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Full Length Research Paper

Variations of horizontal stiffness of laminated rubber bearings using new boundary conditions

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Base isolation is, in recent times, an accepted design philosophy as an earthquake resistant strategy for structural systems and sensitive instruments. Predicting the behavior of laminated rubber bearings, usually obtained from Haringx's theory, has been developed by many researchers. They have proposed a nonlinear, mechanical model for multilayer elastomeric bearings. However, in past theoretical and experimental studies, the effects of rotation in the bottom and top ends of bearings have been neglected. In this study, an analytical method is presented and formulated by considering the rotation of the top and bottom ends of multilayer rubber bearings, as new boundary conditions. According to these rotations, the horizontal stiffness of laminated rubber bearings, which is the one of the most important characteristics of bearings, will change. Comparisons of theoretical and experimental results show that the present analysis model has a good accuracy for analyzing laminated rubber bearings. Examples are presented to demonstrate the validity of the development method in predicting the horizontal stiffness of laminated elastomeric bearings with specified geometric parameters. The results of this study have shown that the horizontal stiffness of laminated rubber bearings will increase or decrease according different boundary conditions.

Key words: Seismic isolation, earthquake engineering, base isolation, elastomeric bearing, laminated rubber bearing, horizontal stiffness.

INTRODUCTION

The use of isolation equipment has been widely used and accepted as a technical approach for engineering applications during seismic and cycling loads. The most recent and interesting application of these bearings has been in the seismic isolation for buildings and bridge structures. Seismic isolation is an innovative technology for reducing the effects of earthquake ground motion by uncoupling the structure from horizontal components of the earthquake motion. Base isolation reduces not only the effects of earthquake acceleration transmitted to the structure, but it also protects the contents of the building, while simultaneously supporting the gravity weight of the structure (Kelly and Aiken, 1991; Koh and Kelly, 1988). The seismically isolation lengthens the fundamental period of the isolated building rather than same building, if conventionally founded and the dominant period has a strong ground motion (Kelly and Takhirov, 2007).

The laminated rubber bearing is one of the most useful devices for seismically isolation. They are composite elements consisting of thin layers of natural or synthetic rubber, bonded to steel plate. Therefore, the multilayer elastomeric bearing has very high compression stiffness, while retaining the characteristics of the low-shear stiffness of rubber (Chang, 2002; Ravari et al., 2011). The great advantage of elastomeric bearings is that they have no moving parts; they are not subject to corrosion and they are reliable, cheap to manufacture and need no maintenance (Simo and Kelly, 1984).

In base-isolated buildings, the multilayer elastomeric bearings, being as protectors of the superstructure, should sometimes be protected from failure or instability because the failure of rubber bearings may result in serious damage to superstructure. The evaluation of the collapse conditions is an essential step in designing the elastomeric bearing. The collapse of the device can occur

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either by global failure, due to buckling or roll-out of the device (Naeim and Kelly, 1999; Kelly, 1997), or by local rupture, due to tensile rupture of the rubber, through detachment of the rubber from the steel or steel yielding (Shariati et al., 2011; Simo and Kelly, 1984). Therefore, it is necessary to have an accurate knowledge of the global characteristics and behavior of the device under maximum lateral displacement with various boundary conditions.

In the study of the laminated elastomeric isolation bearings, Haringx (1949) first treated the bearing as an equivalent column with a constant cross sectional area, homogeneous and isotropic material, and an equivalent height that included the rubber layers and steel shims. By Haringx's theory, the P–Δ effect on the influence of horizontal stiffness of the bearings under an axial compressive loading was studied. Haringx's theory has shown fairly good agreement with experimental results for moderate amounts of shear strains (Gent, 1964; Tsai and Hsueh, 2001). Haringx's theory was then applied to study stability problems (Stanton et al., 1990) and was used to identify material properties of laminated rubber bearings (Tsai and Hsueh, 2001). Kelly (1997) and Koh and Kelly (1988) proposed a simple mechanical model and Iizuka (2000) suggested a macroscopic model to study the mechanical behaviors and the stability analysis of multilayer elastomeric isolation bearings. Nagarajaiah and Ferrell (1999) and Buckle et al. (2002) verified the simple mechanical model using experimental results. Kelly and Takhirov (2007) investigated tension buckling and related it with compression buckling in the multilayer elastomeric bearings.

As mentioned earlier, many theoretical investigations and experimental efforts in the laminated rubber bearing concept have been based on the model of Haringx's equivalent column. However, all researchers use the same boundary conditions. Thus, movement and rotation equations for a column of bearing have been obtained from Haringx's theory, has following boundary conditions (Kelly, 1997):

y(0)=0, θ1=0, θ2=0, y(h)=ymax

Where y(0) is displacement at the bottom end and θ1 and θ2 are initial rotation at the bottom and top end of a laminated rubber bearing, respectively (h is the height of equivalent column of bearing).

For this reason, at this study, the movement and rotation equations of multilayer rubber bearing have been formulated with supposing the measurable initial rotation of the bottom and top end of a bearing under the following three boundary conditions:

i. Equal rotation at the bottom and top end of a bearing,
ii. Rotation only at the bottom end of a bearing,
iii. Rotation only at the top end of a bearing.

According to the boundary conditions, variations in the horizontal stiffness of laminated rubber bearings have been investigated as a result of this study.

GOVERNING EQUATIONS

In the standard approach, the bearing is assumed to be a beam, and plane section, normal to the undeformed axis before deformation and is assumed to remain plane but not necessarily normal after deformation (Tsai and Kelly, 2005). According to Figure 1, which shows the deformation of a bearing, the lower end of the column is fixed against any displacement, whereas the upper end is allowed to move horizontally and vertically. The lower and upper end can also rotate and these rotations are dependent on the measurable and constant values of θ1 and θ2.

The deformation pattern is defined by two independent variables: y(x) is the lateral displacement on the middle line of the column, and θ(x) is the rotation of a plane originally normal to the x axis, where the x is the central axis of the undeformed rubber column shown in Figure 1(a). The overall shear deformation, γ, is the difference between the rotation of the horizontal axis, $\partial y(x)/\partial x$ and the rotation of normal face, θ(x). Figure 1(b) shows the internal and external forces on the bearing in the deformed position.

When the upper end of the column is subjected to a constant compressive force P and a varied horizontal force V, it will induce the bending moment M(x) and shear force V(x) at the cross-section of the height x as shown in Figure 1(b). The constitutive equations for bending moment M(x) and shear force V(x) in surface x can be expressed as:

$$M(x) = EI_s\left(\frac{d\theta(x)}{dx}\right) \quad (1)$$

$$V(x) = GA_s\gamma(x) \Longrightarrow V(x) = GA_s \quad (2)$$

Where E is Young's modulus, G is shear modulus, I_s is the moment of cross-sectional inertia and A_s is the shear area of the laminated elastomeric bearing. The parameters of EI_s and GA_s which are the mechanical characteristics of a laminated rubber bearing will explained in the study.

When rotations θ(x), θ_1 and θ_2 are small, the equations of equilibrium for bending moment and shear force in the deformed state, shown in Figure 1(b), and using Equations (1) and (2) are:

$$EI_s\left(\frac{d\theta(x)}{dx}\right) + (P + V_1\theta_1)y(x) = (P\theta_1 - V_1)x - M_1 \quad (3)$$

$$GA_s\left[\frac{dy(x)}{dx} - \theta(x)\right] - P\theta(x) = V_1 - P\theta_1 \quad (4)$$

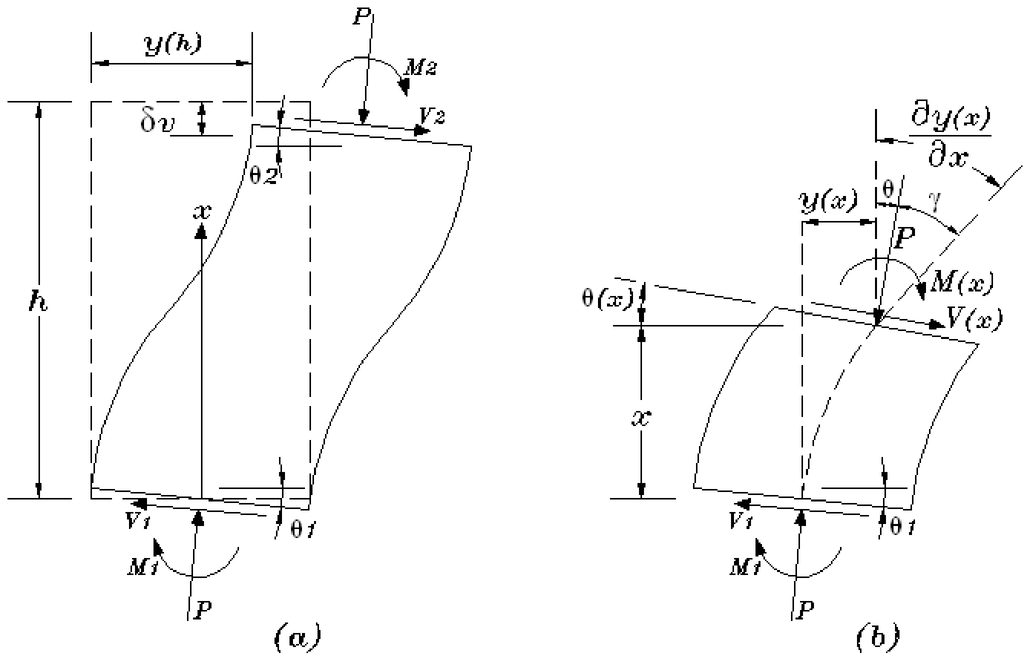

Figure 1. Loading and deformation of laminated rubber bearing at (a) full configuration; (b) bottom part.

The derivative of Equation (3) and substituting into Equations (3) and (4), we obtain two different equations as follows:

$$\frac{\square^2 \square(\square)}{\square\square^2} + \square^2 \square(\square) = \frac{\square^2 (\square\square_1 - \square_1)\square}{(\square + \square_1 \square_1)} - \frac{\square^2 \square_1}{(\square + \square_1 \square_1)} \quad (5)$$

$$\frac{\square^2 \square(\square)}{\square\square^2} + \square^2 \square(\square) = \square^2 \left[\frac{(\square\square_1 - \square_1)}{(\square + \square_1 \square_1)} + \frac{(\square\square_1 - \square_1)(\square_1 \square_1)}{(\square + \square_1 \square_1)(\square\square_{\square} + \square)}\right] \quad (6)$$

Where the parameter *α* is defined by:

$$\square^2 = \frac{(\square\square_{\square} + \square)(\square + \square_1 \square_1)}{\square\square_{\square}\square\square_{\square}} \quad (7)$$

The most general solution for the two differential equations in Equations (5) and (6) are:

$$\square(\square) = \square\square(\square\square) + \square\square(\square\square) + \frac{(\square\square_1 - \square_1)\square}{\square + \square_1 \square_1} - \frac{\square_1}{\square + \square_1 \square_1} \quad (8)$$

$$\square(\square) = \square\square(\square\square) - \square\square(\square\square) + \left[\frac{(\square\square_1 - \square_1)}{\square + \square_1 \square_1} + \frac{(\square\square_1 - \square_1)(\square_1 \square_1)}{(\square + \square_1 \square_1)(\square\square_{\square} + \square)}\right] \quad (9)$$

Boundary conditions

The Constants A and B, and parameters M_1 and y_{max} can be determined from boundary conditions. As shown in Figure 2, the laminated rubber bearing is constrained against displacement at the bottom and is free to measurable rotation at its lower and upper end; however the multilayer rubber bearing in its top end has a lateral displacement (y_{max}).

According to Figure 2, which shows the boundary condition of a laminated rubber bearing, we can find the constants of Equations (8) and (9) under the following three different conditions:

I. Equal rotation at the bottom and top end of a bearing [y(0)=0, θ_1=θ_2=variable],
ii. Rotation only at the top end of a bearing [y(0)=0, θ_1=0, θ_2= variable],
iii. Rotation only at the bottom end of a bearing [y(0)=0, θ_1= variable, θ_2=0].

Horizontal stiffness is one of the most important parameters of multilayer rubber bearings. Kelly (1997) stated that if the load carried by a bearing is comparable to the buckling load, then the simple formula for horizontal stiffness, $K_H = GA/$, may need to be modified. During lateral loading, when the resulting displacement at the top end of a bearing (y_{max}) is computed according the earlier given equations, the horizontal stiffness, K_H is given by:

$$\square_{\square} = \frac{\square}{\square_{\square}} \quad (10)$$

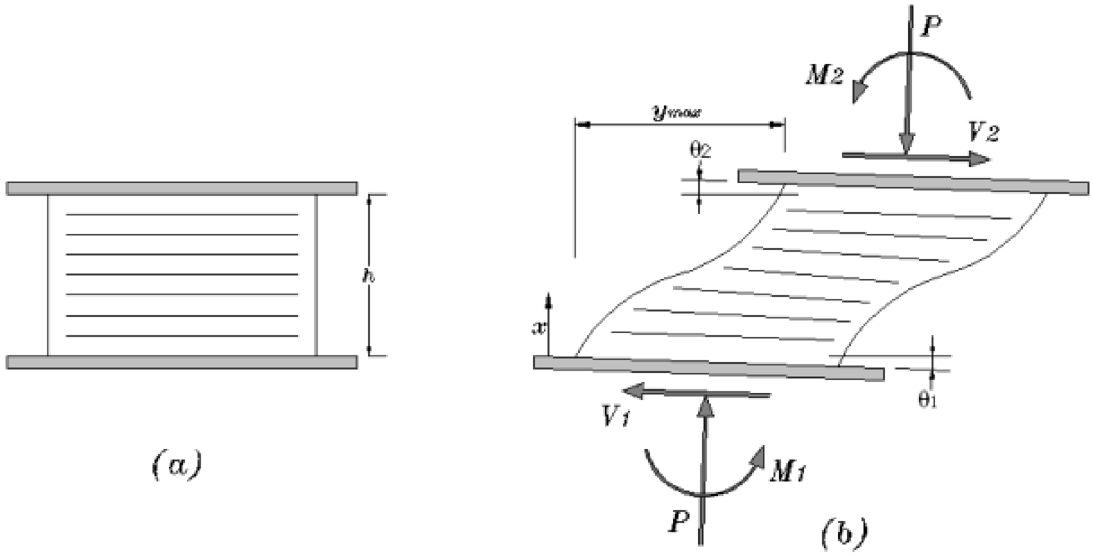

Figure 2. Boundary condition for a bearing (a) before loading (b) after loading.

PROPERTY IDENTIFICATION

As mention earlier, the mechanical properties of multilayer rubber isolators, including GA_s and EI_s as shear and bending stiffness of the composite system, must be recognized and verified.

Shear stiffness of a multilayer rubber bearing (GA_s)

For recognizing the shear stiffness of composite bearings (GA_s), the total shear area, A, of a bearing would be defined by the following basis (Chang, 2002):

$$A = A_1 + A_2 \frac{t_r}{t_s + t_r} \quad (11)$$

Where A_1 is the area of steel shims and A_2 is the cross sectional areas of the rubber cover. The A_1 and A_2 have been shown in Figure 4. The shear area of the cover is included because of its bonding with the end plates. Multiplying the shear area by factor of $h/$, is needed to account for the fact that the steel shim does not deform in the composite system (Kelly, 1997; Tsai and Hsueh, 2001), where h is the total height of bearing (rubber plus steel) and t_r is the total thickness of rubber. Therefore, the shear stiffness of multilayer rubber isolators is obtained as follows:

$$GA_s = G\left(A_1 + A_2 \frac{t_r}{t_s + t_r}\right)\frac{h}{t_r} \quad (12)$$

Bending stiffness of a multilayer rubber bearing (EI_s)

For a single circular rubber layer bonded between two rigid plates, the flexural stiffness for a circular rubber layer of radius R, proposed by Gent (2001) is:

$$(EI)_{eff} = EI\left(1 + \frac{2}{3}S^2\right) \quad (13)$$

Where E is the Young's modulus, I is the effective moment of inertia of the cross section ($I = \pi R^4/$) and S is shape factor of bearing defined as:

$$S = \frac{loaded\ area}{force\text{-}free\ area} \quad (14)$$

Shape factor (S) is a dimensionless parameter of the aspect ratio of the single layer of elastomer. For a circular elastomeric bearing, the shape factor is $R/$ where R is the radius of steel shim and t is the thickness of each layer of rubber.

The rubber is assumed to be an incompressible material (E=3G), where G is shear modulus of a laminated rubber bearing. For multilayer rubber bearings, the stiffness must also be modified and increased to account for the presence of the steel shims. This is shown as follows:

$$EI_s = (EI)_{eff}\frac{h}{t_r} \quad (15)$$

Numerical example and bearing properties

In this study, a computer program has been written for

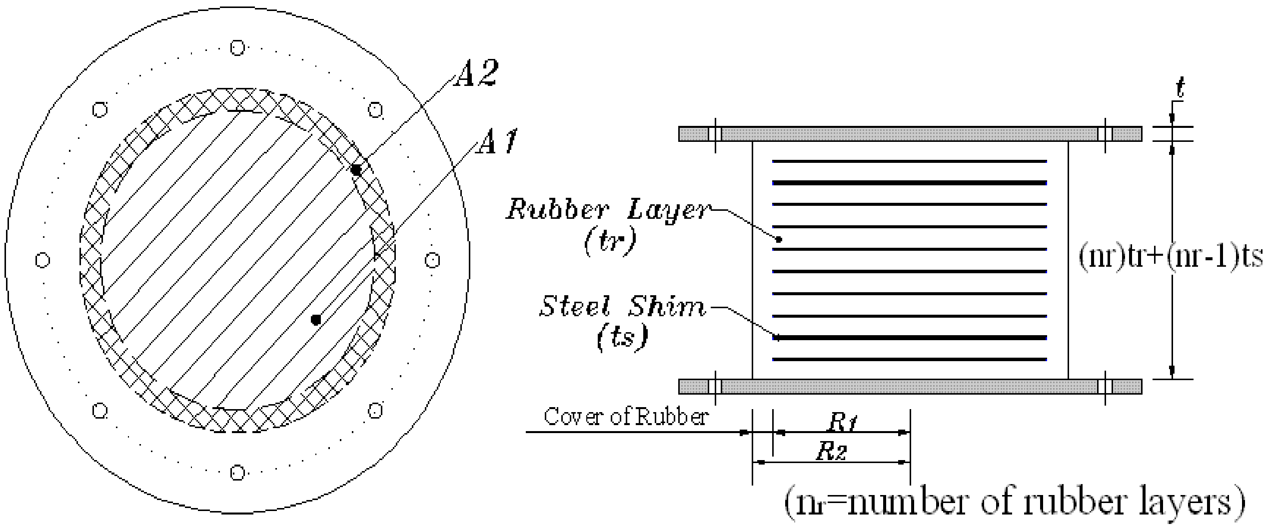

Figure 3. Typical circular multilayer rubber bearing.

calculating the variations in horizontal stiffness of laminated rubber bearings. The program can cover different boundary conditions for circular bearing. Circular multilayer elastomeric isolation bearings, as shown in Figure 3, are used in the following numerical study. For the bearing, the following dimensions are considered: the steel shim radius R_1=140 mm, with an additional 10 mm of protective rubber covering, for a total radius of R_2=150 mm, the number of rubber layers n_r=20 with thickness t_r=10 mm, the total rubber thickness is t=200 mm, the number of steel shims n_s=19 with thickness t_s=2 mm, and each one of the top and bottom end steel plates is 21 mm. The shear modulus of the rubber material G is 0.611 MPa. A vertical compressive force P and a horizontal force V_2 are applied at the top surface, to be given in each case (Figure 2).

EFFECTS OF INITIAL ROTATIONS ON THE VARIATIONS OF HORIZONTAL STIFFNESS

According to the properties mentioned previously, the results of numerical study have been shown as follows, with the results also being compared with experimental data. According to the various boundary conditions, the effects of the consideration of the initial rotation on the horizontal stiffness of multilayer rubber bearings have been investigated as a result of this study.

It is known that the P-Δ effect causes the horizontal stiffness to decrease, with an increase in the compression force (Kelly, 1997; Tsai and Hsueh, 2001; Chang, 2002). In the past, researches in the stability issue of laminated rubber bearings show that the variation in the horizontal stiffness has been calculated without any initial rotation on the ends of a bearing. A theoretical curve, calculated from Haringx's theory and experimental data presented by Tsai and Hsueh (2001) is shown in Figure 4. With consideration of $\theta_1=\theta_2=0$ in the earlier equations, this figure reinforced that the presented results are in good agreement with the theoretical curve and are consistent with experimental data.

Figure 4 clearly shows the effects of considering the rotation as a boundary condition on the horizontal stiffness of a bearing. The variation of horizontal stiffness has been shown due to the three different initial rotations including: 1-$\theta_1=\theta_2=0.02$(rad) 2-$\theta_1=0$, $\theta_2=0.02$(rad) 3-$\theta_1=0.02$(rad), $\theta_2=0$. This figure reveals that the amounts of horizontal stiffness is increased or decreased, although buckling load and horizontal stiffness without compression force are proportionately equal due to the different boundary conditions. Figure 4 shows that by considering of 1-θ_1 = θ_2= variable and 2-θ_1 = 0, θ_2 = variable, as two boundary conditions, an increase in the horizontal stiffness is the result. However, when consideration of 3-θ_1=variable, θ_2=0 has been made, the results show a decrease in the horizontal stiffness rather than condition of $\theta_1 = \theta_2 = 0$.

It is known that the shear force-displacement curve goes through a maximum as the horizontal displacement increases, under constant axial load. The shear force and horizontal displacement, at which the maximum occurs, decreases with an increasing axial load (Nagarajaiah and Ferrell, 1999; Buckle et al., 2002). The effects of rotation as a boundary condition on the horizontal displacement of a laminated rubber bearing, under variation amounts of shear force and constant axial force, are clearly evident in Figure 5. In this analysis, for showing the effects of rotation, the axial load is considered as 350 kN and the rotations as 0.02 Radian. According to Figure 5, considering of 1-$\theta_1 = \theta_2$ = variable as boundary condition

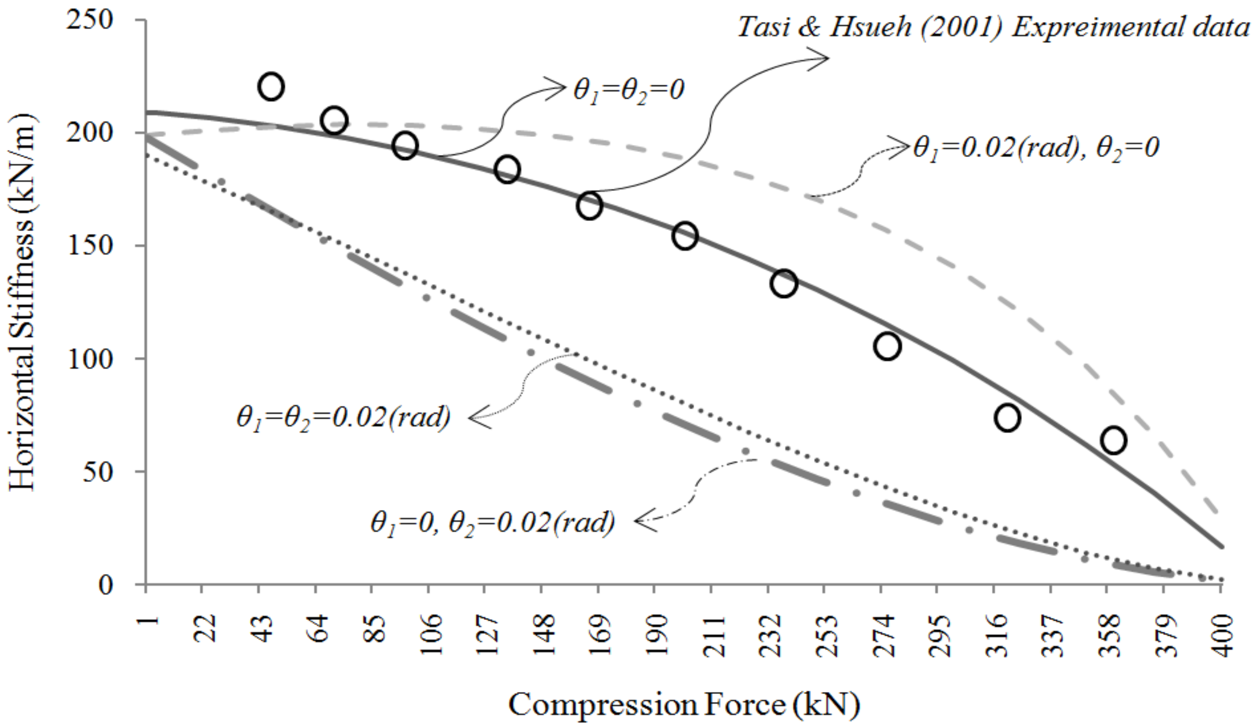

Figure 4. Comparisons of theoretical and experimental results of horizontal stiffness with different boundary conditions.

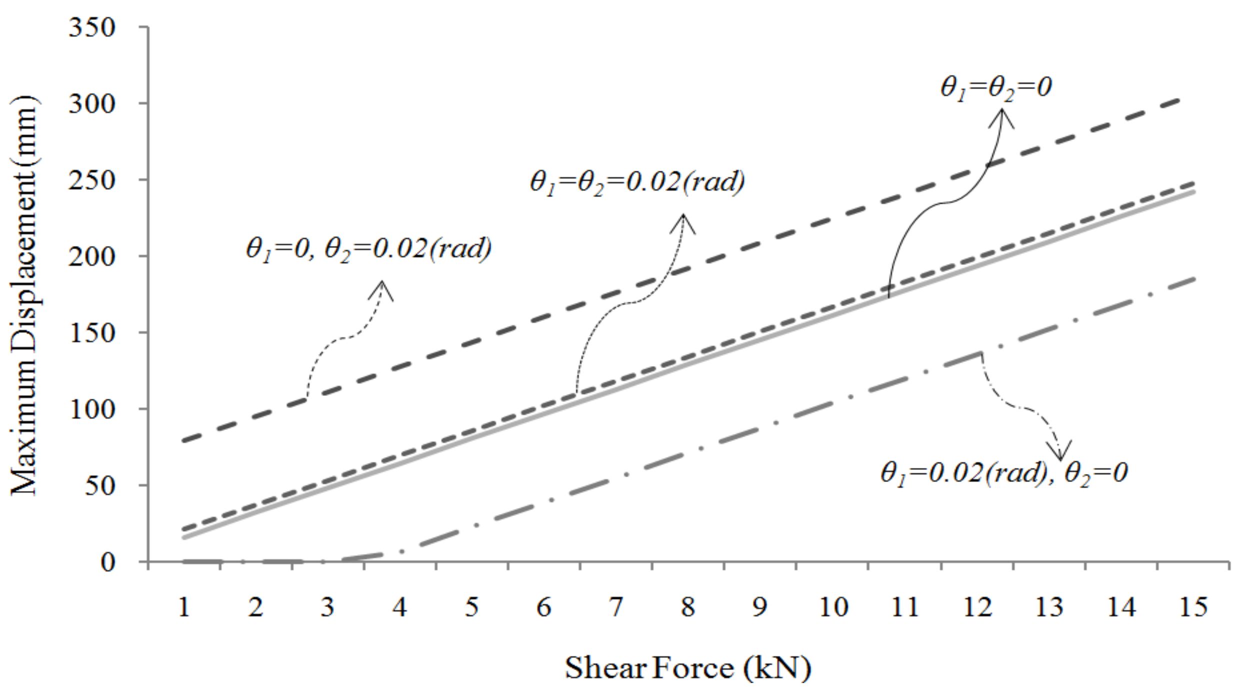

Figure 5. Shear force-displacement curves of bearing with different boundary conditions.

does not have any obviously difference towards to the base condition, including $\theta_1=\theta_2=0$. Whereas the boundary condition of 2-$\theta_1=0$, $\theta_2=$variable causes an increase in the horizontal displacement and in contrast with consideration of 3-θ_1=variable, θ_2 = 0 is made considerable decrease on the horizontal displacement rather than condition of θ1 = θ_2 = 0.

CONCLUSION

An analytical model of multilayer elastomeric isolation bearings has been developed based on the Haringx's theory. Using the initial rotations of a bearing as a new boundary condition, allowed the movement and rotation equations of bearings to be formulated. The variations in horizontal stiffness of laminated rubber bearings have been investigated as a result of this study. Three statements of the initial rotation are considered in the ends of bearings as boundary conditions including: 1-θ_1 = θ_2 = variable, 2-θ_1 = 0, θ_2 = variable and 3-θ_1 = variable, θ_2 = 0 and the results are compared with basic condition: θ_1 = θ_2 = 0.

The developed numerical analytical model satisfactorily predicts the behavior observed in the test results of the bearings. The important conclusions of this study are as follows. The consideration of 1-θ_1 = θ_2 = variable and 2-θ_1 = 0, θ_2 = variable, as two boundary conditions have resulted in an increase in the horizontal stiffness, whereas consideration of 3-θ_1 = variable, θ_2 = 0 has resulted in a decrease in the horizontal stiffness rather than condition of θ_1 = θ_2 = 0. It has also been shown that the buckling load and horizontal stiffness without compression force have insignificantly been changed by using different boundary conditions. It is shown that a consideration of the variables in the initial rotations is necessary for effectively predicting the maximum displacement of bearings. The calibration and verification of the analytical model is based on a limited set of test results; hence further investigations are needed.

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