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Improved *(G'/G)*- expansion method for constructing exact traveling wave solutions for a nonlinear PDE of nanobiosciences

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In the present paper, the improved (G'/G) - expansion method combined with suitable transformations is used to construct many exact solutions involving parameters of a nonlinear equation describing the nano-ionic currents along microtublues. As a result, hyperbolic function solutions, trigonometric function solutions and rational function solutions with parameters are obtained. When these parameters are taken as special values, some solitary wave solutions and the periodic wave solution are derived from the exact solutions. This method can be employed for many other nonlinear evolution equations in mathematical physics and engineering. Comparisons between our results and the well-known results are given.

Key words: Improved (G'/G) - expansion method, exact traveling wave solutions, solitary and periodic solutions, nano-ionic currents along microtublues, nanobiosciences.

INTRODUCTION

Nonlinear phenomena play crucial roles in applied mathematics and physics. Exact solutions for nonlinear partial differential equations (PDEs) play an important role in many phenomena in such as fluid mechanics, hydrodynamics, optics, plasma and physics and so on. Many powerful methods have been presented, such as the inverse scattering transform method (Ablowitz and Clarkson, 1991), the bilinear method (Hirota, 1971; Ma, 2011), the Painleve expansion method (Weiss et al., 1983; Kudryashov, 1988, 1990, 19991), the Backlund truncated method (Miura, 1978), the exp-function method (He and Wu, 2006; Yusufoglu, 2008; Bekir, 2009, 2010; Aslan, 2011a; Ma and Zhu, 2012), the tanh-function method (Abdou, 2007; Fan, 2000; Zhang and Xia, 2008; Yusufoglu and Bekir, 2008), the Jacobi elliptic function method (Chen and Wang, 2005; Liu et al., 2001; Lu, 2005), the (G'/G)-expansion method (Wang et al., 2008; Zhang, 2008; Zhang et al., 2008; Zayed 2009, 2010; Bekir, 2008; Ayhan and Bekir, 2012; Aslan, 2010, 2011b, 2012a,b), the generalized Riccati equation mapping method (Zhu, 2008; Zayed and Arnous, 2013; Ma and Fuchssteiner,1996; Ma et al., 2007; Ma and Lee, 2009), local fractional variation iteration method (Yang and Baleanu, 2013), local fractional series expansion method (Yang et al., 2013) and so on.

The objective of this paper is to apply the improved (G'/G) - expansion method combined with suitable transformations to construct many exact solutions involving parameters of the nonlinear PDE of special interest in nanobiosciences namely, the following transmission line model for nano-ionic currents along microtublues (Sekulic et al., 2011; Sataric et al., 2010):

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$$\frac{L^2}{3}u_{xxx}+\frac{Z^{3/2}}{L}(\chi G_0-2\delta C_0)uu_t+2u_x+\frac{ZC_0}{L}u_t+\frac{1}{L}(RZ^{-1}-G_0Z)u=0 \qquad (1)$$

where $R=0.34\times10^{9}\Omega$ the resistance of the elementary rings (ER) $L=8\times10^{-9}m$, $C_0=1.8\times10^{-15}F$ is the total maximal capacitance of the ER. $G_0=1.1\times10^{-13}si$ is the conductance of pertaining nano-pores (NPS) and $Z=5.56\times10^{10}\Omega$ is the characteristic impedance of the system. The parameters δ *and* χ describe the nonlinearity of ER capacitor and conductance of NPS in ER respectively. The physical details of the derivation of Equation (1) can be elaborated in Sataric et al. (2010). Recently, Equation (1) has been discussed by using the modified extended tanh-function method (Sekulic et al., 2011) and by using the improved Riccati equation mapping method (Zayed et al., 2013) where its exact solutions have been found. Comparison between our results and the well - known results obtained in Sekulic et al. (2011) and Zayed et al. (2013) will be investigated in the discussions and conclusions part of this work.

DESCRIPTION OF THE IMPROVED (G'/G) - EXPANSION METHOD

Suppose that we have the following nonlinear evolution equation:

$$F(u,u_t,u_x,u_{tt},u_{xx},...)=0 \qquad (2)$$

where *F* is a polynomial in *u*(*x*, *t*) and its partial derivatives, in which the highest order derivatives and the nonlinear terms are involved. In the following, we give the main steps of this method (Liu et al., 2001; Lu, 2005) as follows:

Step 1

We use the traveling wave transformation

$$u(x,t)=u(\xi),\quad \xi=kx+\omega t \qquad (3)$$

to reduce Equation (2) to the following ordinary differential equation (ODE):

$$P(u,u',u'',...)=0 \qquad (4)$$

where k,ω are constants. Here, P is a polynomial in $u(\xi)$ and its total derivatives, while the dashes denote the derivatives with respect to ξ.

Step 2

We assume that Equation 4 has the formal solution:

$$u(\xi)=\sum_{i=-m}^{m}a_i\left(\frac{G'(\xi)}{G(\xi)}\right)^i \qquad (5)$$

where a_i $(i=-m,...,m)$ are constants to be determined later and $G(\xi)$ satisfies the following linear ODE:

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \qquad (6)$$

where λ and μ are constants.

Step 3

The positive integer *m* in (5) can be determined by balancing the highest-order derivatives with the nonlinear terms appearing in Equation 4.

Step 4

We **s**ubstitute (5) along with Equation 6 into Equation 4 to obtain polynomials in $\left(\frac{G'}{G}\right)^i,(i=0,\pm1,\pm2,...)$. Equating all the coefficients of these polynomials to zero, yields a set of algebraic equations which can be solved by using the Maple to find the unknowns a_i,k,ω.

Step 5

Since the solutions of Equation 6 are well-known for us, then we have the following ratios:

(i)If $\lambda^2-4\mu>0$, we have

$$\frac{G'(\xi)}{G(\xi)}=-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left[\frac{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right] \qquad (7)$$

(ii)If $\lambda^2-4\mu<0$, we have

$$\frac{G'(\xi)}{G(\xi)}=-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left[\frac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right] \qquad (8)$$

(iii)If $\lambda^2 - 4\mu = 0$, we have

$$\frac{G'(\xi)}{G(\xi)} = -\frac{\lambda}{2} + \frac{c_1}{c_1 + c_2\xi} \qquad (9)$$

where c_1 and c_2 are arbitrary constants.

Step 6

We substitute the values of a_i, k, ω as well as the ratios (7)-(9) into (5) to obtain the exact solutions of Equation (2).

MANY FAMILIES OF EXACT TRAVELING WAVE SOLUTIONS FOR EQUATION (1)

Here, we apply the proposed method of description of the improved (G'/G) - expansion method part of this work to find many families of exact traveling wave solutions of Equation (1). To this end, we use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = \frac{1}{L}x - \frac{c}{\tau}t \qquad (10)$$

where $\tau = RC_0 = 0.6 \times 10^{-6}s,$ and c is the dimensionless velocity of the wave, to reduce Equation (1) into the following ODE:

$$u''' + Acuu' + (6 - Bc)u' + Cu = 0 \qquad (11)$$

Where

$$A = \frac{3Z^{3/2}}{\tau}(2\delta C_0 - \chi G_0),\ B = \frac{3ZC_0}{\tau},\ C = 3(RZ^{-1} - G_0 Z).$$

By balancing u''' *with* uu', we have m = 2. Hence, the formal solution of Equation (11) takes the form

$$u(\xi) = a_2\left(\frac{G'}{G}\right)^2 + a_1\left(\frac{G'}{G}\right) + a_0 + a_{-1}\left(\frac{G'}{G}\right)^{-1} + a_{-2}\left(\frac{G'}{G}\right)^{-2} \qquad (12)$$

where $a_2,\ a_1, a_0,\ a_{-1}, a_{-2}$ are parameters to be determined later, such that $a_{-2} \neq 0$ or $a_2 \neq 0$. Inserting (12) with the aid of Equation (6) into Equation (11), we get the following system of algebraic equations:

$$\left(\frac{G'}{G}\right)^{-5}: \quad 24a_{-2}\mu^3 + 2a_{-2}^2\mu Ac = 0,$$

$$\left(\frac{G'}{G}\right)^{5}: \quad 24a_2 + 2a_2^2 Ac = 0,$$

$$\left(\frac{G'}{G}\right)^{-4}: \quad 54a_{-2}\mu^2\lambda + 6a_{-1}\mu^3 + Ac(2a_{-2}^2\lambda + 3a_{-1}a_{-2}\mu) = 0,$$

$$\left(\frac{G'}{G}\right)^{4}: \quad 54a_2\lambda + 6a_1 + Ac(2a_2^2\lambda + 3a_1a_2) = 0,$$

$$\left(\frac{G'}{G}\right)^{-3}: \quad 40a_{-2}\mu^2 + 38a_{-2}\lambda^2\mu + 12a_{-1}\lambda\mu^2 + Ac(2a_{-2}^2 + 3a_{-1}a_{-2}\lambda + a_{-1}^2\mu + 2a_0a_{-2}\mu) + 2a_{-2}\mu(6 - Bc) = 0,$$

$$\left(\frac{G'}{G}\right)^{3}: \quad 40a_2\mu + 38a_2\lambda^2 + 12a_1\lambda + Ac(2a_2^2\mu + 3a_1a_2\lambda + a_1^2 + 2a_0a_2) + 2a_2(6 - Bc) = 0,$$

$$\left(\frac{G'}{G}\right)^{-2}: \quad 52a_{-2}\lambda\mu + 8a_{-1}\mu^2 + 7a_{-1}\lambda^2\mu + 8a_{-2}\lambda^3 + Ac(3a_{-1}a_{-2} + a_1a_{-2}\mu + a_{-1}^2\lambda + 2a_0a_{-2}\lambda + a_0a_{-1}\mu)$$
$$+ (2a_{-2}\lambda + a_{-1}\mu)(6 - Bc) + Ca_{-2} = 0,$$

$$\left(\frac{G'}{G}\right)^{2}: \quad 52a_2\lambda\mu + 8a_1\mu + 7a_1\lambda^2 + 8a_2\lambda^3 + Ac(3a_1a_2\mu + a_{-1}a_2 + a_1^2\lambda + 2a_0a_2\lambda + a_0a_1)$$
$$+ (2a_2\lambda + a_1)(6 - Bc) - Ca_2 = 0,$$

$$\left(\frac{G'}{G}\right)^{-1}: \quad 16a_{-2}\mu + 8a_{-1}\lambda\mu + 14a_{-2}\lambda^2 + a_{-1}\lambda^3 + Ac(a_1a_{-2}\lambda + a_{-1}^2 + 2a_0a_{-2} + a_0a_{-1}\lambda)$$
$$+ (2a_{-2} + a_{-1}\lambda)(6 - Bc) + Ca_{-1} = 0,$$

$$\left(\frac{G'}{G}\right): \quad 16a_2\mu^2 + 8a_1\lambda\mu + 14a_2\lambda^2\mu + a_1\lambda^3 + Ac(a_{-1}a_2\lambda + a_1^2\mu + 2a_0a_2\mu + a_0a_1\lambda)$$
$$+ (2a_2\mu + a_1\lambda)(6 - Bc) - Ca_1 = 0,$$

$$\left(\frac{G'}{G}\right)^{0}: \quad 6a_{-2}\lambda + 2a_{-1}\mu + a_{-1}\lambda^2 - a_1\lambda^2\mu - 6a_2\lambda\mu^2 - 2a_1\mu^2$$
$$+ Ac(a_1a_{-2} - a_{-1}a_2\mu + a_0a_{-1} - a_0a_1\mu) - (a_1\mu - a_{-1})(6 - Bc) + Ca_0 = 0$$

By solving the above algebraic equations with the aid of Maple or Mathematica, we have the following cases:

Case 1

$$\lambda = \lambda,\ \ \mu = \mu,\ c = \frac{(\lambda^2 + 8\mu + 6)}{B - a_0A},\ a_0 = a_0,\ a_1 = \frac{-12\lambda(B - a_0A)}{A(\lambda^2 + 8\mu + 6)}, \qquad (13)$$
$$a_2 = \frac{-12(B - a_0A)}{A(\lambda^2 + 8\mu + 6)},\ a_{-1} = 0,\ a_{-2} = 0$$

Case 2

$$\lambda = 0,\ \ \mu = \mu,\ c = \frac{8\mu + 6}{B - a_0A},\ a_0 = a_0,\ a_1 = 0, a_{-2} = \frac{-12\mu^2(B - a_0A)}{A(8\mu + 6)},\ a_{-1} = 0,\ a_2 = 0 \qquad (14)$$

Case 3

$$\lambda = 0,\ \ \mu = \frac{3}{4a_2}(2a_0 - a_2 - 2(\frac{B}{A})),\ c = \frac{-12}{a_2A},\ a_0 = a_0,\ a_1 = 0, \qquad (15)$$
$$a_{-2} = \frac{9}{16a_2}\left[2a_0 - a_2 - 2(\frac{B}{A})\right]^2,\ a_{-1} = 0,\ a_2 \neq 0$$

Exact solutions of Equation (1) for case 1

Substituting (13) into (12) and using (7) – (9), we have the following exact solutions for the model (1):

(i)If $\lambda^2 - 4\mu > 0$ (Hyperbolic function solutions),we have the exact solution

$$u(x,t) = a_0 - \frac{3(\lambda^2-4\mu)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{\left[\frac{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right]^2\right\} + \frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) \tag{16}$$

If we set $c_1=0$ and $c_2 \neq 0$ in (16), we have the solitary solution

$$u_1(x,t) = a_0 + \frac{3}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{\lambda^2\operatorname{sech}^2\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+4\mu\tanh^2\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)\right\} \tag{17}$$

while if we set $c_2=0$ and $c_1 \neq 0$ in (16), we have the solitary solution

$$u_2(x,t) = a_0 + \frac{3}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{-\lambda^2\operatorname{cosech}^2\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+4\mu\coth^2\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)\right\} \tag{18}$$

If $c_2 \neq 0$ and $c_1^2 < c_2^2$, then we have the solitary solution

$$u_3(x,t) = a_0 - \frac{3(\lambda^2-4\mu)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\tanh^2\left(\xi_0+\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+\frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) \tag{19}$$

Where $\xi_0 = \tanh^{-1}\left(\frac{c_1}{c_2}\right)$, while if $c_1 \neq 0$ and $c_1^2 < c_2^2$, then we have the solitary solution

$$u_4(x,t) = a_0 - \frac{3(\lambda^2-4\mu)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\tanh^2\left(\xi_0+\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+\frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) \tag{20}$$

where $\xi_0 = \coth^{-1}\left(\frac{c_2}{c_1}\right)$.

(ii) If $\lambda^2 - 4\mu < 0$ (Trigonometric function solutions), we have the exact solution

$$u(x,t) = a_0 + \frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) - \frac{3(4\mu-\lambda^2)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\times \left\{\left[\frac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right]\right\}^2 \tag{21}$$

If we set $c_2=0$ and $c_1 \neq 0$ in (21), we have the periodic solution

$$u_1(x,t) = a_0 + \frac{3}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{\lambda^2\sec^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)-4\mu\tan^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)\right\} \tag{22}$$

while if we set $c_1=0$ and $c_2 \neq 0$ in (21), we have the periodic solution

$$u_2(x,t) = a_0 + \frac{3}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{\lambda^2\csc^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)-4\mu\cot^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)\right\} \tag{23}$$

If $c_1 \neq 0$ and $c_1^2 > c_2^2$, then we have the periodic solution

$$u_3(x,t) = a_0 + \frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) - \frac{3(4\mu-\lambda^2)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left[\tan^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}-\xi_0\right)\right] \tag{24}$$

where $\xi_0 = \tan^{-1}\left(\frac{c_2}{c_1}\right)$,

while if $c_2 \neq 0$ and $c_2^2 > c_1^2$, then we have the periodic solution

$$u_4(x,t) = a_0 + \frac{3\lambda^2}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right) - \frac{3(4\mu-\lambda^2)}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left[\cot^2\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}-\xi_0\right)\right] \tag{25}$$

where $\xi_0 = \cot^{-1}\left(\frac{c_1}{c_2}\right)$.

(iii) If $\lambda^2 - 4\mu = 0$ (Rational function solutions), we have

$$u(x,t) = a_0 + \frac{3}{A}\left(\frac{B-a_0A}{\lambda^2+8\mu+6}\right)\left\{\lambda^2 - 4\left(\frac{c_2}{c_1+c_2\xi}\right)^2\right\} \tag{26}$$

where $\xi = \frac{1}{L}x - \left(\frac{\lambda^2+8\mu+6}{B-a_0A}\right)\frac{t}{\tau}$ and c_1 , c_2 are arbitrary constants.

Exact solutions of Equation (1) for case 2

Substituting (14) into (12) and using (7) – (9), we have the following exact solutions for the model (1):

(i) If $\lambda^2-4\mu>0$ (Hyperbolic function solutions), we have the exact solution

$$u(x,t)=a_0+\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\left\{\frac{c_1\cosh\left(\xi\sqrt{-\mu}\right)+c_2\sinh\left(\xi\sqrt{-\mu}\right)}{c_1\sinh\left(\xi\sqrt{-\mu}\right)+c_2\cosh\left(\xi\sqrt{-\mu}\right)}\right\}^{-2} \tag{27}$$

If we set c_1=0 and $c_2\neq 0$ in (27), we have the solitary solution

$$u_1(x,t)=a_0+\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\coth^2\left(\sqrt{-\mu}\,\xi\right) \tag{28}$$

while if we set c_2=0 and $c_1\neq 0$ in (27), we have the solitary solution

$$u_2(x,t)=a_0+\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tanh^2\left(\sqrt{-\mu}\,\xi\right) \tag{29}$$

If $c_2\neq o$ and $c_2^2>c_1^2$, then we have the solitary solution

$$u_3(x,t)=a_0+\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tanh^2\left(\xi_0+\sqrt{-\mu}\,\xi\right) \tag{30}$$

where $\xi_0=\coth^{-1}\left(\frac{c_1}{c_2}\right)$

while if $c_1\neq 0$ and $c_1^2>c_2^2$ then we have the solitary solution

$$u_4(x,t)=a_0+\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tanh^2\left(\xi_0+\sqrt{-\mu}\,\xi\right) \tag{31}$$

where $\xi_0=\tanh^{-1}\left(\frac{c_2}{c_1}\right)$.

(ii)If $\lambda^2-4\mu<0$ (Trigonometric function solutions), we have the exact solution

$$u(x,t)=a_0-\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\left\{\frac{-c_1\sin\left(\sqrt{\mu}\xi\right)+c_2\cos\left(\sqrt{\mu}\xi\right)}{c_1\cos\left(\sqrt{\mu}\xi\right)+c_2\sin\left(\sqrt{\mu}\xi\right)}\right\}^{-2} \tag{32}$$

If we set c_2=0 and $c_1\neq 0$ in (32), we have the periodic solution

$$u_1(x,t)=a_0-\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\cot^2\left(\sqrt{\mu}\xi\right) \tag{33}$$

while if c_1=0 and $c_2\neq 0$ in (32), we have the periodic solution

$$u_2(x,t)=a_0-\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tan^2\left(\sqrt{\mu}\xi\right) \tag{34}$$

If we set $c_1\neq 0$ and $c_1^2>c_2^2$ then we have the periodic solution:

$$u_3(x,t)=a_0-\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tan^2\left(\xi_0+\sqrt{\mu}\xi\right) \tag{35}$$

where $\xi_0=\cot^{-1}\left(\frac{c_2}{c_1}\right)$,

while if $c_2\neq 0$ and $c_2^2>c_1^2$ then we have the periodic solution

$$u_4(x,t)=a_0-\frac{12\mu}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\tan^2\left(\xi_0+\sqrt{\mu}\xi\right) \tag{36}$$

where $\xi_0=\tan^{-1}\left(\frac{c_1}{c_2}\right)$.

(iii)If $\lambda^2-4\mu=0$ (Rational function solutions), we have

$$u(x,t)=a_0-\frac{12\mu^2}{A}\left(\frac{B-a_0A}{8\mu+6}\right)\left\{\frac{c_2}{c_1+c_2\xi}\right\}^{-2} \tag{37}$$

where $\xi=\frac{1}{L}x-\left(\frac{8\mu+6}{B-a_0A}\right)\frac{t}{\tau}$.

Exact solutions of Equation (1) for case 3

Substituting (15) into (12) and using (7) – (9), we have the following exact solutions for the model (1):

(i)If $\lambda^2-4\mu>0$ (Hyperbolic function solutions), we have the exact solution

$$u(x,t)=a_0-a_2\mu\left[\frac{c_1\cosh\left(\sqrt{-\mu}\xi\right)+c_2\sinh\left(\sqrt{-\mu}\xi\right)}{c_1\sinh\left(\sqrt{-\mu}\xi\right)+c_2\cosh\left(\sqrt{-\mu}\xi\right)}\right]^2 -a_2\mu\left[\frac{c_1\cosh\left(\sqrt{-\mu}\xi\right)+c_2\sinh\left(\sqrt{-\mu}\xi\right)}{c_1\sinh\left(\sqrt{-\mu}\xi\right)+c_2\cosh\left(\sqrt{-\mu}\xi\right)}\right]^{-2} \quad (38)$$

If we set $c_1=0$ and $c_2 \neq 0$ in (38) we have the solitary solution

$$u_1(x,t)=a_0-a_2\mu\left\{\tanh^2\left(\sqrt{-\mu}\xi\right)+\coth^2\left(\sqrt{-\mu}\xi\right)\right\} \quad (39)$$

while if we set $c_2=0$ and $c_1 \neq 0$ in (38) we have the same solitary solution (39).

(ii)If $\lambda^2-4\mu<0$ (Trigonometric function solutions), we have the exact solution

$$u(x,t)=a_0+a_2\mu\left[\frac{-c_1\sin\left(\sqrt{\mu}\xi\right)+c_2\cos\left(\sqrt{\mu}\xi\right)}{c_1\cos\left(\sqrt{\mu}\xi\right)+c_2\sin\left(\sqrt{\mu}\xi\right)}\right]^2 +a_2\mu\left[\frac{-c_1\sin\left(\sqrt{\mu}\xi\right)+c_2\cos\left(\sqrt{\mu}\xi\right)}{c_1\cos\left(\sqrt{\mu}\xi\right)+c_2\sin\left(\sqrt{\mu}\xi\right)}\right]^{-2} \quad (40)$$

If we set $c_2=0$ and $c_1 \neq 0$ in (40) we have the periodic solution

$$u_1(x,t)=a_0+a_2\mu\left[\tan^2\left(\sqrt{\mu}\xi\right)+\cot^2\left(\sqrt{\mu}\xi\right)\right] \quad (41)$$

while if we set $c_1=0$ and $c_2 \neq 0$ in (40) we have the same periodic solution (41).

(iii)If $\lambda^2-4\mu=0$ (Rational function solutions), we have

$$u(x,t)=a_0+a_2\left(\frac{c_2}{c_1+c_2\xi}\right)^2+a_2\mu^2\left(\frac{c_2}{c_1+c_2\xi}\right)^{-2} \quad (42)$$

where $\xi=\frac{1}{L}x+\left(\frac{12}{a_2A}\right)\frac{t}{\tau}$.

DISCUSSION

In this article, we have applied the improved (G'/G) - expansion method to find many exact solutions as well as many solitary wave solutions and periodic solutions (16) - (42) of the nonlinear model (1) which are of special interests in nanobiosciences, namely the transmission line models for nano-ionic currents along microtublues. On comparing our results obtained in this article with the well-known results obtained in Sekulic et al. (2011) and Zayed et al. (2013) we deduce that some of these results are in agreement together, while the others are new which are not discussed elsewhere.

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