t)&=1.04835\cos\frac{2}{\sqrt{\pi}}t-0.0564708\cos\frac{6}{\sqrt{\pi}}t+\\&+0.0112942\cos\frac{10}{\sqrt{\pi}}t-0.00403363\cos\frac{14}{\sqrt{\pi}}t+\\&+0.00188236\cos\frac{18}{\sqrt{\pi}}t-0.00102674\cos\frac{12}{\sqrt{\pi}}t\end{aligned} \tag{54}$$

For A=10, we obtain

$$C_1^*=-0.140281$$

In this case, the first-order approximate solution will be

$$\begin{aligned}\tilde{u}_1(t)&=10.5005\cos\frac{2}{\sqrt{10\pi}}t-0.584506\cos\frac{6}{\sqrt{10\pi}}t-\\&-0.0417504\cos\frac{10}{\sqrt{10\pi}}t+0.0194835\cos\frac{14}{\sqrt{10\pi}}t-\\&-0.0106274\cos\frac{18}{\sqrt{10\pi}}t+0.116901\cos\frac{12}{\sqrt{10\pi}}t\end{aligned} \tag{55}$$

Case 2

In the second case, we consider the second-order approximate solution (51).

In the case A=1, from Equation (27) we obtain:

$$C_1=-0.815614211, C_2=-0.351951443, C_3=-0.598628474$$

From Equations (47) and (48) we obtain

$$\lambda=-0.123043117\ ,\ \omega_0=1.07247211\ ,\ \tilde{\omega}=1.110714944$$

The second order approximate solution (51) becomes:

$$\tilde{u}_2(t) = 1.030088562\cos\tilde{\omega}t - 0.033468588\cos 3\tilde{\omega}t + \\ + 0.00146585\cos 5\tilde{\omega}t + 0.009978953\cos 7\tilde{\omega}t - \\ - 0.008820567\cos 9\tilde{\omega}t + 0.001398542\cos 11\tilde{\omega}t \quad (56)$$

In the case A=10, we obtain:

$$C_1 = -0.815614211\ ,\ C_2 = 5.287986428\ ,\ C_3 = 3.774015449$$

$$\lambda = -0.0123043117\ ,\ \omega_0 = 0.339145459\ ,\ \tilde{\omega} = 0.351238905$$

In this case, the second order approximate solution becomes:

$$\tilde{u}_2(t) = 10.30035768\cos\tilde{\omega}t - 0.401767921\cos 3\tilde{\omega}t + \\ + 0.1745208\cos 5\tilde{\omega}t - 0.103175516\cos 7\tilde{\omega}t + \\ + 0.026982622\cos 9\tilde{\omega}t - 0.00930889\cos 11\tilde{\omega}t \quad (57)$$

DISCUSSION

In order to emphasize the accuracy and effectiveness of the proposed method, a graphical analysis is performed in what follows.

A comparison of OHAM results with numerical integration results obtained by using a fourth-order Runhe-Kutta method is presented in Figures 1 to 4 for both first-order and second-order approximate solutions in two cases.

Figures 1 and 2 show the comparison between the present solutions and the numerical integration results for the first-order and second-order solutions, respectively, in the case A=1.

Figures 3 and 4 show the comparison between the present solutions and the numerical integration results for the first-order and second-order solutions, respectively, in the case A=10.

One can be seen from the above figures that the first-order approximate solutions obtained by OHAM show moderate accuracy, but the second-order solutions are very accurate, since they are nearly identical with the solutions given by the numerical method, which proves the effectiveness of the method. Additionally, we remark that the exact values of the frequencies are ω_{ex}=1.110720735 for the case A=1 and ω_{ex}=0.351240736 in case A=10, which means that very good approximations have been found also for these frequencies.

Conclusions

In this work, a new approach for finding solutions to some nonlinear oscillations is proposed. A version of the

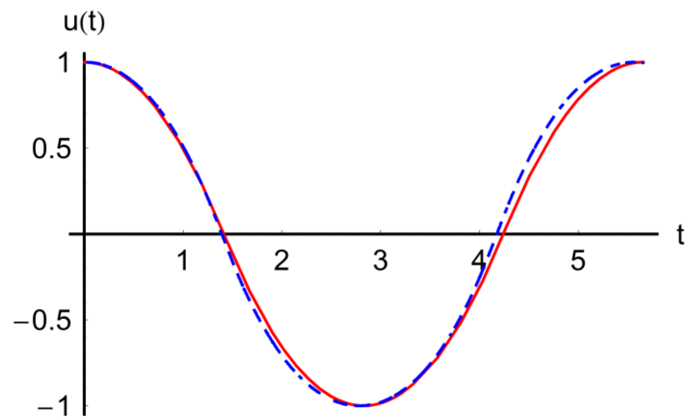

Figure 1. Comparison between the first-order approximate solution (54) and the numerical solution for A=1. ______ numerical solution; _ _ _ _ approximate solution.

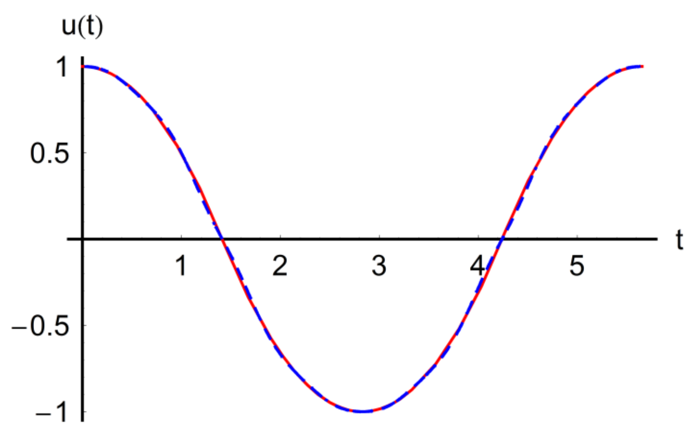

Figure 2. Comparison between the second-order approximate solution (56) and the numerical solution for A=1. ______ numerical solution; _ _ _ _ approximate solution.

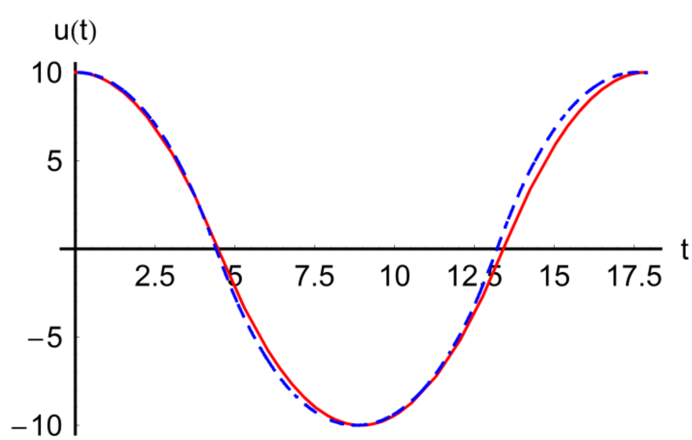

Figure 3. Comparison between the first-order approximate solution (55) and the numerical solution for A=10. ______ numerical solution; _ _ _ _ approximate solution.

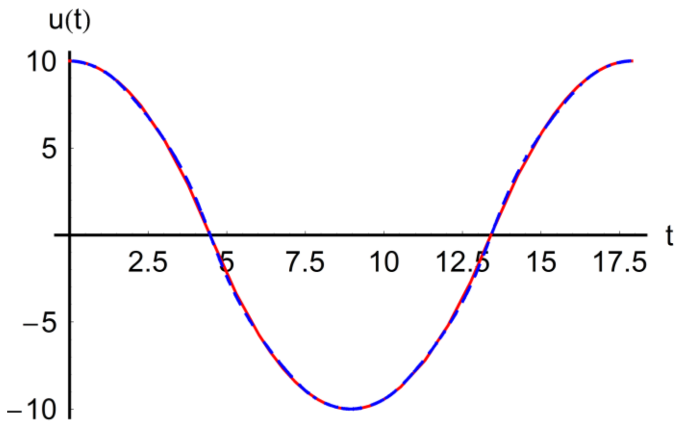

Figure 4. Comparison between the second-order approximate solution (57) and the numerical solution for A=10. ______ numerical solution; _ _ _ _ approximate solution.

optimal homotopy asymptotic method is employed to propose a new analytic approximate solution for some nonlinear oscillations with discontinuities. The validity of the procedure is illustrated on an antisymetric constant force oscillator. Our procedure is valid even if the considered nonlinear equation does not contain any small or large parameter. The arbitrary parameter λ is determined by applying the "principle of minimal sensitivity". The OHAM provides us with a simple way to optimally control and adjust the convergence of solutions and can give very good approximations in a few terms. The convergence of the approximate solution given by OHAM is determined by the auxiliary function h(τ,p). The error of the approximate solutions rapidly decreases if the number of iterations increases.

This version of the method proves to be very rapid, effective and accurate. We proved the accuracy of the results by comparing the solution obtained through the proposed method with the solution obtained via numerical integration using a fourth-order Runge-Kutta method. This work shows one step in the attempt to develop a new nonlinear analytical technique in the absence of small or large parameters.

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