

*Full Length Research Paper*

# Impacts of random scrap rate on production system in supply chain environment with a specific shipping policy

Feng-Tsung Cheng<sup>1</sup>, Kuang-Ku Chen<sup>2</sup>, Yuan-Shyi Peter Chiu<sup>3</sup> and Huei-Hsin Chang<sup>4\*</sup>

<sup>1</sup>Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung 407, Taiwan.

<sup>2</sup>Department of Accounting, National Changhua University of Education, Changhua 500, Taiwan.

<sup>3</sup>Department of Industrial Engineering and Management, Chaoyang University of Technology, Wufong, Taichung 413, Taiwan.

<sup>4</sup>Department of Finance, Chaoyang University of Technology, Wufong, Taichung 413, Taiwan.

Accepted 13 July, 2011

**This paper examines impacts of random scrap rate on production system in supply chain environment with a specific  $n+1$  distribution policy. The purpose of introducing such a specific shipping policy is to cut down inventory holding costs for both producer and customer in vendor-buyer supply chain environment. Mathematical modeling along with Hessian matrix equations is used to simultaneously determine the optimal policies for production lot-size as well as number of shipments first. Then impacts of random scrap rate on the proposed system are analyzed. Numerical example is provided to show its practical usage and to demonstrate impacts of scrap rate on the system.**

**Key words:** Imperfect production system, random scrap rate, replenishment lot size.

## INTRODUCTION

In manufacturing processes, random scrap items produced are inevitable (Nahmias, 2009). Impacts of scrap on production system must be studied specifically in order to achieve optimization of manufacturing system in terms of total production costs. Rosenblatt and Lee (1986) studied an economic production quantity (EPQ) model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of imperfect quality items are produced. Approximate solutions of optimal lot size were derived by them. Schwaller (1988) examined economic order quantity model by adding both fixed and variable inspection costs for finding and removing a known proportion of defective items in incoming lots. Wee (1993) developed and formulated an economic production policy for deteriorating items with partial back-ordering. Two numerical examples are used to illustrate the theory and

computational results indicated that the proposed policy leads to lower cost. Cheung and Hausman (1997) presented an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. Trade-off between investing in the two options (PM and SS) were examined, and optimality conditions under which either one or both strategies should be implemented are provided to minimize the associated cost function.

Kim et al. (2001) investigated optimal production run length and inspection schedules in a deteriorating production process. They assumed that a production process is subject to a random deterioration from the in-control state to the out-of-control state and thus produces some proportion of defective items. An optimal production run length and an optimal number of inspections that minimize total costs are derived accordingly. Grosfeld-Nir and Gerchak (2002) considered multistage production systems where defective units can be reworked repeatedly at every stage. They showed that a multistage system where only one of the stages requires

\*Corresponding author. E-mail: chs@cyut.edu.tw.

a set-up can be reduced to a single-stage system. They proved that it is best to make the "bottle-neck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems. Chiu (2003) studied optimal lot size for an imperfect quality finite production rate model with rework and backlogging (Barlow and Proschan, 1965; Chiu and Chiu, 2006; Chiu et al., 2007; Koçyiğit et al., 2009; Wazed et al., 2009; Chen et al., 2010; Chiu, 2010; Chiu et al., 2010a; Wazed et al., 2010a and b; Chen and Chiu, 2011).

In real-life supply chains environment, multiple deliveries of finished products are commonly used by vendor to supply items to its buyer. Goyal (1977) examined an integrated production-inventory model for single supplier-single customer case. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. An example was provided to illustrate his proposed method. Many studies have since been carried out to address various aspects of supply chain optimization. Schwarz et al. (1985) treated the system fill-rate of a one-warehouse N-identical retailer distribution system as a function of warehouse and retailer safety stock. Approximation model was developed from a prior study to maximize system fill-rate subject to a constraint on system safety stock. Properties of fill-rate policy lines are suggested and they can be used to provide managerial insight into system optimization. Hill (1996) examined a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers, while minimizing total cost of purchasing, manufacturing and stockholding. Sarker and Khan (1999) considered a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. A general cost model was formulated considering both raw materials and finished products. Then, using this model, a simple procedure was developed to determine an optimal ordering policy for procurement of raw materials as well as the manufacturing batch size, to minimize the total cost of meeting customer demands in time. Abdul-Jalbar et al. (2005) studied a multistage distribution/inventory system with a central warehouse and N retailers. Customer demand arrives at each retailer at a constant rate. The retailers replenish their inventories from the warehouse, which in turn orders from an outside supplier. It is assumed that shortages are not allowed and lead times are negligible. The goal was to determine policies which minimize the overall cost in the system. A heuristic procedure to compute near-optimal policies is presented with computational results on several randomly generated problems. Sarker and Diponegoro (2009) studied optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a producer

and multiple non-identical buyers. They assume that manufacturer procures raw materials from suppliers, converts them to finished products and ships the products to each buyer at a fixed-interval of time over a finite planning horizon. Their objective was to determine the production start time, the initial and ending inventory, the cycle beginning and ending time, the number of orders of raw materials in each cycle, and the number of cycles for a finite planning horizon so as to minimize the system cost (Goyal and Nebebe, 2000; Sarker and Khan, 2001; Chiu et al., 2009a,b,c; Chiu and Ting, 2010; Diponegoro and Sarker, 2006; Chiu et al., 2010b,c,d,e; Chen et al., 2011).

This paper reexamines and extends model of Chiu et al. (2009b) by simultaneously determining the optimal production lot size as well as optimal number of shipments for such a specific supply chain system with scrap and a cost lessening  $n+1$  delivery policy. The joint effects of the random scrap rate and  $n+1$  delivery policy on the production system are investigated.

## MATERIALS AND METHODS

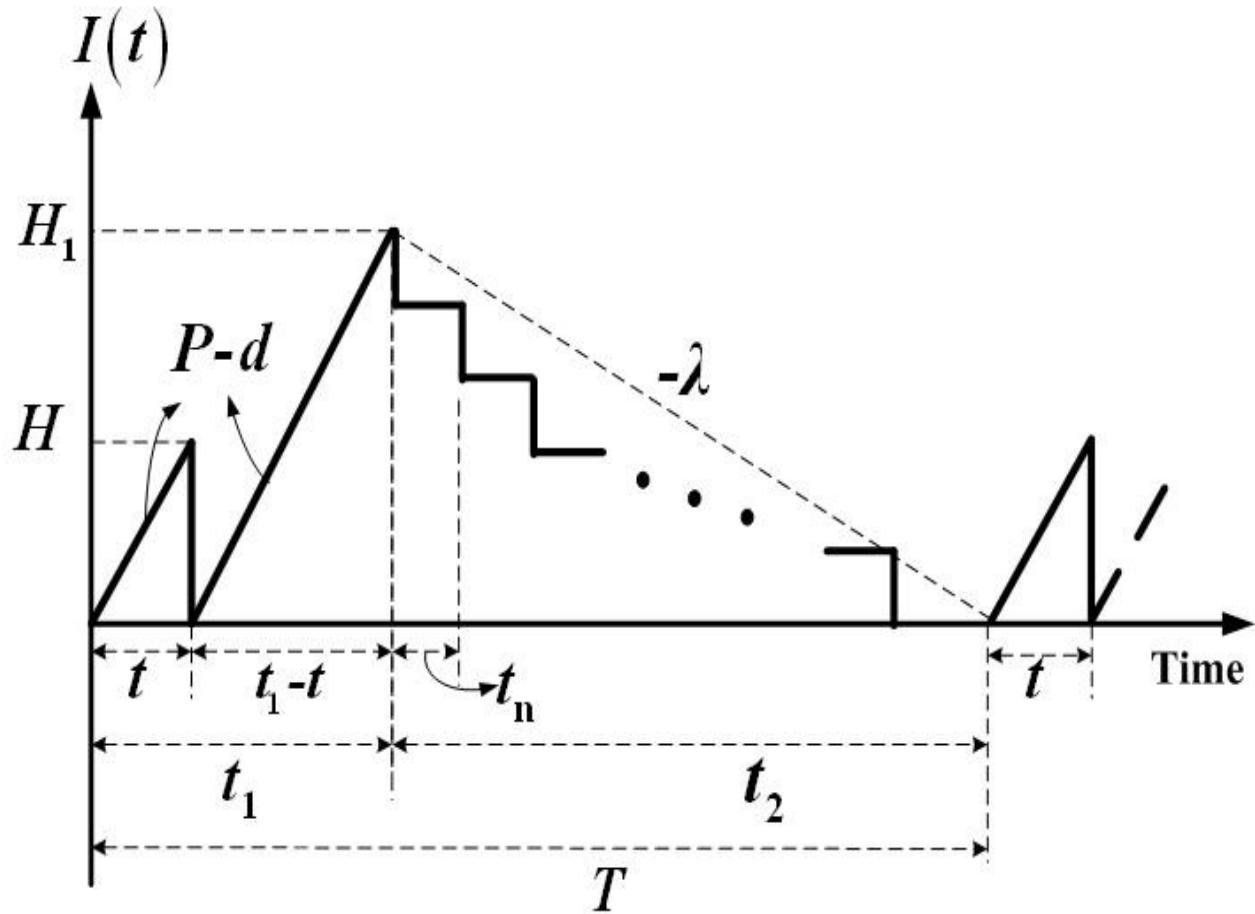
### Description and mathematical modeling

Suppose in a supply chain environment, a vendor-buyer integrated production system may produce  $x$  portion of random scrap items at a production rate  $d$ . Under regular operating schedule, the constant production rate  $P$  is larger than the sum of demand rate  $\lambda$  and production rate of defective items  $d$ . That is:  $(P-d-\lambda)>0$ ; where  $d$  can be expressed as  $d = Px$ . Cost parameters in this study include unit manufacturing production cost  $C$ , setup cost  $K$  per production run, vendor's unit holding cost  $h$ , buyer's unit holding cost  $h_2$ , disposal cost per scrap item  $C_s$ , delivery cost  $C_T$  per item and a fixed delivery cost  $K_1$  per shipment. Additional notation is listed as follows:

$n$  = number of fixed quantity installments of finished batch to be delivered to customer during  $t_2$ , one of the decision variables to be determined for each cycle,  $Q$  = replenishment lot size, another decision variable to be determined for each cycle,  $T$  = cycle length,  $H$  = the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time  $t_1$ ,  $H_1$  = maximum level of on-hand inventory in units when regular production ends,  $t$  = the production time needed for producing enough perfect items for satisfying product demand during the production uptime  $t_1$ ,  $t_1$  = the production uptime for the proposed model,  $t_2$  = time required for delivering the remaining perfect quality finished products,  $t_n$  = a fixed interval of time between each installment of products delivered during  $t_2$ ,  $I$  = demand during production time  $t$ , that is,  $I = \lambda t$ .  $D$  = demand during production uptime  $t_1$ , that is,  $D = \lambda t_1$ .

$l(t)$  = on-hand inventory of perfect quality items at time  $t$ ,  $TC(Q,n)$  = total production-inventory-delivery costs per cycle for the proposed model,  $TC_1(Q,n)$  = total production-inventory-delivery costs per cycle for the special case model,  $E[TCU(Q,n)]$  = the long-run average costs per unit time for the proposed model,  $E[TCU_1(Q,n)]$  = the long-run average costs per unit time for the special case.

Figure 1 depicts the producer's on-hand inventory of perfect quality items of the proposed model. Under the proposed  $n+1$  delivery policy, an initial installment of finished products is delivered to customer for satisfying the demand during uptime  $t_1$ . At the end of production, fixed quantity  $n$  installments of the rest of finished items are delivered to customer at a fixed interval of time. Figure 2 models illustrates customer's on-hand inventory levels of the proposed



**Figure 1.** Vendor's on-hand inventory of perfect quality items for the proposed model with random scrap rate and (n+1) delivery policy (Chen et al., 2011).

TC(Q,n) of the proposed model consists of the setup cost, variable manufacturing cost, variable disposal cost, (n+1) fixed and variable shipping cost, holding cost for perfect quality items during  $t_1$ , holding cost for scrap items during  $t_1$ , vendor's holding cost for finished goods during the delivery time  $t_2$  (Figure 1), and buyer's holding cost (to Figure 2).

$$\begin{aligned}
 TC(Q,n) &= K + CQ + C_s[xQ] + (n+1)K_1 + C_TQ(1-x) \\
 &+ h \left[ \frac{Ht}{2} + \frac{H_1(t_1-t)}{2} + \frac{dt_1}{2}(t_1) \right] + h \left[ \left( \frac{n-1}{2n} \right) H_1 t_2 \right] \\
 &+ h_2 \left[ \frac{H(t_1)}{2} + n \left( \frac{D+2I}{2} \right) t_n \right]
 \end{aligned} \tag{1}$$

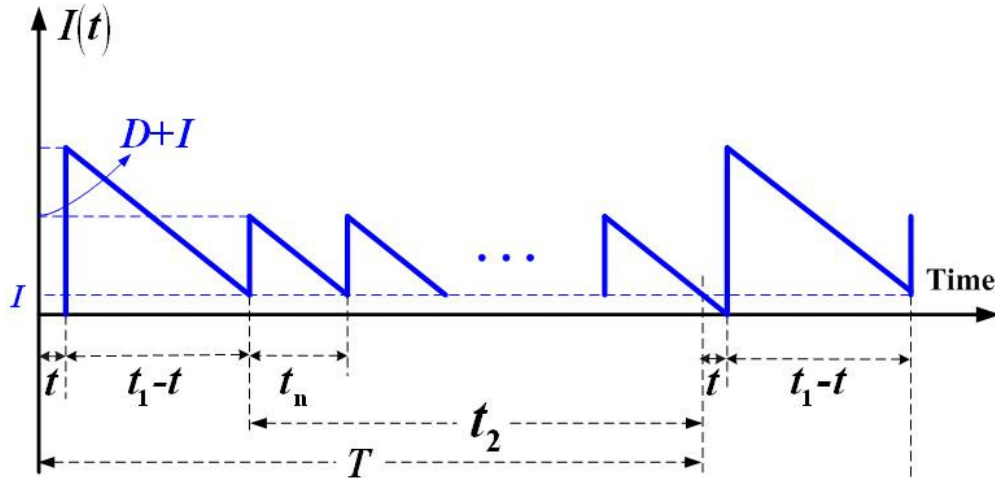
For derivation of the very last term (that is, the inventory quantity associated with buyer's holding cost  $h_2$ ), one can refer to Figure 2, and first considers the area in the largest triangle, then areas in n pieces of trapezoids.

Taking into account of the randomness of scrap rate  $x$ , one can use the expected values of  $x$  in cost analysis, and with further derivations one obtains  $E[TCU(Q,n)]$  as:

$$\begin{aligned}
 \frac{E[TC(Q,n)]}{E[T]} &= \frac{C\lambda}{1-E(x)} + \frac{[(n+1)K_1 + K]\lambda}{Q[1-E(x)]} + \frac{C_s E(x)\lambda}{1-E(x)} + C_T\lambda \\
 &+ \frac{hQ}{2} \left\{ \frac{2\lambda^3}{P^3[1-E(x)]} E\left(\frac{1}{1-x}\right) - \frac{\lambda^2}{P^2[1-E(x)]} + [1-E(x)] - \frac{\lambda[1-2E(x)]}{P[1-E(x)]} \right\} \\
 &\quad \left\{ -\left(\frac{1}{n}\right) \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \right\} \\
 &+ \frac{h_2 Q}{2n} \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \\
 &+ h_2 Q \lambda^2 \left[ \frac{1}{P^2} \left[ E\left(\frac{1}{1-x}\right) + \frac{1}{[1-E(x)]} \right] - \frac{\lambda}{P^3[1-E(x)]} E\left(\frac{1}{1-x}\right) \right]
 \end{aligned} \tag{2}$$

**Convexity of the total cost function**

In order to derive the optimal production-shipment policy for the proposed model, one must first prove  $E[TCU(Q,n)]$  is convex. The Hessian matrix equations (Rardin, 1998) are employed here to verify whether the following condition (that is, Equation 3) holds or not.



**Figure 2.** Buyer's on-hand inventory level for the proposed model with random scrap rate and (n+1) delivery policy.

$$[Q \ n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q,n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} > 0 \quad (3)$$

From Equation (2), one obtains the following terms:

$$\begin{aligned} \frac{\partial E[TCU(Q,n)]}{\partial Q} &= -\frac{[(n+1)K_1 + K]\lambda}{Q^2(1-E(x))} \\ &+ \frac{h}{2} \left\{ \frac{2\lambda^2}{P^3[1-E(x)]} E\left(\frac{1}{1-x}\right) - \frac{\lambda^2}{P^2[1-E(x)]} + [1-E(x)] \right. \\ &\left. - \frac{\lambda[1-2E(x)]}{P[1-E(x)]} - \left(\frac{1}{n}\right) \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \right\} \\ &+ \frac{h_2}{2n} \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \\ &+ h_2 \lambda^2 \left[ \frac{1}{P^2} \left[ E\left(\frac{1}{1-x}\right) + \frac{1}{[1-E(x)]} \right] - \frac{\lambda}{P^3[1-E(x)]} E\left(\frac{1}{1-x}\right) \right] \end{aligned} \quad (4)$$

$$\frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} = \frac{2[(n+1)K_1 + K]\lambda}{Q^3(1-E(x))} \quad (5)$$

$$\begin{aligned} \frac{\partial E[TCU(Q,n)]}{\partial n} &= \frac{K_1 \lambda}{Q[1-E(x)]} \\ &- \frac{Q}{2n^2} (h_2 - h) \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \end{aligned} \quad (6)$$

$$\frac{\partial^2 E[TCU(Q,n)]}{\partial n^2} = \frac{Q}{n^2} (h_2 - h) \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \quad (7)$$

$$\begin{aligned} \frac{\partial^2 E[TCU(Q,n)]}{\partial n \partial Q} &= \frac{-K_1 \lambda}{Q^2[1-E(x)]} \\ &- \frac{1}{2n^2} (h_2 - h) \left[ [1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \end{aligned} \quad (8)$$

Substituting Equations (4) to (8) in (3), one has:

$$[Q \ n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q,n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\lambda(K_1 + K)}{Q[1-E(x)]} > 0 \quad (9)$$

Equation (9) is resulting positive, because K, K<sub>1</sub>, λ, (1-E[x]), and Q are all positive. Hence, E[TCU(Q,n)] is a strictly convex function for all Q and n different from zero. Convexity of E[TCU(Q,n)] is proved and the minimum of E[TCU(Q,n)] exists.

## RESULTS

### Simultaneous determination of the production-shipment policy

Once convexity of E[TCU(Q,n)] is proved, for deriving the optimal lot size Q\* and the optimal number of shipments n\* one can differentiate E[TCU(Q,n)] with respect to Q and with respect to n, and solve the linear system of Equations (4) and (6) by setting these partial derivatives equal to zero. With further derivations one obtains Q\* and n\* respectively as follows:

$$Q^* = \sqrt{\frac{2\lambda[(n+1)K_1 + K]}{h \left\{ \frac{2\lambda^2}{P^3} E\left(\frac{1}{1-x}\right) - \frac{\lambda^2}{P^2} + [1-E(x)] - \frac{\lambda[1-2E(x)]}{P} \right\}} + \frac{(h_2 - h)}{n} \left\{ [1-E(x)] - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right\} + h_2 \lambda^2 \left\{ \frac{2}{P^2} E\left(\frac{1}{1-x}\right) \left[ [1-E(x)] - \frac{\lambda}{P} + \frac{1}{P^2} \right] \right\}} \quad (10)$$

And

$$n^* = \sqrt{\frac{(K_1 + K)(h_2 - h) \left[ 1 - E(x) \right]^2 - \frac{2\lambda}{P} \left[ 1 - E(x) \right] + \frac{\lambda^2}{P^2}}{K_1 \left\{ h \left[ \frac{2\lambda^3}{P^3} \cdot E \left( \frac{1}{1-x} \right) - \frac{\lambda^2}{P^2} + \left[ 1 - E(x) \right]^2 - \frac{\lambda \left[ 1 - 2E(x) \right]}{P} \right\} + h_2 \lambda^2 \left[ \frac{2}{P^2} E \left( \frac{1}{1-x} \right) \left[ 1 - E(x) \right] - \frac{\lambda}{P} + \frac{1}{P^2} \right]}}}} \quad (11)$$

One notes that optimal number of shipments  $n^*$  only takes on integer value, while Equation (11) results likely in real number. One should use 2 adjacent integers from the original result of Equation (11), plugging into Equation (10) to obtain their corresponding  $Q$ s. Then substituting each pair of  $Q$  and  $n$  in  $E[TCU(Q,n)]$  (that is, Equation 2) to compare and select whichever integer  $n$  (and its corresponding  $Q$ ) that gives the minimal value of total cost as our optimal production-shipment policy.

**DISCUSSION**

**Analysis of the similar model without considering random scrap rate**

Suppose random scrap rate is not considered, then one has  $E[TCU(Q,n)]$  as:

$$\frac{E[TC(Q,n)]}{E[T]} = C\lambda + \frac{[(n+1)K_1 + K]\lambda}{Q} + C_T\lambda + \frac{hQ}{2} \left\{ \frac{2\lambda^3}{P^3} - \frac{\lambda^2}{P^2} + \left( 1 - \frac{\lambda}{P} \right) - \left( \frac{1}{n} \right) \left[ 1 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right] \right\} + h_2 Q \lambda^2 \left[ \frac{2}{P^2} - \frac{\lambda}{P^3} \right] + \frac{h_2 Q}{2n} \left[ 1 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right] \quad (12)$$

Similarly one proves convexity of  $E[TCU(Q,n)]$  as follows:

$$[Q \ n] \cdot \left( \frac{\frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2}}{\frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n}} \quad \frac{\frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n}}{\frac{\partial^2 E[TCU(Q,n)]}{\partial n^2}} \right) \cdot [Q] = \frac{2\lambda(K_1 + K)}{Q} > 0 \quad (13)$$

Equation (13) is resulting positive, because  $K, K_1, \lambda,$  and  $Q$  are all positive. Hence,  $E[TCU(Q,n)]$  is a strictly convex function for all  $Q$  and  $n$  different from zero. Again, with further derivations one can derive jointly the optimal lot size  $Q^*$  and optimal number of shipments  $n^*$  as follows:

$$Q^* = \sqrt{\frac{2\lambda[(n+1)K_1 + K]}{h \left\{ \frac{2\lambda^3}{P^3} - \frac{\lambda^2}{P^2} + \left( 1 - \frac{\lambda}{P} \right) \right\} + \frac{(h_2 - h)}{n} \left\{ 1 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right\} + h_2 \lambda^2 \left\{ \frac{2}{P^2} \left[ \left( 1 - \frac{\lambda}{P} \right) \right] + \frac{1}{P^2} \right\}}} \quad (14)$$

And

$$n^* = \sqrt{\frac{(K_1 + K)(h_2 - h) \left[ 1 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right]}{K_1 \left\{ h \left[ \frac{2\lambda^3}{P^3} - \frac{\lambda^2}{P^2} + \left( 1 - \frac{\lambda}{P} \right) \right] + h_2 \lambda^2 \left[ \frac{2}{P^2} \left( 1 - \frac{\lambda}{P} \right) + \frac{1}{P^2} \right] \right\}}} \quad (15)$$

To cope with the integer value of  $n$ , similar solution procedure (as stated earlier) should be applied to this special case for deriving the optimal  $(Q^*, n^*)$ .

**Numerical example with further discussion**

To enable comparisons of the models with/without random scrap, and  $n$  versus  $n+1$  delivery policy, the same example as in Chiu et al. (2009b) is used here. Assuming a manufacturing system has an annual production rate of 60,000 units, and a flat demand rate of 3,400 units per year. During the production, a random scrap rate is assumed to be uniformly distributed over interval  $[0, 0.3]$ . Cost parameters include  $K = \$20,000$  per run;  $C = \$100$ ;  $C_S = \$20$  per scrap item;  $h = \$20$  per item per year;  $K_1 = \$4,400$  per shipment;  $C_T = \$0.1$  per item delivered; and  $h_2 = \$80$  per item kept at the customer's end per unit time.

From computation of Equation (11) one obtains  $n^* = 3.84$ . To determine the optimal integer value of  $n$ , two adjacent integer numbers and its corresponding lot-sizes (Equation 10) are plugged in Equation (2) respectively. Results are  $E[TCU(Q = 3073, n = 3)] = \$509,702$  and  $E[TCU(Q = 3455, n = 4)] = \$509,012$ . Therefore, one obtains the optimal production lot size  $Q^* = 3455$ , the optimal number of deliveries  $n^* = 4$ , and the long-run expected cost  $E[TCU(Q^*, n^*)] = \$509,012$ .

Impact of random scrap rate on production system is analyzed as follows. Suppose that scrap rate  $x = 0$ . From computation of Equation (15) one obtains  $n^* = 3.91$ . To determine the optimal integer value of  $n$ , two adjacent integer numbers and its corresponding lot-sizes (Equation 14) are plugged in Equation (12) respectively. Results are  $E[TCU(Q = 2609, n = 3)] = \$437,803$  and  $E[TCU(Q = 2938, n = 4)] = \$436,987$ . Therefore, one obtains the optimal production lot size  $Q^* = 2938$ , the optimal number of deliveries  $n^* = 4$ , and the long-run expected cost  $E[TCU(Q^*, n^*)] = \$436,987$ .

This study proposes a  $(n^*+1 = 5)$  delivery policy versus  $n^* = 4$  as was used in Chiu et al. (2009b), where they had optimal  $Q^* = 2652$  and the long-run expected cost  $E[TCU(Q^*, n^*)] = \$512,047$ . In theory, one would think that it costs more to have one extra shipping cost. However, total savings on inventory holding costs (both from vendor and buyer) offsets this extra expense. As a result, excluding the variable production cost ( $\lambda C$ ) and set up cost, there is a total saving of 2.1%.

Comparing the proposed model with versus without random scrap rate, one realizes that a total savings of  $\$72,025$  when there is no scrap rate in production. It is a

reduction of 14.15% in total cost.

## Conclusions

This paper investigates the impacts of random scrap rate on production system in supply chain environment with a specific  $n+1$  distribution policy. Mathematical modeling along with Hessian matrix equations is used to simultaneously determine the optimal policies for production lot-size as well as number of shipments first. Impacts of random scrap rate on the proposed system are then analyzed. Numerical example is provided to show its practical usage.

The research results depict that the  $n+1$  delivery policy can outperform  $n$  shipping method in terms of savings in stock holding costs from both ends of vendor and buyer. Further, impacts of random scrap rate on production system are significant in terms of extra expenses in assuring product's quality. In summary, such a real-life system must be specifically studied in order to obtain more insights of the system parameters.

## REFERENCES

- Abdul-Jalbar B, Gutiérrez J, Sicilia J (2005). Integer-ratio policies for distribution/inventory systems. *Int. J. Prod. Econ.*, 93-94: 407-415.
- Barlow RE, Proschan F (1965). *Mathematical Theory of Reliability*, John Wiley & Sons, New York, USA.
- Chen KK, Chiu YSP, Hwang MH (2010). Integrating a cost reduction delivery policy into an imperfect production system with repairable items. *Int. J. Phys. Sci.*, 5(13): 2030-2037.
- Chen KK, Chiu SW (2011). Replenishment lot size and number of shipments for EPQ model derived without derivatives. *Math. Comput. Appl.*, 16(3): 753-760.
- Chen KK, Chiu SW, Yang JC (2011). Incorporating a cost lessening distribution policy into a production system with random scrap rate. *Afr. J. Bus. Manage.*, 5(5): 1927-1935.
- Cheung KL, Hausman WH (1997). Joint determination of preventive maintenance and safety stocks in an unreliable production environment. *Nav. Res. Logistics.*, 44: 257-272.
- Chiu YSP (2003). Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Eng. Optimiz.*, 35(4): 427-437.
- Chiu SW, Chiu YSP (2006). Mathematical modeling for production system with backlogging and failure in repair. *J. Sci. Ind. Res.*, 65(6): 499-506.
- Chiu SW, Lin HD, Cheng CB, Chung CL (2009b). Optimal production-shipment decisions for the finite production rate model with scrap. *Int. J. Eng. Model.*, 22(1-4): 25-34.
- Chiu SW (2010). Robust planning in optimization for production system subject to random machine breakdown and failure in rework. *Comput. Oper. Res.*, 37(5): 899-908.
- Chiu SW, Cheng CB, Wu MF, Yang JC (2010a). An Algebraic Approach for Determining the Optimal Lot Size for EPQ Model with Rework Process. *Math. Comput. Appl.*, 15(3): 364-370.
- Chiu SW, Chen KK, Yang, JC (2009c). Optimal replenishment policy for manufacturing systems with failure in rework, backlogging, and random breakdown. *Math. Comp. Model. Dyn. Sys.*, 15(3): 255-274.
- Chiu YSP, Chen KK, Ting CK, Chiu V (2007). Economic lot sizing with imperfect rework derived without derivatives. *Int. J. Eng. Model.*, 20: 61-65.
- Chiu YSP, Chiu SW, Li CY, Ting CK (2009a). Incorporating multi-delivery policy and quality assurance into economic production lot size problem. *J. Sci. Ind. Res.*, 68(6): 505-512.
- Chiu YSP, Chen KK, Cheng FT, Wu MF (2010b). Optimization of the finite production rate model with scrap, rework and stochastic machine breakdown, *Comput. Math. Appl.*, 59(2): 919-932.
- Chiu YSP, Chen KK, Chang HH (2010c). Solving an economic production lot size problem with multi-delivery policy and quality assurance using an algebraic approach. *J. Sci. Ind. Res.*, 69(12): 926-929.
- Chiu YSP, Cheng FT, Chang HH (2010d) Remarks on optimization process of manufacturing system with stochastic breakdown and rework. *Appl. Math. Lett.* 23(10): 1152-1155.
- Chiu YSP, Lin CAK, Chang HH, Chiu, V. (2010e) Mathematical modeling for determining economic batch size and optimal number of deliveries for EPQ model with quality assurance. *Math. Comp. Model. Dyn. Sys.*, 16(4): 373-388.
- Chiu YSP, Ting CK (2010) A note on 'Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns'. *Eur. J. Oper. Res.*, 201(2): 641-643
- Diponegoro A, Sarker BR (2006). Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries. *Comput. Oper. Res.*, 33: 2387-2404.
- Goyal SK (1977). Integrated inventory model for a single supplier-single customer problem. *Int. J. Prod. Res.*, 15: 107-111.
- Goyal SK, Nebebe F (2000). Determination of economic production-shipment policy for a single- vendor-single-buyer system. *Eur. J. Oper. Res.*, 121(1): 175-178.
- Koçyiğit F, Yanikoğlu E, Yılmaz AS, Bayrak M (2009). Effects of power quality on manufacturing costs in textile industry. *Sci. Res. Essays.*, 4(10): 1085-1099.
- Nahmias S (2009). *Production and Operations Analysis*. McGraw-Hill Co. Inc., New York, USA.
- Rardin RL (1998). *Optimization in Operations Research*, Prentice-Hall, New Jersey, USA.
- Rosenblatt MJ, Lee HL (1986). Economic production cycles with imperfect production processes. *IIE Trans.*, 18: 48-55.
- Hill RM (1996). Optimizing a production system with a fixed delivery schedule. *J. Oper. Res. Soc.*, 47: 954-960.
- Sarker RA, Khan LR (1999). Optimal batch size for a production system operating under periodic delivery policy. *Comput. Ind. Eng.*, 37(4): 711-730.
- Sarker BR, Diponegoro A (2009). Optimal production plans and shipment schedules in a supply-chain system with multiple suppliers and multiple buyers. *Eur. J. Oper. Res.*, 194(3): 753-773.
- Sarker RA, Khan LR (2001). An optimal batch size under a periodic delivery policy. *Int. J. Syst. Sci.* 32(9): 1089-1099.
- Schwaller R (1988). EOQ under inspection costs. *Prod. Inventory Manage.*, 29: 22-24.
- Schwarz LB, Deuermeyer BL, Badinelli RD (1985). Fill-rate optimization in a one-warehouse N-identical retailer distribution system. *Manage. Sci.*, 31(4): 488-498.
- Wazed MA, Ahmed S, Yusoff N (2009). Impacts of common components on production system in an uncertain environment. *Sci. Res. Essays.*, 4(12): 1505-1517.
- Wazed MA, Ahmed S, Nukman Y (2010a). Impacts of quality and processing time uncertainties in multistage production system. *Int. J. Phys. Sci.*, 5(6): 814-825.
- Wazed MA, Ahmed S, Nukman Y (2010b). Analysis of the impacts of common components in multistage production system under uncertain conditions: Application of Taguchi method. *Sci. Res. Essays.*, 5(24): 3814-3825.
- Wee HM (1993). Economic production lot size model for deteriorating items with partial backordering. *Comput. Ind. Eng.*, 24(3): 449-458.